

Information Flow in Wireless Networks

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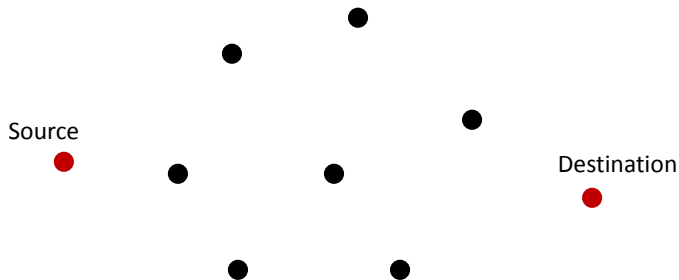
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Acknowledgements

- Andrew Thangaraj
- Bama Muthuramalingam

Information Flow Problem



- Wireless network of nodes
- Single source, single or multiple destinations
- Information rate maximization
- Per node power constraint

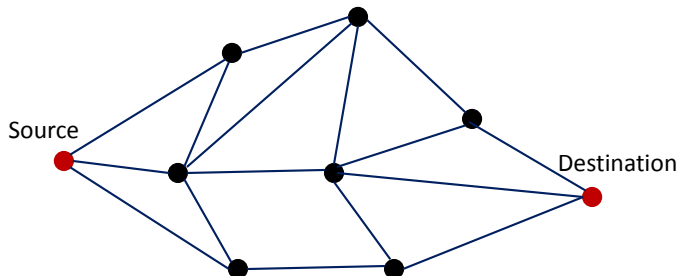
Outline

- Wired Networks
 - ▶ Max-flow min-cut theorem
 - ▶ Network coding
- Wireless Networks
 - ▶ Broadcast and interference
 - ▶ Interference Avoidance Approach
 - ▶ Information-theoretic Approach
 - ★ Cut-Set Bounds
 - ★ Flow optimization
 - ★ Approximate capacity
- Summary

Wired Networks

Single Source - Single Destination

Wired Network as a Graph



- Graph $G = (V, E)$, V : set of nodes (vertices), E : set of links (edges)
- Each edge (i, j) associated with a capacity C_{ij}

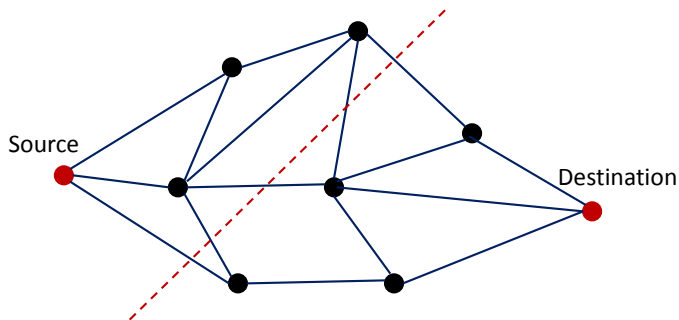
Flow

- Given G , assign $\{x_{ij}\}$ such that:
- $x_{ij} \geq 0$
- Rate constraints: $x_{ij} \leq C_{ij} \quad \forall i, j$
- Flow constraints:

$$\sum_i x_{ji} - \sum_i x_{ij} = \begin{cases} f & j = s \text{ (Source)} \\ -f & j = t \text{ (Destination)} \\ 0 & \text{else.} \end{cases} \quad \forall j$$

- f is the **value of the flow** from s to t
- Maximum flow can be found using linear programming

Cut and Cut Capacity



Cut with respect to s and t

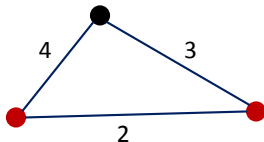
- Partitions V into S and S^c with $s \in S$, $t \in S^c$
- Cut Capacity (sum of capacities of edges from S to S^c):

$$C(S, S^c) = \sum_{i \in S, j \in S^c} C_{ij}$$

Max-Flow Min-Cut Theorem

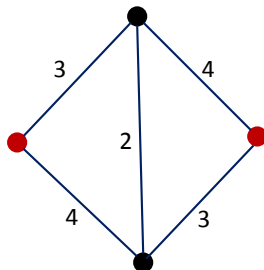
For a given G , the maximum value of flow from s to t is equal to the minimum value of the capacities of all cuts in G that separate s from t .

- Examples



Cut capacities: 6, 5

Min-cut : 5

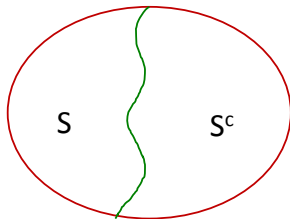


Cut capacities: 7, 7, 8, 10

Min-cut : 7

Proof Outline (Directed Graph)¹

Part 1: Show that $f \leq C(S, S^c)$ for any cut



$$\sum_{p \in S, i \in G} x_{pi} - \sum_{i \in S, p \in G} x_{pi} = f$$

Since $\sum_{p \in S, i \in S} x_{pi} - \sum_{i \in S, p \in S} x_{pi} = 0$, $\sum_{p \in S, i \in S^c} x_{pi} - \sum_{i \in S, p \in S^c} x_{pi} = f$

$$\Rightarrow f \leq \sum_{p \in S, i \in S^c} x_{pi} \leq \sum_{p \in S, i \in S^c} C_{pi} = C(S, S^c)$$

¹ N. Deo, Graph Theory with Applications to Engineering and Computer Science, Prentice Hall India, 1974.

An undirected graph can be converted into a directed graph by replacing each undirected edge by two directed edges.

Proof Outline

Part 2: There exists a flow $f_0 = C(S_0, S_0^c)$ for some cut

- Step 1: Consider flow pattern corresponding to maximum flow
- Step 2: Define S_0 as:
 - ▶ $s \in S_0$
 - ▶ If $i \in S_0$ and either $x_{ij} < C_{ji}$ or $x_{ji} > 0$, then $j \in S_0$.
- Step 3: Show $t \in S_0^c$
- Step 4: Show $f_0 = C(S_0, S_0^c)$

Wired Networks

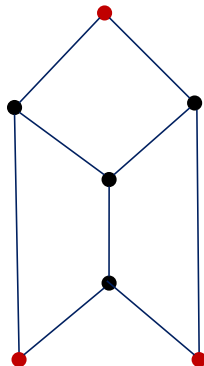
Single Source - Multiple Destinations

Multicast and Network Coding²

- Source s , L destinations t_1, t_2, \dots, t_L
- All destinations want the same information
- Let f_k denote the maximum flow possible from s to t_k
- Maximum multicast rate

$$f = \min_k f_k$$

- Routing is not enough, network coding is required



²R. Ahlswede, N. Cai, S-Y. R. Li, R. W. Yeung, "Network Information Flow," IEEE Transactions on Information Theory, vol. 46, no. 4, pp. 1204-1216, July 2000.

Multicast Flow Optimization

$$\max_{\{x_{ij}^{(k)}\}} f$$

- Flow constraints:

$$\sum_i x_{ji}^{(k)} - \sum_i x_{ij}^{(k)} = \begin{cases} f & j = s \text{ (Source)} \\ -f & j = t \text{ (Destination)} \\ 0 & \text{else.} \end{cases} \quad \forall k, j$$

- Rate constraints:

$$x_{ij}^{(k)} \leq C_{ij} \quad \forall k, i, j$$

- $x_{ij}^{(k)}$: Flow in (i, j) towards destination t_k

Network Codes

- Random α -codes³
- Linear codes⁴
- Random linear network codes⁵
- Network codes exist for every feasible flow solution⁶

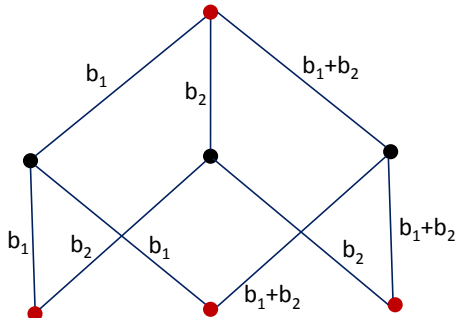
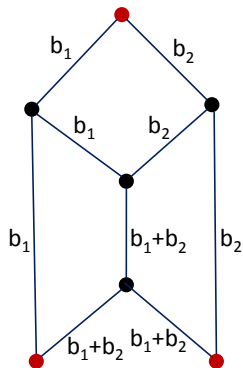
³R. Ahlswede, N. Cai, S-Y. R. Li, R. W. Yeung, "Network Information Flow," IEEE Transactions on Information Theory, vol. 46, no. 4, pp. 1204-1216, July 2000.

⁴S-Y. Li, R. Yeung, N. Cai, "Linear Network Coding," IEEE Transactions on Information Theory, vol. 49, no. 2, pp. 371-381, 2003.

⁵T. Ho, M. Medard, R. Koetter, D. R. Karger, M. Effros, J. Shi, B. Leong "A Random Linear Network Coding Approach to Multicast," IEEE Transactions on Information Theory, vol. 52, no. 10, pp. 4413-4430, 2006.

⁶D. S. Lun, N. Ratnakar, M. Medard, R. Koetter, D. R. Karger, T. Ho, E. Ahmed, "Minimum-cost multicast over coded packet networks," IEEE Transactions on Information Theory, vol. 52, no. 6, pp. 2608-2623, 2006.

Examples

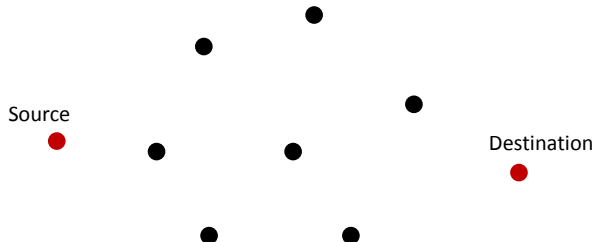


- Links with unit capacity

Wireless Networks

Single Source - Single/Multiple Destinations

Wireline Networks vs. Wireless Networks

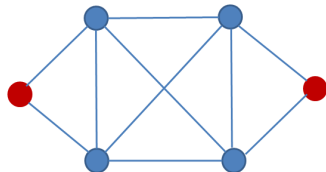


- Wireline networks
 - ▶ Links are independent
 - ▶ Graph model natural
- Wireless networks
 - ▶ Single shared resource → Broadcast nature, Interference
 - ▶ Links are dependent → Cross-layer optimization

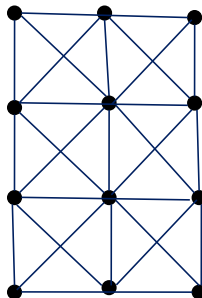
Wireless Network as a Graph

Many possibilities

- Complete graph: All nodes connected to all others
- Finite transmission range model



Two-stage relay network



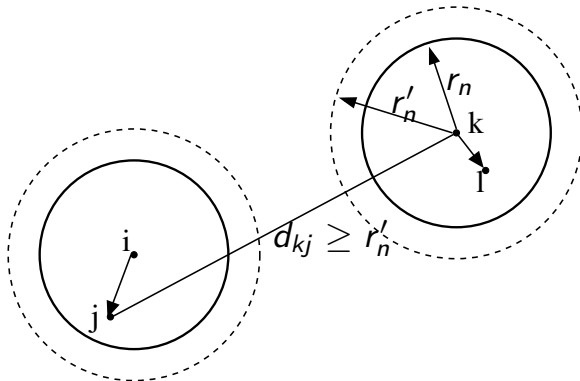
Wireless Networks

Interference Avoidance Approach

Interference Avoidance Model for Links

- Protocol model to **avoid interference** between links⁷

- ▶ Check transmission range: $d_{ij} \leq r_n$, $d_{kl} \leq r_n$
- ▶ Check interference range: $d_{kj} \geq r'_n$, $d_{il} \geq r'_n$

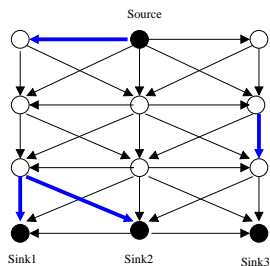


- Link activation constraints can be extended for broadcast hyperarcs⁸

⁷ P. Gupta *et al.*, The Capacity of Wireless Networks, IEEE Transactions on Information Theory, Mar. 2000

⁸ Park *et al.*, Performance of network coding in adhoc networks, in Proc. of IEEE-MILCOM 2006

Interference Avoidance (IA) with Broadcast Hyperarcs

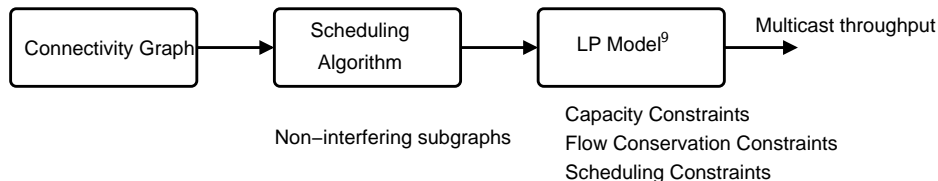


A non-interfering subgraph

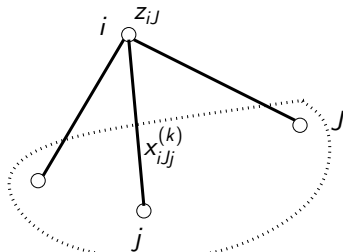
- A collection of non-interfering hyperarcs forms a non-interfering subgraph
- All non-interfering subgraphs can be generated using:
 - ▶ Conflict graph scheduling^a

^aJain *et al* "Impact of interference on multihop wireless networks," Mobicom 2003

Optimization Model



- \mathcal{A} : Set of non-interfering subgraphs
- (i, J) : Broadcast hyperarc
- $x_{ij}^{(k)}$: Flow from i to $j \in J$ using hyperarc (i, J) towards sink t_k
- z_{iJ} : Average rate at which packets are injected by i in (i, J)



⁹Lun *et al*, Performance of network coding in adhoc networks, MILCOM 2006

Flow Optimization Model: Wireless Networks

$$\max_{\{\lambda_m\}, \{z_{iJ}\}, \{x_{ij}^{(k)}\}} f$$

- Scheduling constraints: $\sum_m \lambda_m \leq 1$

- Rate constraints:

$$\sum_{j \in J} x_{ij}^{(k)} \leq z_{iJ} \quad \forall (i, J) \in \mathcal{A}, k$$

$$z_{iJ} \leq \sum_m \lambda_m C_m(i, J)$$

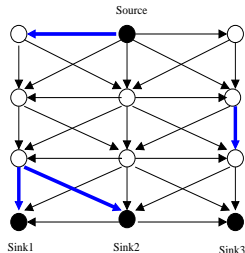
- Flow constraints:

$$\sum_{(i,J) \in \mathcal{A}} \sum_{j \in J} x_{ij}^{(k)} - \sum_{(j,I) \in \mathcal{A}} \sum_{i \in I} x_{ji}^{(k)} = \begin{cases} f & i = s \text{ (Source)} \\ -f & i = t \text{ (Destination)} \\ 0 & \text{else.} \end{cases} \quad \forall k, i$$

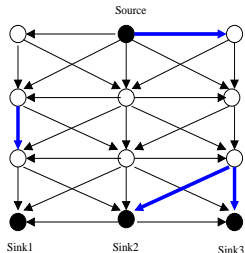
- $\lambda_m \geq 0, x_{ij}^{(k)} \geq 0, z_{iJ} \geq 0$

IA Solution for 4 x 3 Grid Network

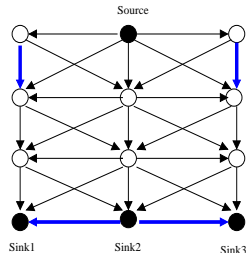
$$\lambda_1 = 1/3$$



$$\lambda_2 = 1/3$$



$$\lambda_3 = 1/3$$



- $f = 2/3$ packets per time unit

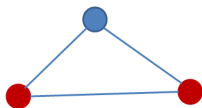
Wireless Networks

Information-theoretic Approach

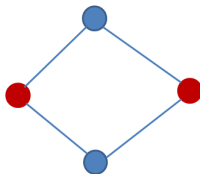
Wireless Relay Networks: What is known/unknown?

Single source-destination pair Gaussian relay networks

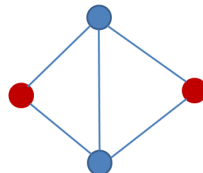
- Capacity unknown for arbitrary topology
- Cut-set upper bound
- Achievable rates for specific protocols and topologies
- Approximate capacity



Three terminal network



Diamond Channel



Diamond Channel
with Interfering Relays

Wireless Relaying: Assumptions and Results

Duplex	SNR	Cooperation	Topology
Full	Large	MIMO	Arbitrary, Directed
Half	All	Limited	Restricted
		No MIMO	Arbitrary

- Both, Large SNR, MIMO, Arbitrary directed¹⁰
 - ▶ Constant gap to capacity
- Both, Large SNR, MIMO, Arbitrary¹¹
 - ▶ Diversity-multiplexing trade-off
- Half duplex, All SNR, Limited, Restricted¹²
 - ▶ Rates close to capacity
- Half duplex, All SNR, No MIMO, Restricted¹³
 - ▶ Constant gap to capacity

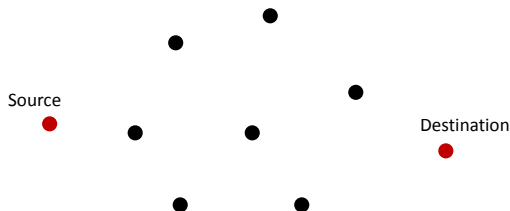
¹⁰ A. S. Avestimehr, S. N. Diggavi, and D. N. C. Tse, Wireless network information flow: A deterministic approach, IEEE Transactions on Information Theory, vol. 57, no. 4, pp. 1872-1905, April 2011.

¹¹ K. Sreeram, P. S. Birenjith, P. V. Kumar, "DMT of multi-hop cooperative networks," IEEE ITW, Cairo, Egypt, Jan. 2010.

¹² W. Chang, S. Chung, and Y. Lee, Capacity bounds for alternating twopath relay channels, in Proc. of the Allerton Conference on Communications, Control and Computing, Monticello, Illinois, USA, Sep. 2007, pp. 1149-1155.

¹³ H. Bagheri, A. Motahari, and A. Khandani, On the capacity of the halfduplex diamond channel, in Proc. of IEEE International Symposium on Information Theory, Austin, USA, June 2010, pp. 649-653.

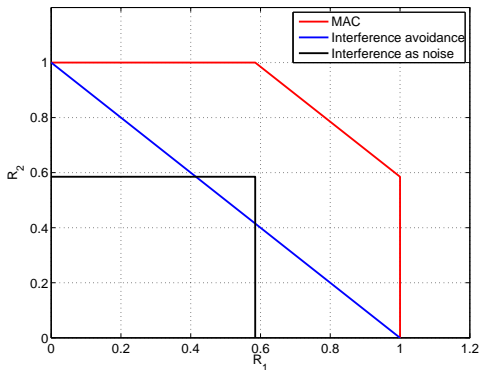
Relay Networks



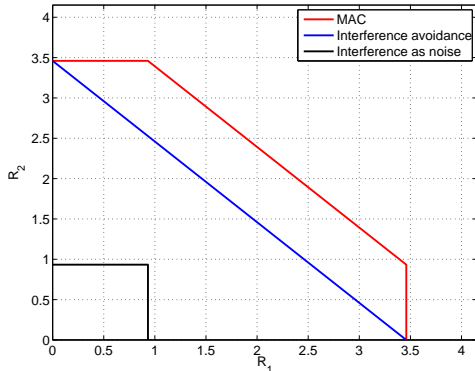
- Interference processing/decoding
 - ▶ Decode strong interference and cancel
 - ▶ Joint decoding of interfering signals
- Processing at the relays
 - ▶ More general than decode and forward and network coding
 - ▶ Relay can transmit any encoded function of received signal

Interference Processing

- Gaussian Multiple Access Channel

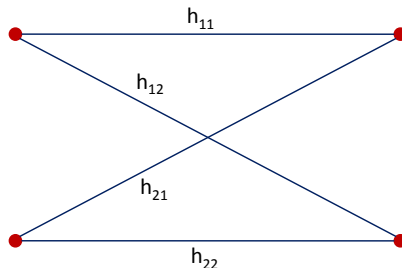


$P_1 = 0$ dB, $P_2 = 0$ dB



$P_1 = 10$ dB, $P_2 = 10$ dB

Interference Channel

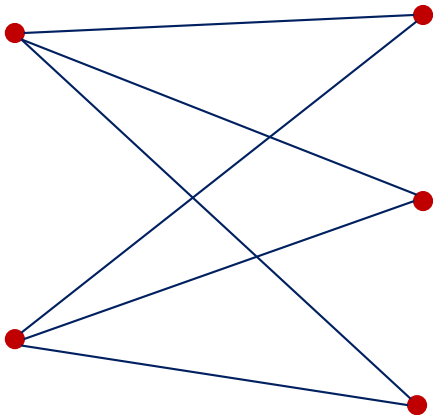


- Two transmit-receive pairs
- Strong interference: Decode interference and cancel¹⁴
- Weak interference: Treat interference as noise¹⁵

¹⁴A. B. Carleial, A case where interference does not reduce capacity, IEEE Trans. Inform. Theory, vol. IT-21, pp. 569-570, Sept. 1975.

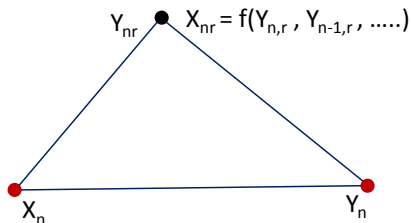
¹⁵V. Annapureddy and V. Veeravalli, Gaussian interference networks: Sum capacity in the low-interference regime and new outer bounds on the capacity region, IEEE Transactions on Information Theory, vol. 55, no. 7, pp. 3032-3050, July 2009.

Interference Networks



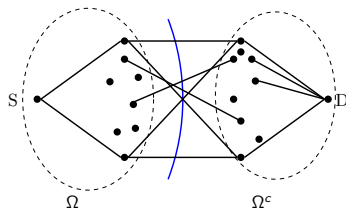
2 x 3 Interference Network

Processing at the Relays



- Transmit signal = $f(\text{past received signals})$
- Link model with an associated rate
 - ▶ Decode-and-forward (DF) + Network coding
- Other general models
 - ▶ Amplify-and-forward (AF)
 - ▶ Compress-and-forward (CF)
 - ▶ Quantize-map-and-forward
 - ▶ ...

Cut-Set Bound



- Full Duplex Network¹⁶

$$R \leq \min_{\Omega} I(X^{\Omega}; Y^{\Omega^c} | X^{\Omega^c}) \text{ for some } p(x_1, x_2, \dots, x_N)$$

- Half Duplex Network¹⁷

$$R \leq \sup_{\lambda_k} \min_{\Omega} \sum_{k=1}^{\mathcal{M}} \lambda_k I(X_{(k)}^{\Omega}; Y_{(k)}^{\Omega^c} | X_{(k)}^{\Omega^c}) \text{ for some } p(x_1, x_2, \dots, x_N | k)$$

¹⁶ T. M. Cover, J. A. Thomas, Elements of Information Theory, John Wiley, 2004.

¹⁷ M. Khojastepour, A. Sabharwal, B. Aazhang, "Bounds on achievable rates for general multiterminal networks with practical constraints", IPSN, pp. 146-161, 2003

Cut Capacity Bounds: Gaussian Relay Networks¹⁸

- Based on MIMO capacity

- ▶ Maximize

$$\log \det (\mathbf{I} + \mathbf{H} \mathbf{K}_X \mathbf{H}^H)$$

subject to $\text{tr}(\mathbf{K}_X) \leq P$

- ▶ MIMO capacity – Water-filling
 - ▶ Easy to compute MIMO capacity bound

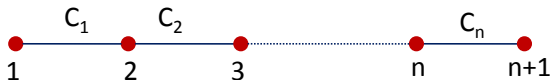
$$\log \det (\mathbf{I} + P N_t \mathbf{H} \mathbf{H}^H)$$

- ▶ Per antenna power constraint
 - ▶ Same input distribution for a state for all cuts

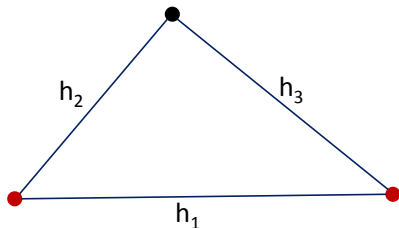
¹⁸M. Bama, “Cut-set Bound for Gaussian Relay Networks,” Available at <http://www.ee.iitm.ac.in/~skrishna/TechRepCUB.pdf>.

Examples: Full-duplex Cut-Set Bound

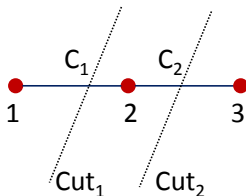
- Linear network (n hops/stages, $n + 1$ nodes)
 - 2^{n-1} cuts, $C_{FD} = \min_n C_n$



- 3-node relay network
 - 2 cuts, $C_{FD} = \min\{C((h_1^2 + h_2^2)P), C((h_1 + h_3)^2P)\}$



Examples: Half-Duplex Cut-Set Bound



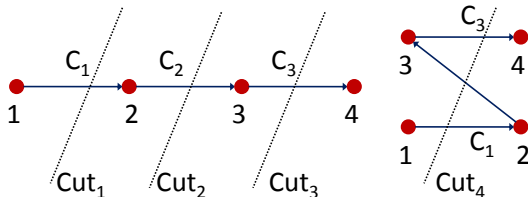
State	Cut ₁	Cut ₂
S_0 (00)	0	0
S_1 (01)	0	C_2
S_2 (10)	C_1	0
S_3 (11)	0	C_2

- Enough to consider S_1 and S_2 ($\lambda_1 + \lambda_2 = 1$)

$$C_{HD} = \max_{\lambda_1, \lambda_2} \min(\lambda_2 C_1, \lambda_1 C_2)$$

- $\lambda_1 C_2 = \lambda_2 C_1$
 $\Rightarrow C_{HD} = \frac{C_1 C_2}{C_1 + C_2}$
- $C_{FD} = \min(C_1, C_2)$

Examples: Half-Duplex Cut-Set Bound



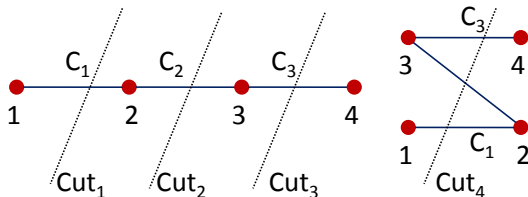
State	Cut ₁	Cut ₂	Cut ₃	Cut ₄
S_0 (000)	0	0	0	0
S_1 (001)	0	0	C_3	C_3
S_2 (010)	0	C_2	0	0
S_3 (011)	0	0	C_3	C_3
S_4 (100)	C_1	0	0	C_1
S_5 (101)	C_1	0	C_3	$C_1 + C_3$
S_6 (110)	0	C_2	0	0
S_7 (111)	0	0	C_3	C_3

- Enough to consider S_2 and S_5 ($\lambda_2 + \lambda_5 = 1$)

$$\max_{\lambda_2, \lambda_5} \min(\lambda_2 C_2, \lambda_5 C_1, \lambda_5 C_3)$$

- $C_{HD} = \min_n \frac{C_{n-1} C_n}{C_{n-1} + C_n}$
- $C_{FD} = \min_n C_n$

Undirected network with more than 2 stages/hops



State	Cut ₁	Cut ₂	Cut ₃	Cut ₄
S_0 (000)	0	0	0	0
S_1 (001)	0	0	C_3	C_3
S_2 (010)	0	C_2	0	0
S_3 (011)	0	0	C_3	C_3
S_4 (100)	C_1	0	0	C_1
S_5 (101)	C_1	0	C_3	$C_1 + C_3$
S_6 (110)	0	C_2	0	0
S_7 (111)	0	0	C_3	C_3

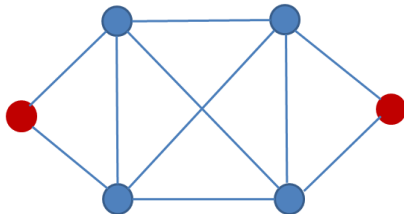
- Interference from node 3 to node 2
- Dirty paper coding (DPC) if interference is known non-causally
- Knowing interference not always possible

Wireless Networks

Information-theoretic Approach and Flow Optimization

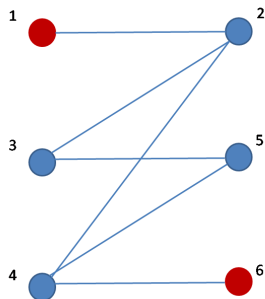
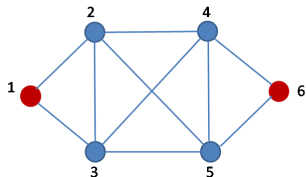
Our Focus

- Half-duplex
- All SNR
- No MIMO/Limited cooperation
- Restricted, arbitrary
- Decode-and-Forward



Two-stage relay network

States of a Half-Duplex Network



- Each node: Transmit, Receive, or Idle
- Each state is an interference network

Relaying Scheme

- Two components: Scheduling and Coding
- Scheduling of states
 - ▶ Which states help in information flow?
 - ▶ What is the best time-sharing of these states?
- Coding for a given state
 - ▶ Which encoding and decoding scheme should be used?
 - ▶ Choice of operating point in capacity region

Scheduling: Choice of States

All States

- Complexity

Interference Avoidance

- Only one node can transmit at any time

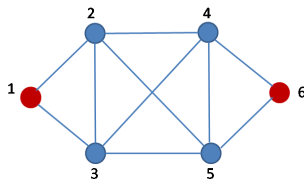
Interference Processing

- Source should be in transmit mode
- Destination should be in receive mode
- Relays should be in both transmit and receive modes
 - ▶ Required for information flow
- Atleast two node-disjoint paths required for source to be transmitting in all chosen states

Coding for a State

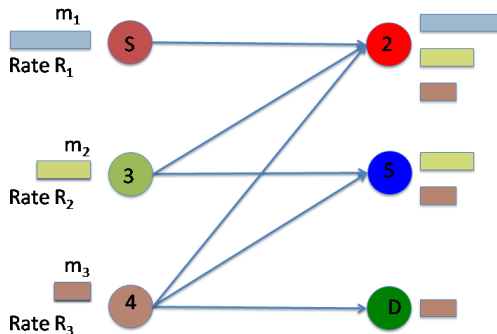
- $M \times N$ interference network [Carleial1978]
- Possible message from each transmitter to each subset of receivers
 - ▶ $M(2^N - 1)$ possible rates
- M -user Interference channel
 - ▶ M possible messages (M rates)
- Achievable rate regions based on
 - ▶ Superposition
 - ▶ Successive interference cancellation
 - ▶ Dirty paper coding

Two-Path Two-State Schedule



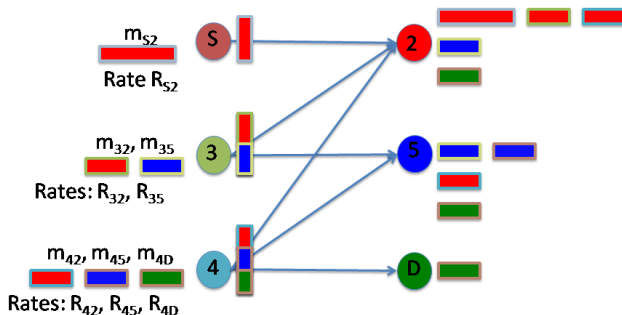
- Shortest (three-hop) paths connecting S and D
 - ▶ Path P1: $S \rightarrow 2 \rightarrow 4 \rightarrow D$
 - ▶ Path P2: $S \rightarrow 3 \rightarrow 5 \rightarrow D$
 - ▶ Path P3: $S \rightarrow 2 \rightarrow 5 \rightarrow D$
 - ▶ Path P4: $S \rightarrow 3 \rightarrow 4 \rightarrow D$.
- Only two pairs of node-disjoint paths: (P1, P2) and (P3, P4).
- States from (P1, P2):
 - ▶ State S1: Nodes S, 3, 4 transmit, Nodes 2, 5, D receive
 - ▶ State S2: Nodes S, 2, 5 transmit, Nodes 3, 4, D receive

Common Broadcast (CB)



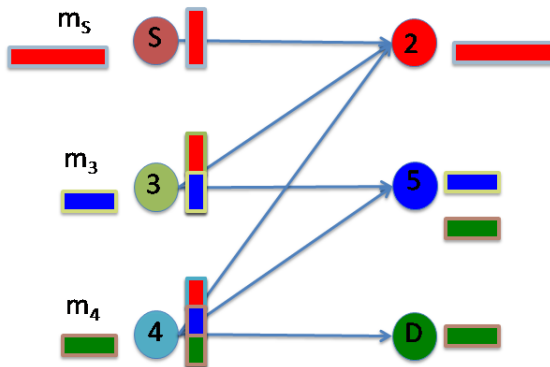
- Rate limited by weakest link
- Receivers employ SIC/MAC decoding

Superposition Coding (SC)



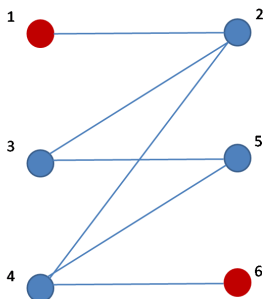
- Transmitters send superposed codewords
- Constraints involve power allocation parameters (non-linear)
- Larger rate region than CB

Dirty paper coding (DPC) at the source



- Source: origin for all messages; knows m_3 and m_4
- Source does DPC to eliminate interference at receiver 2
- Can be combined with CB or SC at other transmitters

Coding for the Two-Stage Relay Example



DPC-SC

- State S1: Nodes S (1), 3, 4 transmit, Nodes 2, 5, D (6) receive
- Node S: Transmit to Node 2 using DPC
- Node 3: Transmit to Node 5
- Node 4: Transmit to Nodes 5 and D using SC

Flow Optimization

- Joint optimization problem

maximize **Rate**
subject to

- Scheduling constraints

- ▶ State k is ON for λ_k units of time
- ▶ Total transmission time is one unit

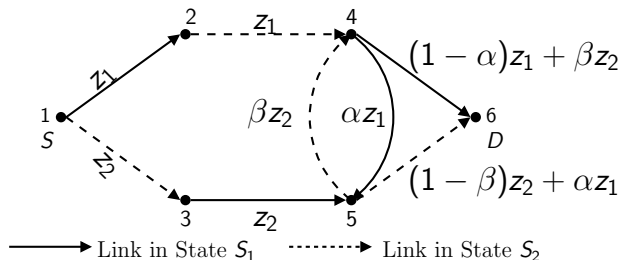
- Rate region constraints

- ▶ appropriate rate region depending on the coding scheme

- Flow constraints

- ▶ Total flow in a link $(i, j) = \sum_k \text{flow in link } (i, j) \text{ in state } k$
- ▶ Outgoing flow from Node i - Incoming flow to Node $i = \text{Rate, if } i = S,$
 $-\text{Rate, if } i = D,$
 $0, \text{ otherwise.}$

Two-Stage Relay Flow optimization: DPC-SC



Two-Stage Relay Flow optimization

$$\max_{0 \leq \lambda_1, \lambda_2, \alpha, \beta \leq 1} R = z_1 + z_2,$$

subject to rate constraints

- Flow in each link less than average rate

$$\begin{aligned} z_1 &\leq \lambda_1 R_{S2}, & z_1 &\leq \lambda_2 R_{24}, & z_2 &\leq \lambda_2 R_{S3}, & z_2 &\leq \lambda_1 R_{35}, \\ (1 - \alpha)z_1 + \beta z_2 &\leq \lambda_1 R_{4D}, & (1 - \beta)z_2 + \alpha z_2 &\leq \lambda_2 R_{5D}, \\ \alpha z_1 &\leq \lambda_1 R_{45}, & \beta z_2 &\leq \lambda_2 R_{54}, \end{aligned}$$

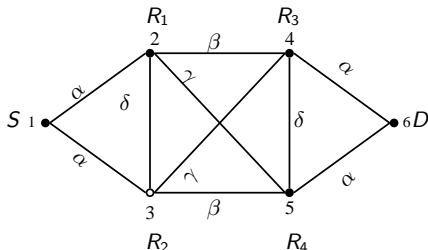
- Scheduling constraint: $0 \leq \lambda_1 + \lambda_2 \leq 1$
- Rates chosen according to rate region of interference network

$$(R_{S2}, R_{35}, R_{45}, R_{4D}) \in \mathcal{R}_1, (R_{S3}, R_{24}, R_{54}, R_{5D}) \in \mathcal{R}_2.$$

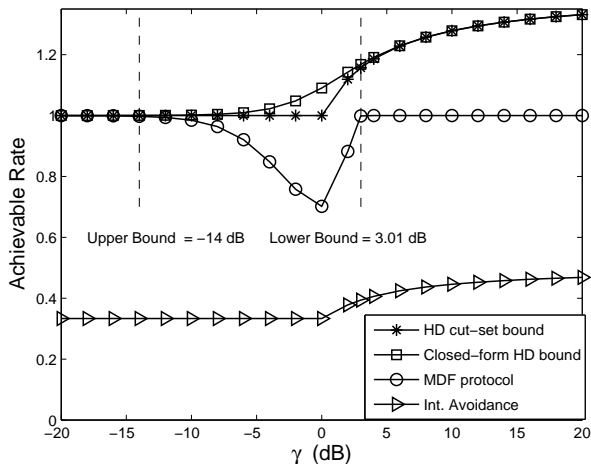
Numerical Results

Parameters:

- Tx power, $P = 3$ units
- Noise variance, $\sigma^2 = 1$
- Variable channel gains
- Case 1: $\alpha = \beta = 1, \gamma = \delta$
- Case 2: $\alpha = \beta = 1.25, \gamma = \delta$

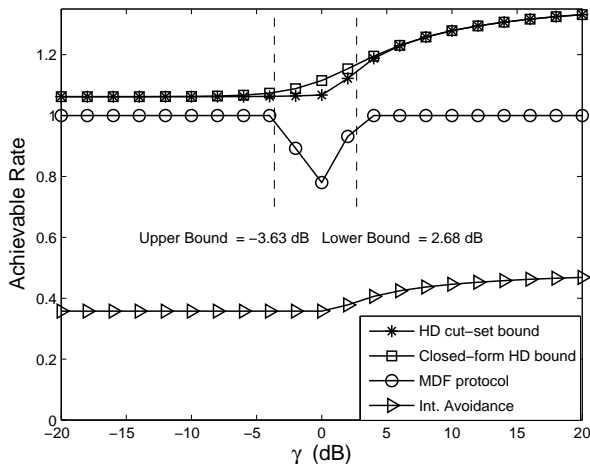


Numerical Results: Case 1



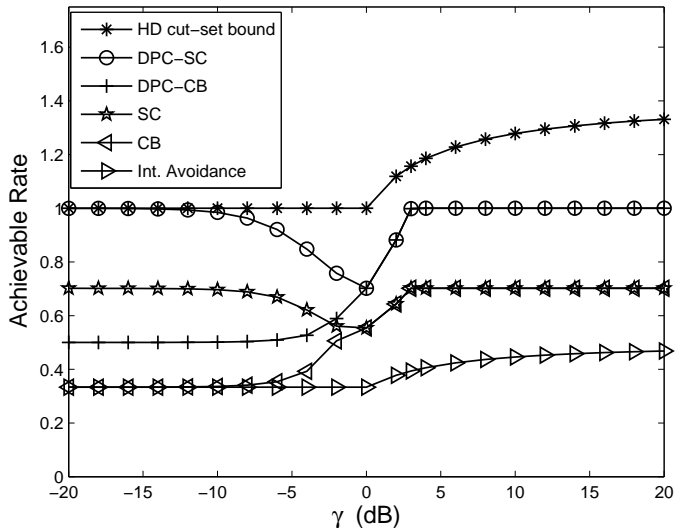
- Achieves cut-set bound in weak interference regime
- Gap from cut-set bound in strong interference regime ≤ 0.33 bits

Numerical Results: Case 2



- Gap from cut-set bound in weak interference regime ≤ 0.06 bits
- Gap from cut-set bound in strong interference regime ≤ 0.33 bits

Comparison of All Schemes: Case 1



Numerical Results: Multicast

Parameters:

- Tx power, $P = 3$ units
- Noise variance, $\sigma^2 = 1$
- Variable channel gains

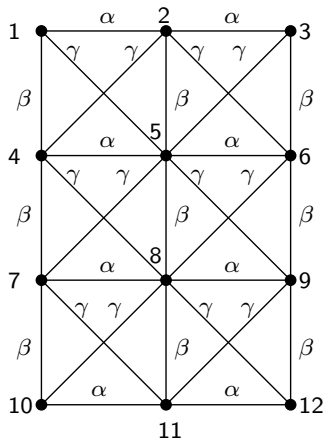
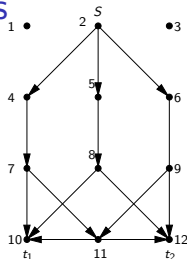
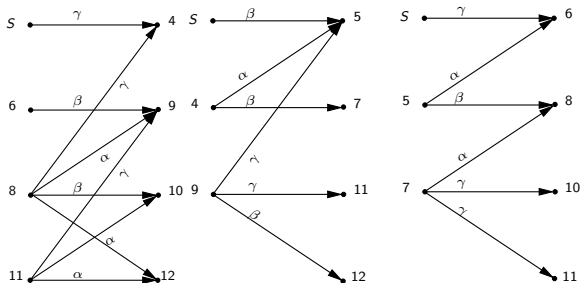


Figure: 4 × 3 Grid Network.

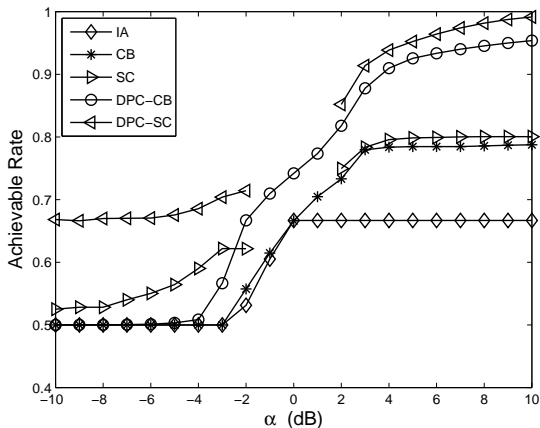
Information Flow Paths



• Three IP states



Performance in Grid Network, $\beta = 1, \gamma = 1$, vary α



- Six IA states, three IP states

Summary of Optimization Formulation

- Flow optimization with more general physical layer
- States of a half-duplex relay network as interference networks
- Scheduling + Coding components
- Scheduling of states using path heuristic
- Interference processing receivers at the relays
- Strong and weak interference conditions on channel gains
 - ▶ Close to cut-set bound

Wireless Networks

Information-theoretic Approach: Approximate Capacity

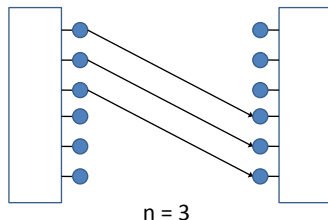
Relay Networks and Approximate Capacity¹⁹

- Achieve rates within a constant gap of cut-set bound
- Gap independent of channel parameters
- Gap not significant at high rate/high SNR

- Deterministic model (approximation)
- Capacity of a deterministic relay network
- Approximate schemes for Gaussian relay networks

¹⁹A. S. Avestimehr, S. N. Diggavi, and D. N. C. Tse, Wireless network information flow: A deterministic approach, IEEE Transactions on Information Theory, vol. 57, no. 4, pp. 1872 1905, April 2011.

Deterministic Model: Point-to-Point



- Signal strength model

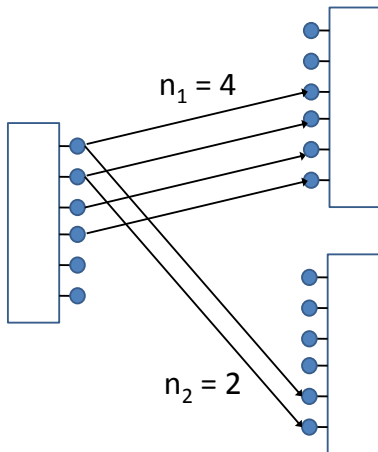
$$y = \sqrt{\text{SNR}}x + z, \quad z \sim N(0, 1), \quad E[x^2] \leq 1$$

$$y \approx 2^n \sum_{i=1}^n x(i)2^{-i} + \sum_{i=1}^{\infty} (x(i+n) + z(i))2^{-i}$$

where $n = \lceil 0.5 \log \text{SNR} \rceil^+$

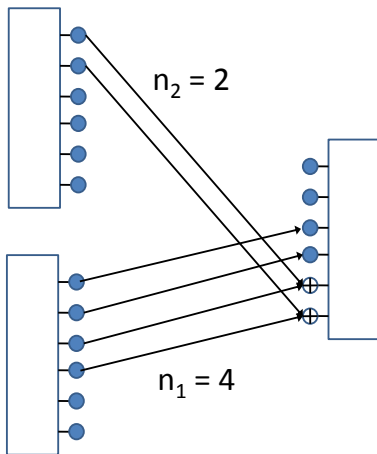
- Most significant n bits received as destination

Deterministic Model: Broadcast



- $R_2 \leq n_2$
- $R_1 + R_2 \leq n_1$

Deterministic Model: Multiple Access



- $R_2 \leq n_2$
- $R_1 + R_2 \leq n_1$

Deterministic Model: Summary

- Component-wise within one bit gap for BC and MAC
 - Not a finite gap for MIMO
 - Models link from transmitter to receiver
-
- Deterministic model for relay network
 - Quantize-map-and-forward strategy
 - Finite gap from cut-set bound

Related Results

- Abstract flow model for deterministic relay networks
- Simpler computable schemes (instead of random coding)^{20 21}

²⁰M. X. Goemans, S. Iwata, and R. Zenklusen, An algorithmic framework for wireless information flow, in Proceedings of Allerton Conference on Communications, Control, and Computing, Sep. 2009.

²¹S. M. S. Yazdi and S. A. Savari, A combinatorial study of linear deterministic relay networks, in Proceedings of Allerton Conference on Communications, Control, and Computing, Sep. 2009.

Other Constant Gap Achieving Schemes

- Noisy Network Coding²²
 - ▶ Vector-quantization of received signal in blocks
- Compress-and-forward²³
 - ▶ Analog of algebraic flow results in deterministic networks for Gaussian networks

²²S. H. Lim; Y. -H. Kim; A.. El Gamal, and S. -Y. Chung. Noisy Network Coding. IEEE Trans. Inform. Theory, vol. 57, no. 5, pp.3132-3152, May 2011.

²³A. Raja and P. Viswanath. Compress-and-Forward Scheme for a Relay Network: Approximate Optimality and Connection to Algebraic Flows Proc. of IEEE ISIT, Aug. 2011.

Multiple Unicast and Polymatroidal Networks^{24 25}

- Wireless network as an undirected polymatroidal network
- Use results on polymatroidal networks
- Polymatroidal Networks
 - ▶ Edge capacity constraints
 - ▶ Joint capacity constraints on set of edges that meet a vertex

²⁴S. Kannan and P. Viswanath. Multiple-Unicast in Fading Wireless Networks: A Separation Scheme is Approximately Optimal. Proc. of IEEE ISIT, Aug. 2011.

²⁵S. Kannan, A. Raja and P. Viswanath. Local Phy + Global Flow: A Layering Principle for Wireless Networks. Proc. of IEEE ISIT, Aug. 2011.

Summary

Summary

- Wired Networks
 - ▶ Unicast: Max-flow min-cut theorem
 - ▶ Multicast: Network coding
- Wireless Networks
 - ▶ Interference management
 - ▶ Interference Avoidance Approach
 - ▶ Interference processing
 - ▶ Flow optimization + Interference processing
 - ▶ Approximate capacity + deterministic models
- Issues
 - ▶ Centralized scheduling + rate selection
 - ▶ Limited topology and channel information