COMBINED SUBCARRIER AND POWER ALLOCATION IN MULTIUSER OFDM

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ABSTRACT

This paper investigates joint subcarrier and power allocation in multiuser Orthogonal Frequency Division Multiplexing (OFDM). Existing algorithms allocate power uniformly across all subcarriers belonging to a particular user. Optimal power allocation algorithms that use water-filling for users have been suggested. But these algorithms are optimal only from the point of view of power allocation. We propose an algorithm that combines subcarrier and power allocation while using water-filling for users. Simulation results have shown that substantial gains can be achieved over existing algorithms.

1. INTRODUCTION

OFDM is one of the most promising techniques for high data rate mobile communications. In OFDM, a multipath channel is split into several lower bandwidth channels to eliminate the problem of Inter-Symbol Interference (ISI) . In the absence of ISI, subcarriers can be thought of as independent additive white Gaussian noise (AWGN) channels. In a multiuser scenario, subcarrier independence can be exploited by allowing multiple users to share an OFDM symbol.

Two classes of resource allocation algorithms exist: fixed resource allocation [7] and dynamic resource allocation [1] [4] [2] [3]. In fixed resource allocation, a resource, subcarrier or time slot, is allocated to a user regardless of the user's current channel conditions. In dynamic resource allocation, subcarriers and time slots are allocated to users adaptively taking into account the users' current channel conditions. Dynamic resource allocation can be margin adaptive [2] or rate adaptive [4] [1] [3]. The margin adaptive technique focuses on minimizing the overall transmit power while achieving individual users' rate constraint. On the other hand, the rate adaptive technique attempts to maximize the overall rate under a total power constraint.

In the subsequent sections, we consider the rate adaptive technique. We attempt to maximize the minimum user's capacity thereby achieving proportional fairness amongst users . Though proportional fairness amongst users is achieved in [4], spectral diversity (by spectral diversity, we refer to different gains on subcarriers for a particular user) is not exploited fully. Instead, power is allocated uniformly over all subcarriers belonging to a particular user. Though [3] exploits spectral diversity of users during power allocation, it is computationally intensive. Our algorithm, on the other hand, performs joint subcarrier and power allocation while exploiting spectral diversity of users.

The organization of this paper is as follows. In section 2, we introduce the multiuser OFDM system model and state the objective. In section 3, we look at existing solutions to the problem and suggest an alternate solution. Simulation results are presented in section 4. Finally, we conclude in section 5.

2. MULTIUSER OFDM SYSTEM MODEL

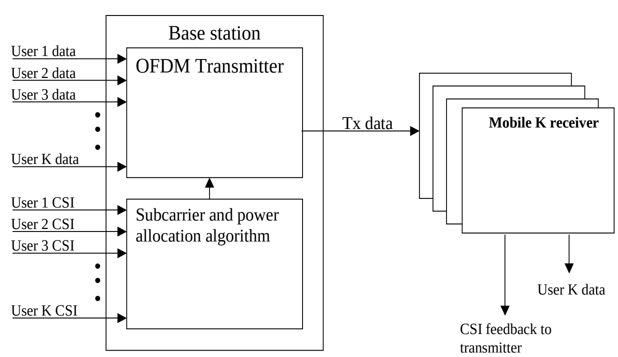

Figure 1: Multiuser OFDM system model

Figure 1 shows the block diagram of a multiuser OFDM system. It is assumed that the base station has channel knowledge of all the users in the system.

Since channel conditions vary over a period of time, this information is updated periodically with the help of feedback channels.

In the rate adaptive technique under consideration, subcarrier and power allocation has to be carried out jointly to achieve the optimal solution. For the sake of simplicity, each subcarrier is allocated to only one user at any instant of time. The optimization problem can be formulated as,

$$\max_{P_{k,n},A_k} \min_k \frac{\gamma_1}{\gamma_k} \sum_{n \in A_k} \log_2(1 + P_{k,n} H_{k,n}) \quad (1)$$

$$\begin{aligned} \text{subject to } & \sum_{n=1}^{N} \sum_{k=1}^{K} P_{k,n} \leq P_{total} \\ & P_{k,n} \geq 0 \text{ for all } k, n \\ & A_1, A_2, ..., A_K \text{ are all disjoint} \\ & A_1 \cup A_2 \cup ... \cup A_K \subseteq \{1, 2, ..., N\} \\ & R_1 : R_2 : .... : R_K = \gamma_1 : \gamma_2 : .... : \gamma_K \\ & \sum_{k=1}^{K} N_k = N \end{aligned}$$

where $H_{k,n} = \frac{|h_{k,n}|^2}{N_0 \frac{B}{N}}$ is the channel gain to noise power ratio for the k^{th} user in the n^{th} subcarrier; K is the total number of users; N is the total number of subcarriers; N_0 is the power spectral density of additive white Gaussian noise; B and P_{total} are the overall available bandwidth and power, respectively; $P_{k,n}$ is the power allocated to the k^{th} user in the n^{th} subcarrier; $h_{k,n}$ is the channel gain for the k^{th} user in the n^{th} subcarrier; N_k is the number of subcarriers allocated to the k^{th} user; A_k is the set of all subcarriers allocated to the k^{th} user. $\{\gamma_1, \gamma_2, ..., \gamma_K\}$ is a set of predetermined constants to ensure proportional fairness amongst users. R_k is the k^{th} user's rate defined as,

$$R_k = \sum_{n \in A_k} \log_2(1 + P_{k,n} H_{k,n}) \quad (2)$$

3. PROPOSED SOLUTION

In this section, we propose a sub-optimal solution to the optimization problem in (1). [4] proposes an iterative sub-optimal subcarrier-power allocation algorithm in which the total available power P_{total} is allocated uniformly over all the subcarriers. Subcarriers allocated to a particular user provide sufficient spectral diversity to warrant the use of water-filling [5] for each user. It is this diversity that [3] exploits during optimal power allocation. The optimal power allocation algorithm is optimal only from the point of view of power allocation. The extent to which [3] achieves the optimal solution in (1) depends largely on the subcarrier allocation algorithm. Furthermore, [3] involves solving a non linear equation resulting in a huge computational overhead.

In the proposed iterative solution that follows, we have combined subcarrier and power allocation. The gain from water-filling, resulting from the spectral diversity of each user, is taken into account during subcarrier allocation. The subcarrier and power allocation strategy is as follows.

1. Initialize $A = \{1, 2, 3, ...N\}$
2. $\forall\, k = 1$ to K, $A_k = \phi$.
3. $\forall\, k = 1$ to K,
   a. $H_k = \max_n H_{k,n}$ for $n \in A$
   b. $N_k = N_k + 1$
   c. $A_k = A_k \cup \{n\}$
   d. $R_k = \log_2\left(1 + \frac{P_{total}}{N} H_k\right)$
   e. $A = A - \{n\}$
4. While $A \neq \phi$,
   a. find i such that $\frac{\gamma_1}{\gamma_i} R_i \leq \frac{\gamma_1}{\gamma_k} R_k \forall\, k = 1$ to K
   b. for the found i, find n such that
      $H_{i,n} \leq H_{i,m}$ where $n, m \in A$
   c. $N_i = N_i + 1$
   d. $A_i = A_i \cup \{n\}$
   e. $A = A - \{n\}$
   f. $R_i = \sum_{n \in A_i} log_2(1 + P_n H_{i,n})$
      where $P_n = \left(\gamma - \frac{1}{H_{i,n}}\right)^+$
      and $\sum_{n \in A_i} P_n = \frac{N_i}{N} P_{total}$

The $f(x) = (x)^+$ operator indicates that $f(x) = 0$ when $x < 0$ and $f(x) = x$ when $x \geq 0$.

The subcarrier and power allocation strategy described above follows the strategy used in [4] except for the rate update equation 4(f). The rate update equation used in [4] is,

$$R_i = \sum_{n \in A_i} \log_2\left(1 + \frac{P_{total}}{N} H_{i,n}\right)$$

It must be noted that the power budget in both algorithms is the same $\sum_{n \in A_i} P_n = \frac{N_i}{N} P_{total}$. While [4] uses uniform power allocation across all subcarriers, our algorithm uses water-filling. In doing so, gains resulting from water-filling for a particular user is used to the advantage of all users in the system.

4. SIMULATION RESULTS

Simulations were carried out for an $N = 128$ subcarrier multiuser OFDM system. Each user in the system is assumed to have a 6-tap sample spaced multipath channel with each tap experiencing independent Rayleigh fading. The energy of the first tap for each user is taken to be one unit. The tap energies of each of the remaining taps is assumed to decay exponentially as in [3]. Noise power is assumed to be one unit in each subcarrier for every user in the system. Let $\gamma_1 : \gamma_2 : \gamma_3 : .... : \gamma_K = 1 : 1 : 1 : ... : 1$ so that the overall rate is maximized while trying to achieve equal rate for all users.

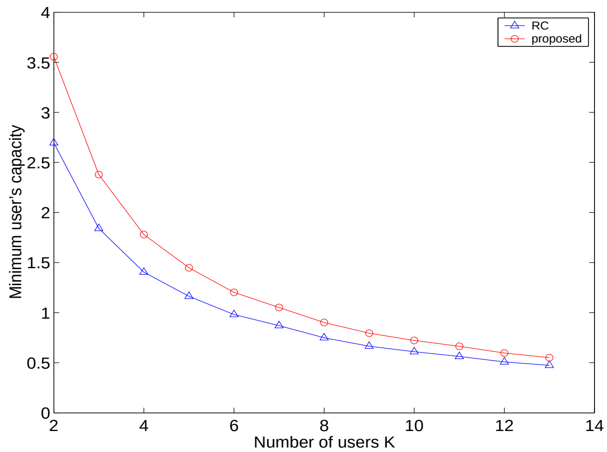

Figure 2: Minimum user's capacity vs. number of users

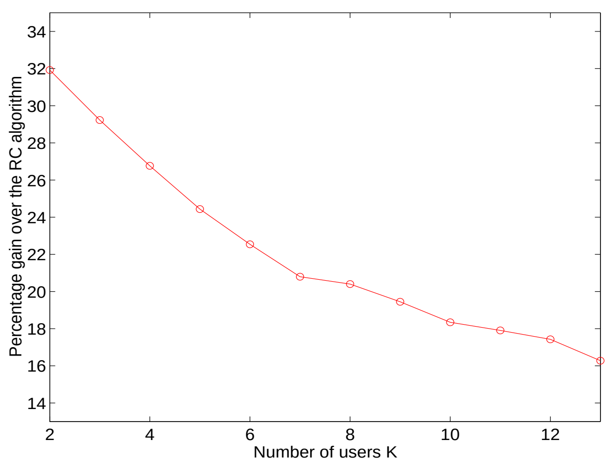

Figure 3: Gain over RC algorithm vs. number of users

The proposed algorithm is compared against the algorithm in [4]. The total power available at the transmitter P_{total} is taken to be 300 units. Throughout our simulations, the algorithm in [4] will be referred to as the RC algorithm. Figure 2 shows the minimum user's capacity being plotted against the number of users in the multiuser system. Our algorithm achieves larger gains over the RC algorithm when the number of users is low. Figure 3 shows that as the number of users in the system increases, the gain achieved over the RC algorithm reduces. This is because an increase in the number of users results in users being allocated lesser number of subcarriers. These subcarriers have similar channel gains as a result of which, allocating power uniformly across all the subcarriers belonging to a particular user will not affect the user's rate adversely. This accounts for the reduced gain of our algorithm over the RC algorithm when the number of users is high.

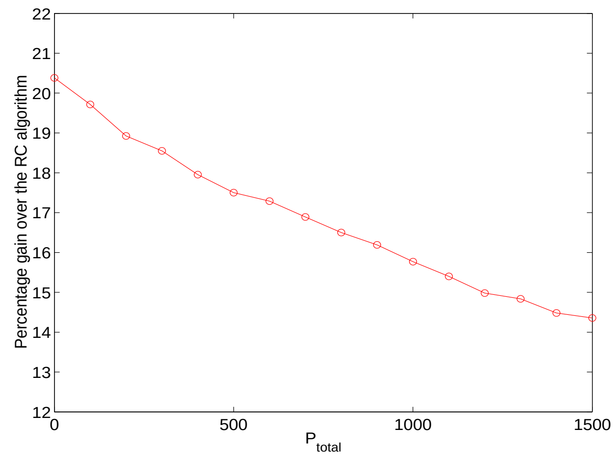

Figure 4: Gain over RC algorithm vs. power

In figure 4, the gain achieved over the RC algorithm for a fixed number of users ($K = 10$) for different values of power is shown. For low values of the total available power P_{total}, our algorithm achieves a larger gain over the RC algorithm. This is because spectral diversity is better exploited when the power available is low. For larger values of power, use of water-filling for a particular user will not yield significant gains over uniform power allocation [6].

5. CONCLUSION

In this paper, we have introduced an algorithm that performs joint subcarrier and power allocation while also exploiting spectral diversity of users in the system. Simulation results have shown that significant gains can be achieved over existing algorithms [4]. Since the spectral diversity of each user is fully exploited through water-filling, the use of computationally intensive algorithms such as [3] can be avoided.

REFERENCES

[1] J. Jang, K. B. Lee,"Transmit power adaptation for multiuser OFDM systems", *IEEE Journal on Selected Areas in Communications,* Vol. 21, no.2, pp. 171-178, February 2003.

[2] C. Y. Wong, R. S. Cheng, K. B. Letaief, R. D. Murch,“Multiuser OFDM with Adaptive Subcarrier, Bit, and Power Allocation”, *IEEE Journal on Selected Areas in Communications,* Vol. 17, no.10, pp. 1747-1758, October 1999.

[3] Z. Shen, J. G. Andrews, B. L. Evans,“Optimal Power Allocation in Multiuser OFDM Systems”, *Proceedings of the IEEE Global Communications Conference, San Francisco,* Vol. 1, pp. 337-341, December 2003.

[4] W. Rhee, J. M. Cioffi,“Increase in Capacity of Multiuser OFDM System Using Dynamic Subchannel Allocation”, *Proceedings of the 51st IEEE Vehicular Technology Conference, Tokyo,* Vol. 2, pp. 1085-1089, Spring 2000.

[5] R. G. Gallager, *Information Theory and Realiable Communication,* New York: Wiley 1968.

[6] T. L. Tung and K. Yao, “Channel Estimation and Optimal Power Allocation for a Multiple-Antenna OFDM System” , *EURASIP Journal on Applied Signal Processing,* Vol. 2002, No. 3, pp. 330-339, March 2002.

[7] E. Lawrey, “Multiuser OFDM” , *Proceedings of the IEEE International Symposium on Signal Processing and its Applications,* Vol. 2, pp. 761-764, August 1999.