Cortical Source Localization of Human Scalp EEG

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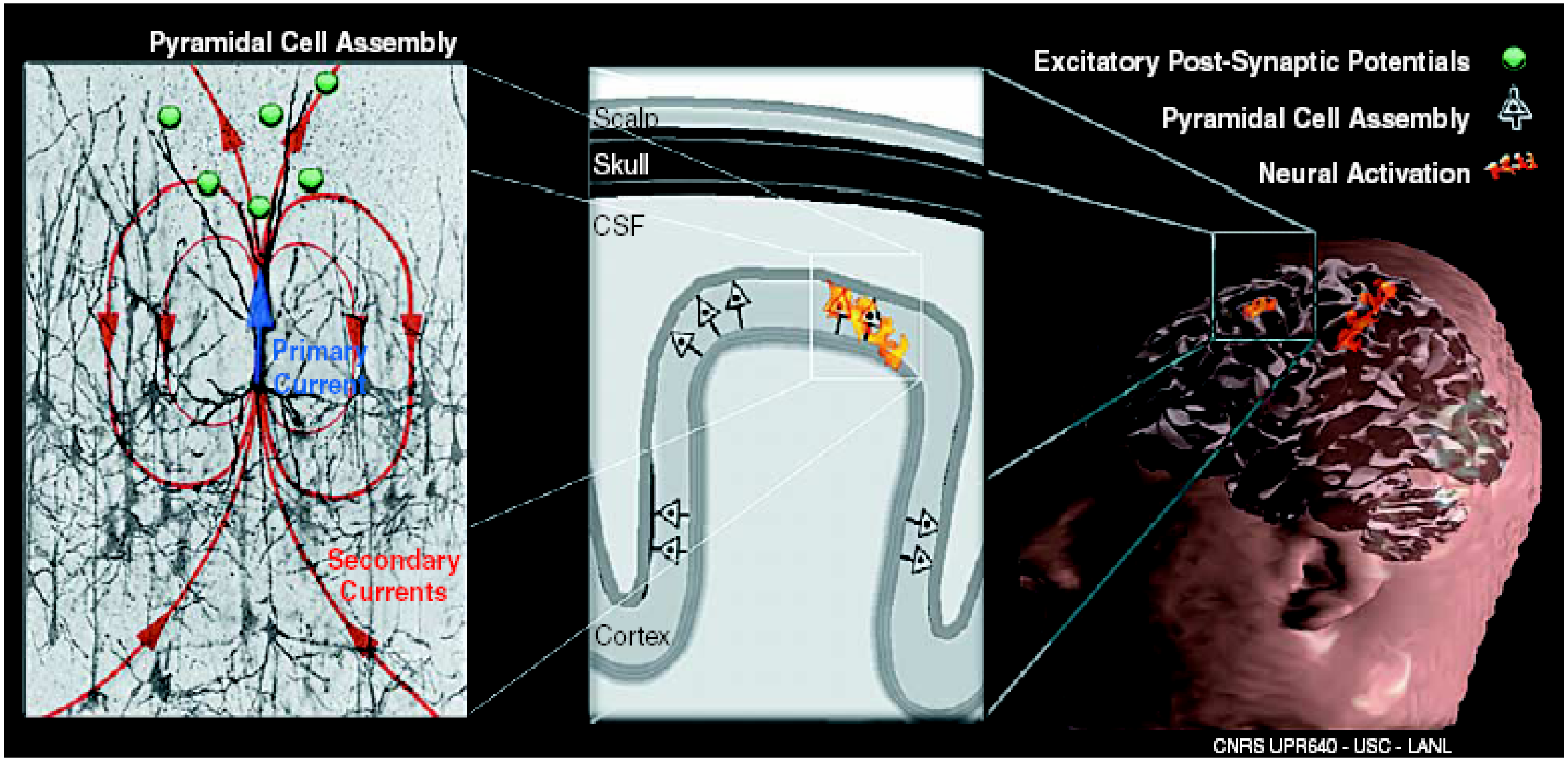
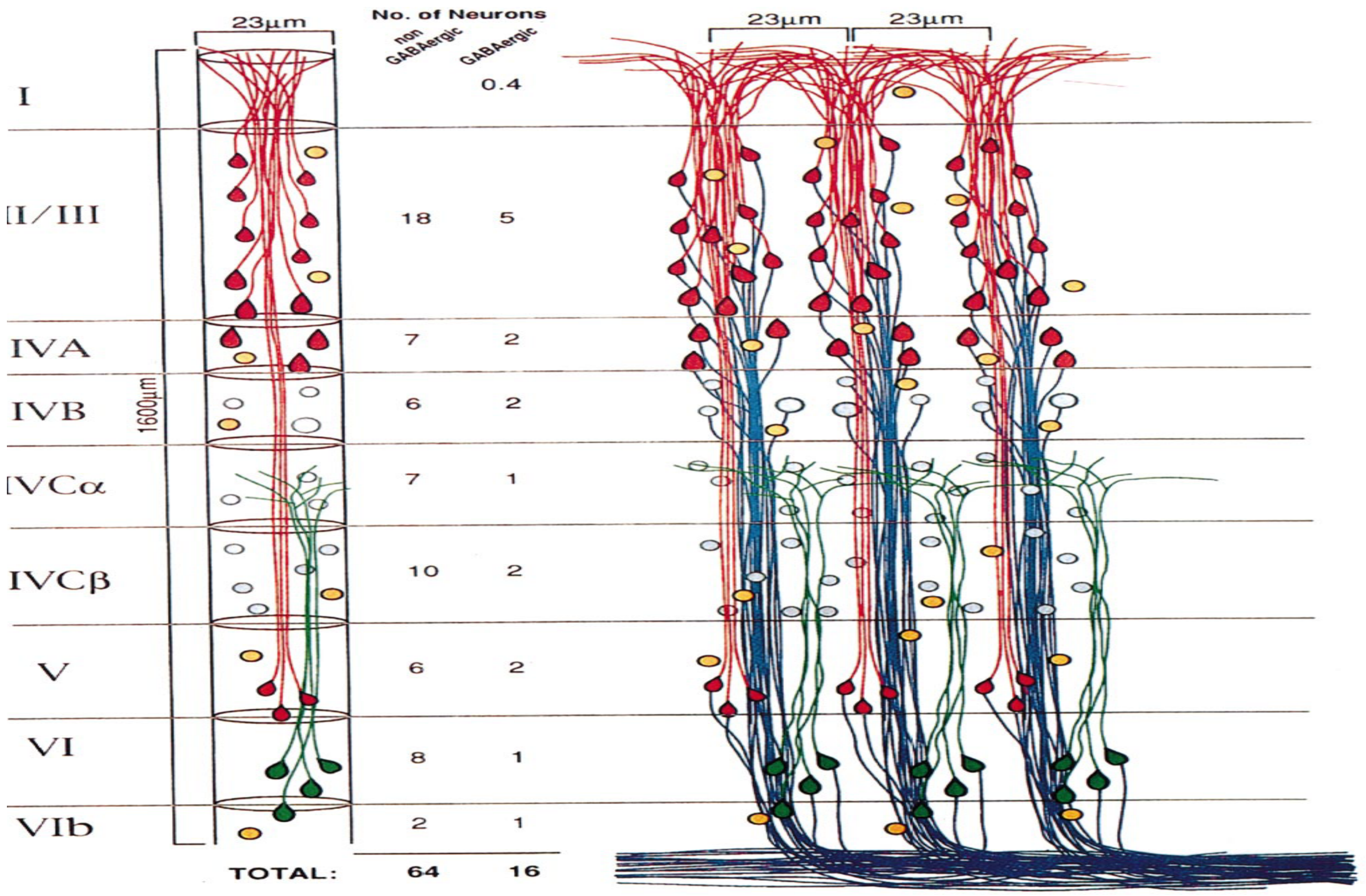
Cortical Basis of Scalp EEG



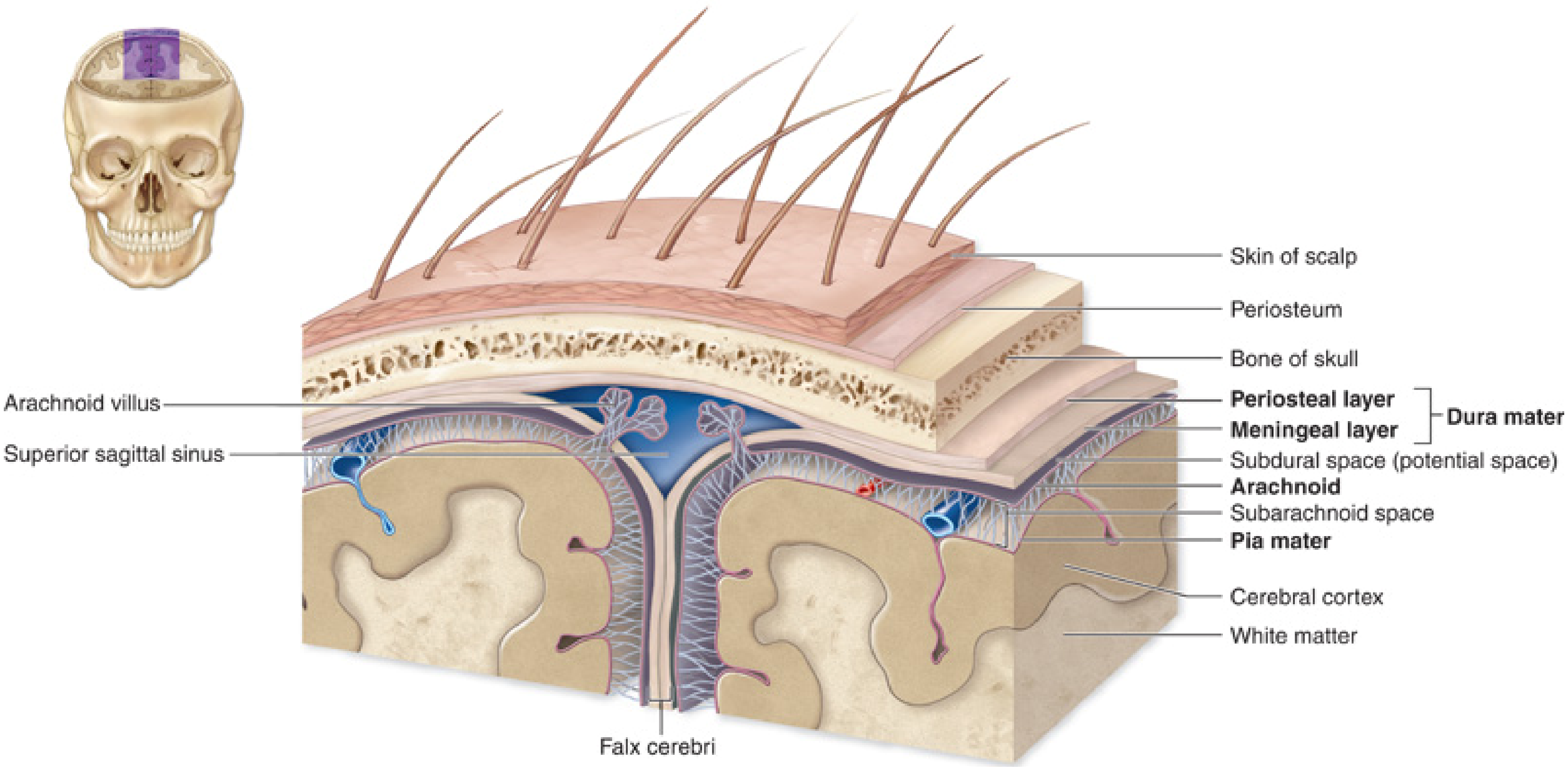
Baillet et al., IEEE Sig. Proc. Mag., Nov 2001, p. 16.

Mountcastle, Brain, 120:701-722, 1997.

Six Layer Cortex



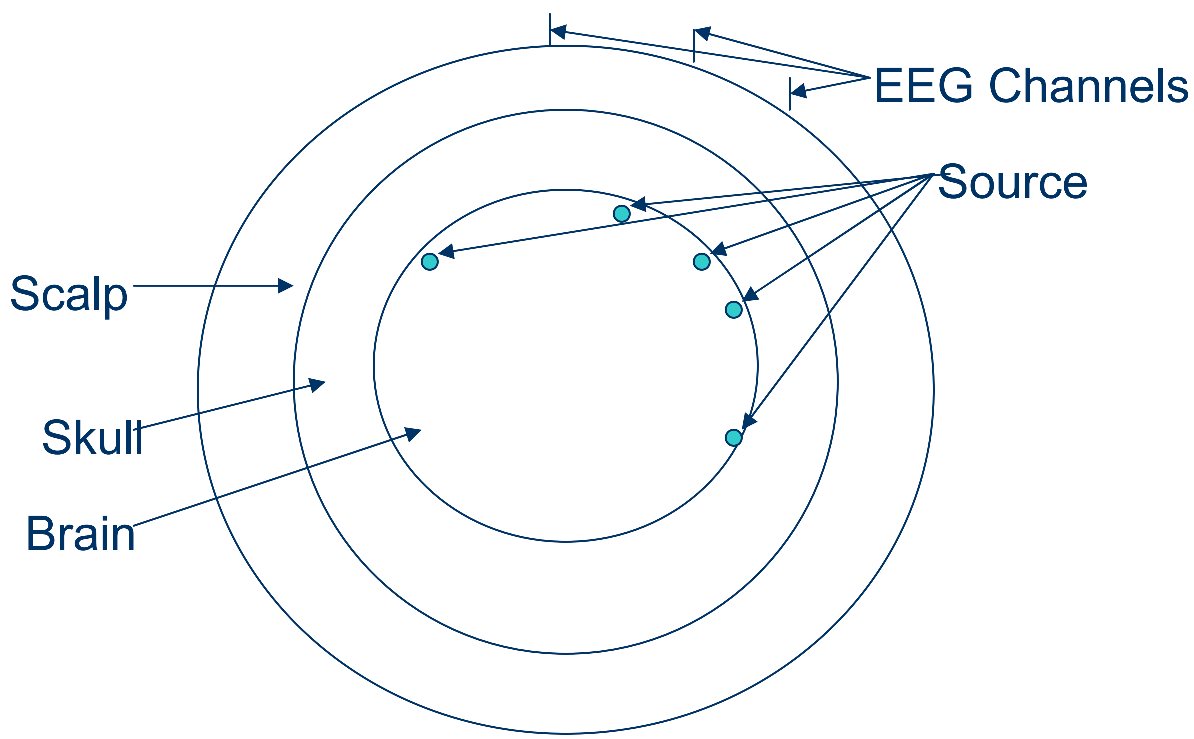
Head Tissue Layers



Source Localization in Two Parts

- Part I : Forward Problem
- Part II : Inverse Problem

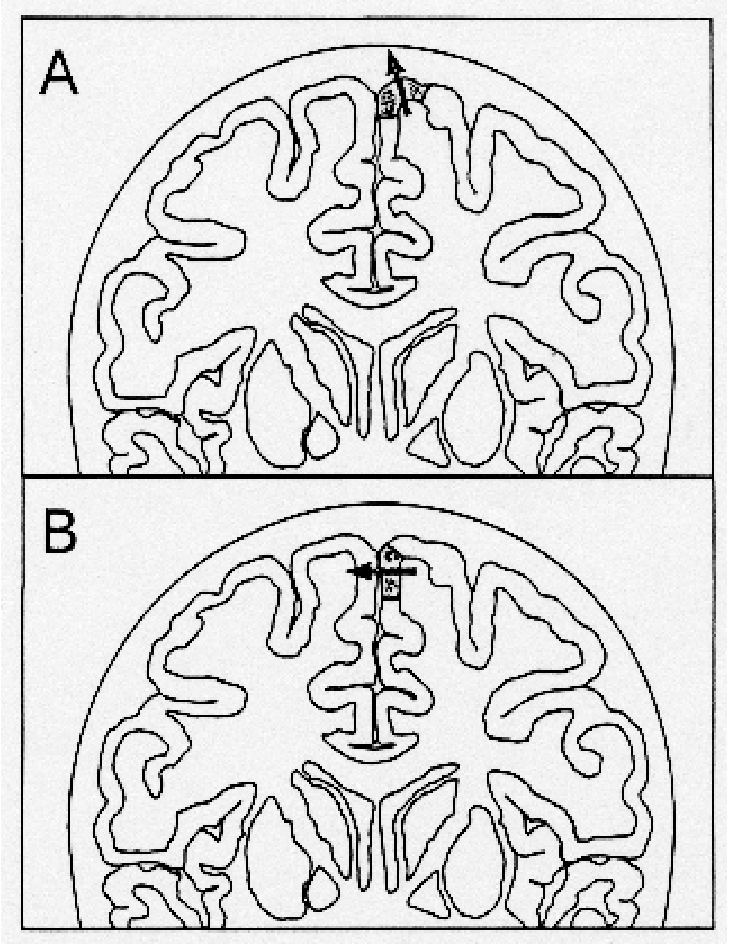
Forward Problem : Schematic Head Model



Source Models

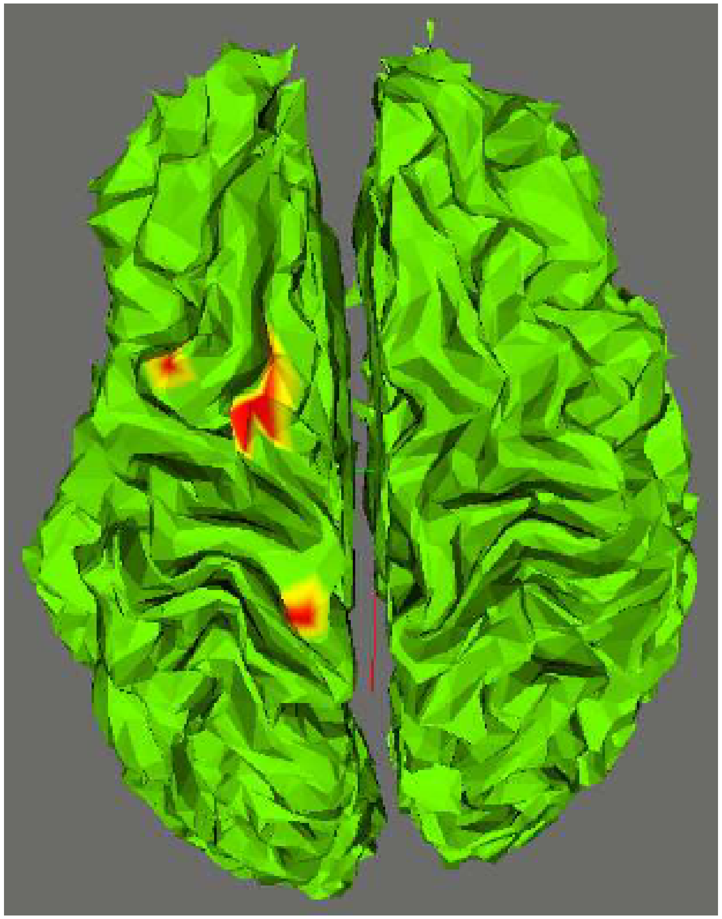
- Dipole Source Model (parametric model)
- Distributed Source Model (nonparametric model)

Dipole Source Model



Majumdar, IEEE Trans. Biomed. Eng., vol. 56(4), p. 1228 – 1235, 2009.

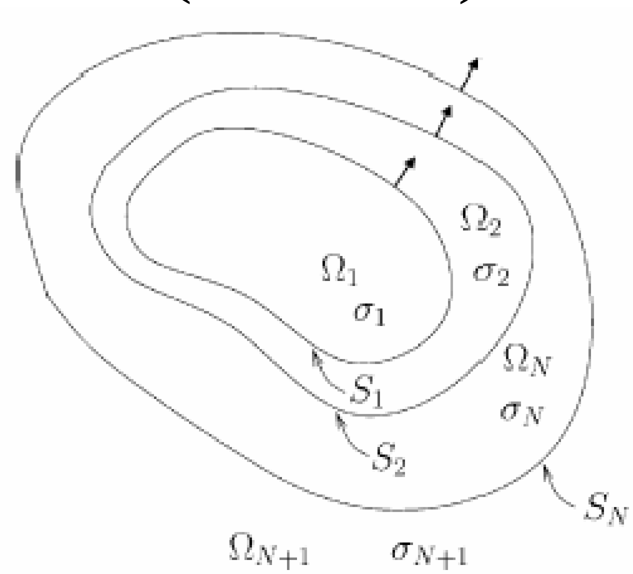
Distributed Source Model

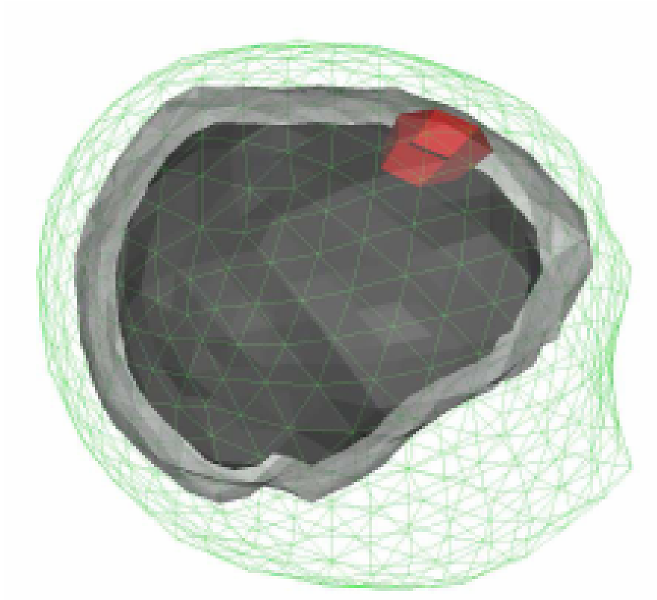


Forward Calculation

- Poisson's equation in the head

$$\nabla \cdot (\sigma \nabla V) = f = \nabla \cdot \mathbf{J}^p$$





Hallez et al., J. NeuroEng. Rehab., 2007, open access.

http://www.jneuroengrehab.com/content/4/1/46

Published Conductivity Values

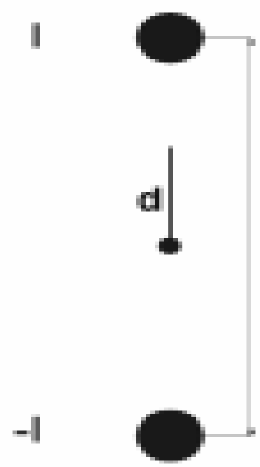
Table 1: The reference values of the absolute and relative conductivity of the compartments incorporated in the human head.

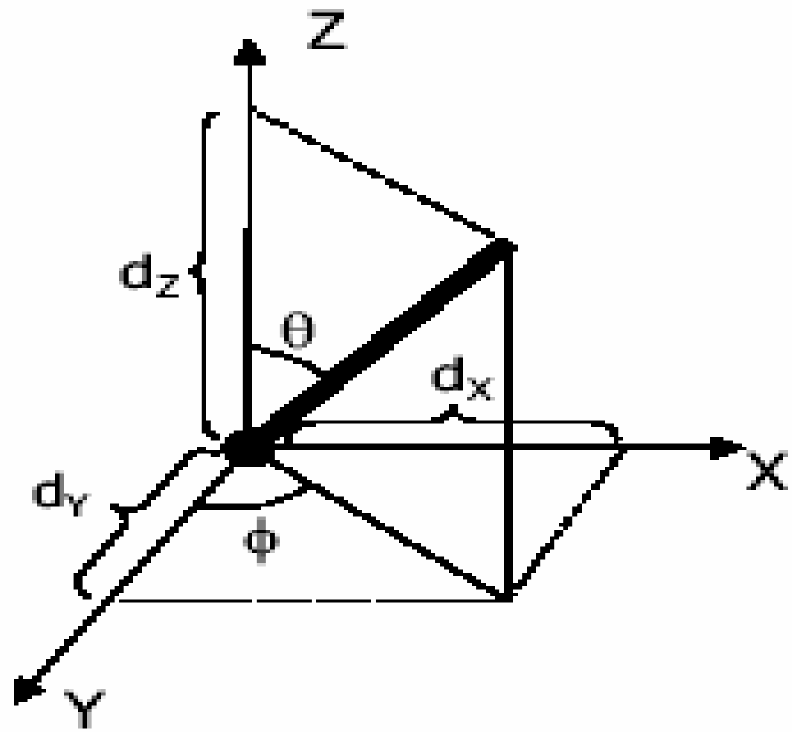
compartments	Geddes & Baker (1967)	Oostendorp (2000)	Gonçalves (2003)	Guttierrez (2004)	Lai (2005)
scalp	0.43	0.22	0.33	0.749	0.33
skull	0.006 – 0.015	0.015	0.0081	0.012	0.0132
cerebro-spinal fluid	-	-	-	1.79	-
brain	0.12 – 0.48	0.22	0.33	0.313	0.33
$\sigma_{scalp}/\sigma_{skull}$	80	15	20–50	26	25

Hallez et al., J. NeuroEng. Rehab., 2007, open access.

http://www.jneuroengrehab.com/content/4/1/46

6 Parameter Dipole Geometry





Potential at any Single Scalp Electrode Due to All Dipoles

$$V(\mathbf{r}) = \sum_i g(\mathbf{r}, \mathbf{r}_{dip_i}, \mathbf{d}_i) = \sum_i g(\mathbf{r}, \mathbf{r}_{dip_i}, \mathbf{e}_{d_i}) d_i$$

r is the position vector of the scalp electrode

$\mathbf{r}_{dip - i}$ is the position vector of the ith dipole

$\mathbf{d}_i$ is the dipole moment of the ith dipole

Potential at All Scalp Electrodes

$$V = \begin{bmatrix} V(r_1) \\ \vdots \\ V(r_N) \end{bmatrix} = \begin{bmatrix} g(r_1, r_{dip_1}, e_{d_1}) & \cdots & g(r_1, r_{dip_p}, e_{d_p}) \\ \vdots & \ddots & \vdots \\ g(r_N, r_{dip_1}, e_{d_1}) & \cdots & g(r_N, r_{dip_p}, e_{d_p}) \end{bmatrix} \begin{bmatrix} d_1 \\ \vdots \\ d_p \end{bmatrix} = G(\{r_j, r_{dip_i}, e_{d_i}\}) \begin{bmatrix} d_1 \\ \vdots \\ d_p \end{bmatrix}$$

For N Electrodes, p Dipoles, T Discrete Time Points

$$\mathbf{V} = \begin{bmatrix} V(\mathbf{r}_1,1) & \cdots & V(\mathbf{r}_1,T) \\ \vdots & \ddots & \vdots \\ V(\mathbf{r}_N,1) & \cdots & V(\mathbf{r}_N,T) \end{bmatrix}$$

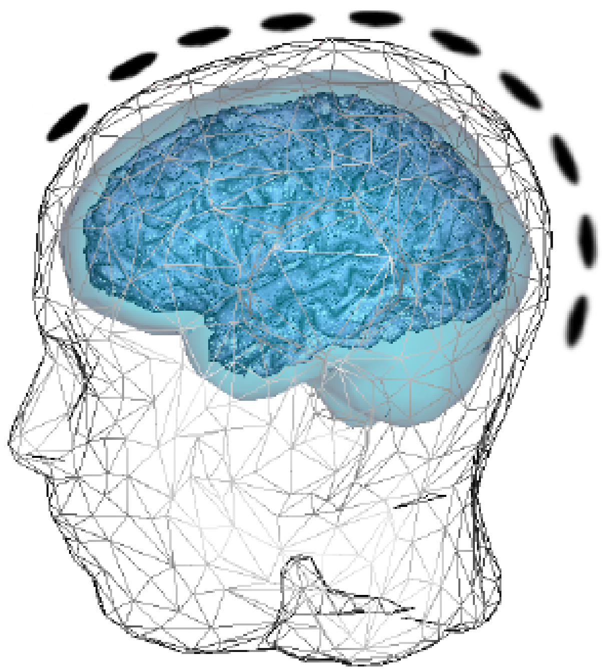
$$= \mathbf{G}(\{\mathbf{r}_j, \mathbf{r}_{dip_i}, \mathbf{e}_{\mathbf{d}_i}\}) \begin{bmatrix} d_{1,1} & \cdots & d_{1,T} \\ \vdots & \ddots & \vdots \\ d_{p,1} & \cdots & d_{p,T} \end{bmatrix} = \mathbf{G}(\{\mathbf{r}_j, \mathbf{r}_{dip_i}, \mathbf{e}_{\mathbf{d}_i}\})\mathbf{D}$$

Generalization

$$\mathbf{V} = \mathbf{G}\mathbf{D} + \mathbf{n}$$

G is *gain matrix*, **n** is additive noise.

EEG Gain Matrix Calculation



[potential at sensors] = [interpolation matrix] x [potential at interfaces]

For detail of potential calculations see Geselowitz, *Biophysical J.*, 7, 1967, 1-11.

Gain Matrix : Elaboration

$$\begin{pmatrix} V_1 \\ V_2 \\ . \\ . \\ V_n \end{pmatrix} = \begin{pmatrix} B_{11} & B_{12} & B_{13} & . & . & . & . & . & . & . & . & . & . & . & B_{1m} \\ B_{21} & B_{22} & B_{23} & & & & & & & & & & & & B_{2m} \\ . & & & & & & & & & & & & & & \\ . & & & & & & & & & & & & & & \\ B_{n1} & B_{n2} & B_{n3} & . & . & . & . & . & . & . & . & . & . & . & B_{nm} \end{pmatrix} \begin{pmatrix} J_1 \\ J_2 \\ J_3 \\ . \\ . \\ . \\ . \\ . \\ . \\ . \\ . \\ . \\ . \\ . \\ J_m \end{pmatrix}$$

Boundary Elements Method for Distributed Source Model

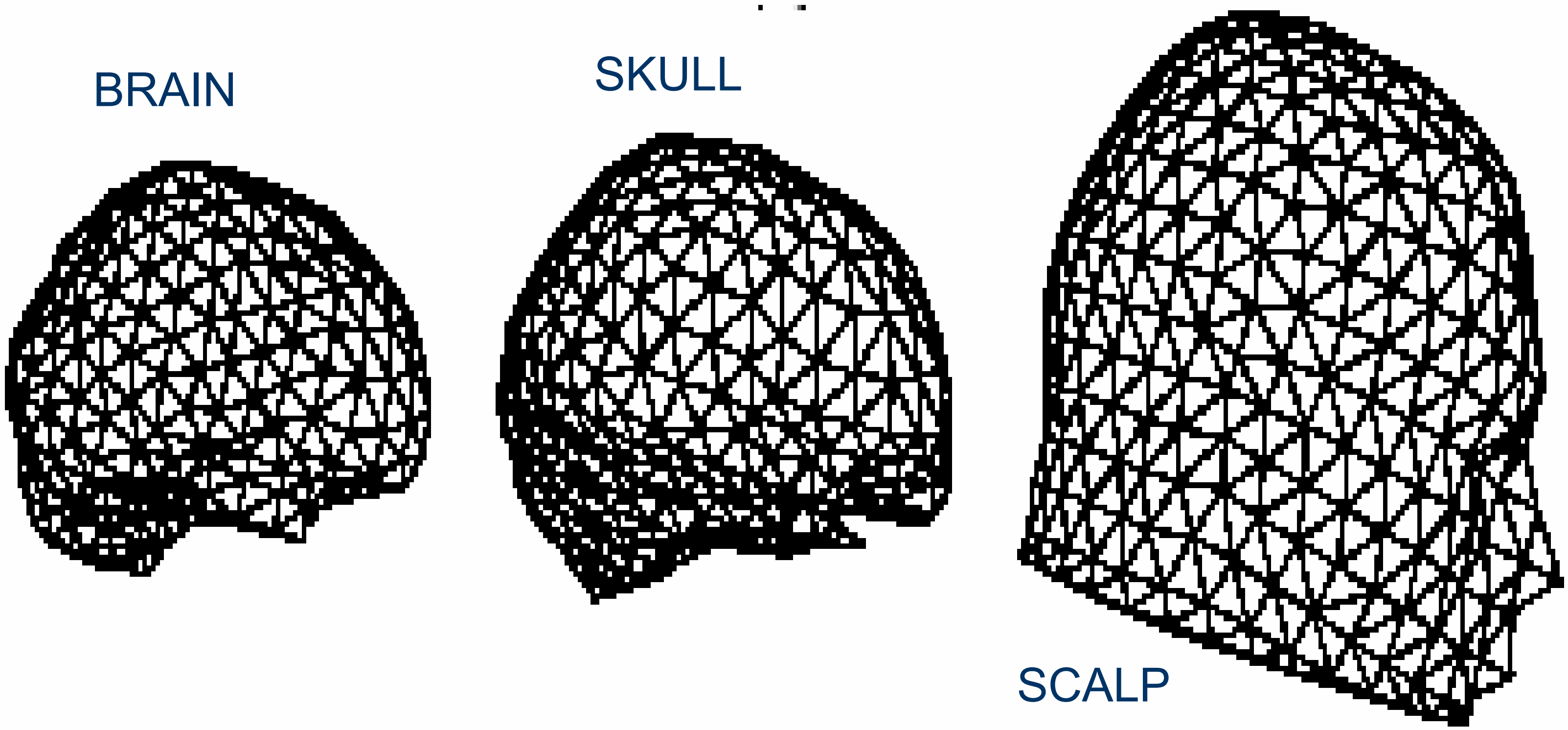
- If $\nabla^2 u = 0$

  on the complement of a smooth surface then u can be completely determined by its values and the values of its derivatives on that surface.

Hallez et al., J. NeuroEng. Rehab., 2007, open access.

http://www.jneuroengrehab.com/content/4/1/46

Nested Head Tissues



Green's Function

$$f_{\Omega_i} = f . 1_{\Omega_i}$$

$$G(\mathbf{r}) = \frac{1}{4\pi \|\mathbf{r}\|}$$

$$v_{\Omega_i}(\mathbf{r}) = -f_{\Omega_i} * G(\mathbf{r})$$

Kybic, et al., *IEEE Trans. Med. Imag.*, vol. 24(1), p. 12 – 28, 2005.

Representation Theorem

Ω be a connected, open, bounded subset of R^3 and $\partial\Omega$ be regular boundary of Ω. $u : R^3 - \partial\Omega \rightarrow R$ is harmonic and $\mathbf{r}|u(\mathbf{r})| < \infty$, $\mathbf{r}(\partial u/\partial \mathbf{r}) = 0$, then if $p(\mathbf{r}) = \partial_{\mathbf{n}} u(\mathbf{r})$ the following holds:

$$
\begin{aligned}
-p &= && +\mathcal{N}[u] && -\mathcal{D}^*[p], && \text{for } \mathbf{r} \notin \partial\Omega \\
u &= && -\mathcal{D}[u] && +\mathcal{S}[p] && \\
-p^{\pm} &= && +\mathcal{N}[u] && +\left(\pm\tfrac{\mathcal{I}}{2} - \mathcal{D}^*\right)[p], && \text{for } \mathbf{r} \in \partial\Omega \\
u^{\pm} &= && \left(\mp\tfrac{\mathcal{I}}{2} - \mathcal{D}\right)[u] && +\mathcal{S}[p] &&
\end{aligned}
$$

Representation Theorem (cont)

I is the identity operator and

$$(\mathcal{D}f)(\mathbf{r}) = \int_{\partial\Omega} \partial_{\mathbf{n}'} G(\mathbf{r}-\mathbf{r}') f(\mathbf{r}')\,\mathrm{d}s(\mathbf{r}')$$

$$(\mathcal{S}f)(\mathbf{r}) = \int_{\partial\Omega} G(\mathbf{r}-\mathbf{r}') f(\mathbf{r}')\,\mathrm{d}s(\mathbf{r}')$$

$$(\mathcal{N}f)(\mathbf{r}) = \int_{\partial\Omega} \partial_{\mathbf{n},\mathbf{n}'} G(\mathbf{r}-\mathbf{r}') f(\mathbf{r}')\,\mathrm{d}s(\mathbf{r}')$$

$$(\mathcal{D}^{*}f)(\mathbf{r}) = \int_{\partial\Omega} \partial_{\mathbf{n}} G(\mathbf{r}-\mathbf{r}') f(\mathbf{r}')\,\mathrm{d}s(\mathbf{r}') .$$

Representation Theorem (cont)

$\partial_{\mathbf{n}}G$ means $\mathbf{n}.\nabla G$, where **n** is normal to an interfacing head tissue surface.

Justification

- Holes in the skull (such as eyes) account for up to 2 cm of error in source localization.
- The closer a source is to the cortical surface the more its effect tends to smear.
- If size of mesh triangles is of the order of the gaps between the surfaces the errors go up rapidly.
- Implemented in OpenMeeg – an open source software (***openmeeg****.gforge.inria.fr/*).

Kybic, et al., *IEEE Trans. Med. Imag.*, vol. 24(1), p. 12 – 28, 2005.

Distributed Source : Gain Matrix

$$
\underbrace{\begin{bmatrix}
(\sigma_1+\sigma_2)N_{11} & -2D^*_{11} & -\sigma_2 N_{12} & D^*_{12} & & & \\
-2D_{11} & (\sigma_1^{-1}+\sigma_2^{-1})S_{11} & D_{12} & -\sigma_2^{-1}S_{12} & & & \\
-\sigma_2 N_{21} & D^*_{21} & (\sigma_2+\sigma_3)N_{22} & -2D^*_{22} & -\sigma_3 N_{23} & D^*_{23} & \\
D_{21} & -\sigma_2^{-1}S_{21} & -2D_{22} & (\sigma_2^{-1}+\sigma_3^{-1})S_{22} & D_{23} & -\sigma_3^{-1}S_{23} & \\
 & & -\sigma_3 N_{32} & D^*_{32} & (\sigma_3+\sigma_4)N_{33} & -2D^*_{33} & \cdots \\
 & & D_{32} & -\sigma_3^{-1}S_{32} & -2D_{33} & (\sigma_3^{-1}+\sigma_4^{-1})S_{33} & \cdots \\
 & & & & \vdots & \vdots & \ddots
\end{bmatrix}}_{A}
\underbrace{\begin{bmatrix} \mathbf{x}_1 \\ \mathbf{y}_1 \\ \mathbf{x}_2 \\ \mathbf{y}_2 \\ \mathbf{x}_3 \\ \mathbf{y}_3 \\ \vdots \end{bmatrix}}_{\mathbf{w}}
=
\underbrace{\begin{bmatrix} \mathbf{b}_1 \\ \mathbf{c}_1 \\ \mathbf{b}_2 \\ \mathbf{c}_2 \\ \mathbf{b}_3 \\ \mathbf{c}_3 \\ \vdots \end{bmatrix}}_{\mathbf{z}}
$$

with

$$(N_{kl})_{ij} = \langle \mathcal{N}_{kl}\phi_j^{(l)}, \phi_i^{(k)} \rangle \qquad (S_{kl})_{ij} = \langle \mathcal{S}_{kl}\psi_j^{(l)}, \psi_i^{(k)} \rangle$$

$$(D_{kl})_{ij} = (D^*_{lk})_{ji} = \langle \mathcal{D}_{kl}\phi_j^{(l)}, \psi_i^{(k)} \rangle$$

$$(\mathbf{b}_k)_i = \left\langle \partial_{\mathbf{n}} v_{\Omega_k} - \partial_{\mathbf{n}} v_{\Omega_{k+1}}, \phi_i^{(k)} \right\rangle \quad (\mathbf{c}_k)_i = \left\langle \sigma_{k+1}^{-1} v_{\Omega_{k+1}} - \sigma_k^{-1} v_{\Omega_k}, \psi_i^{(k)} \right\rangle$$

$$(\mathbf{x}_k)_i = x_i^{(k)} \quad (\mathbf{y}_k)_i = y_i^{(k)}. \tag{28}$$

Gain Matrix (cont)

$$V = \sum_{k=1}^{N_v} \sum_{k'(k)} x_k \varphi'_{k'}$$

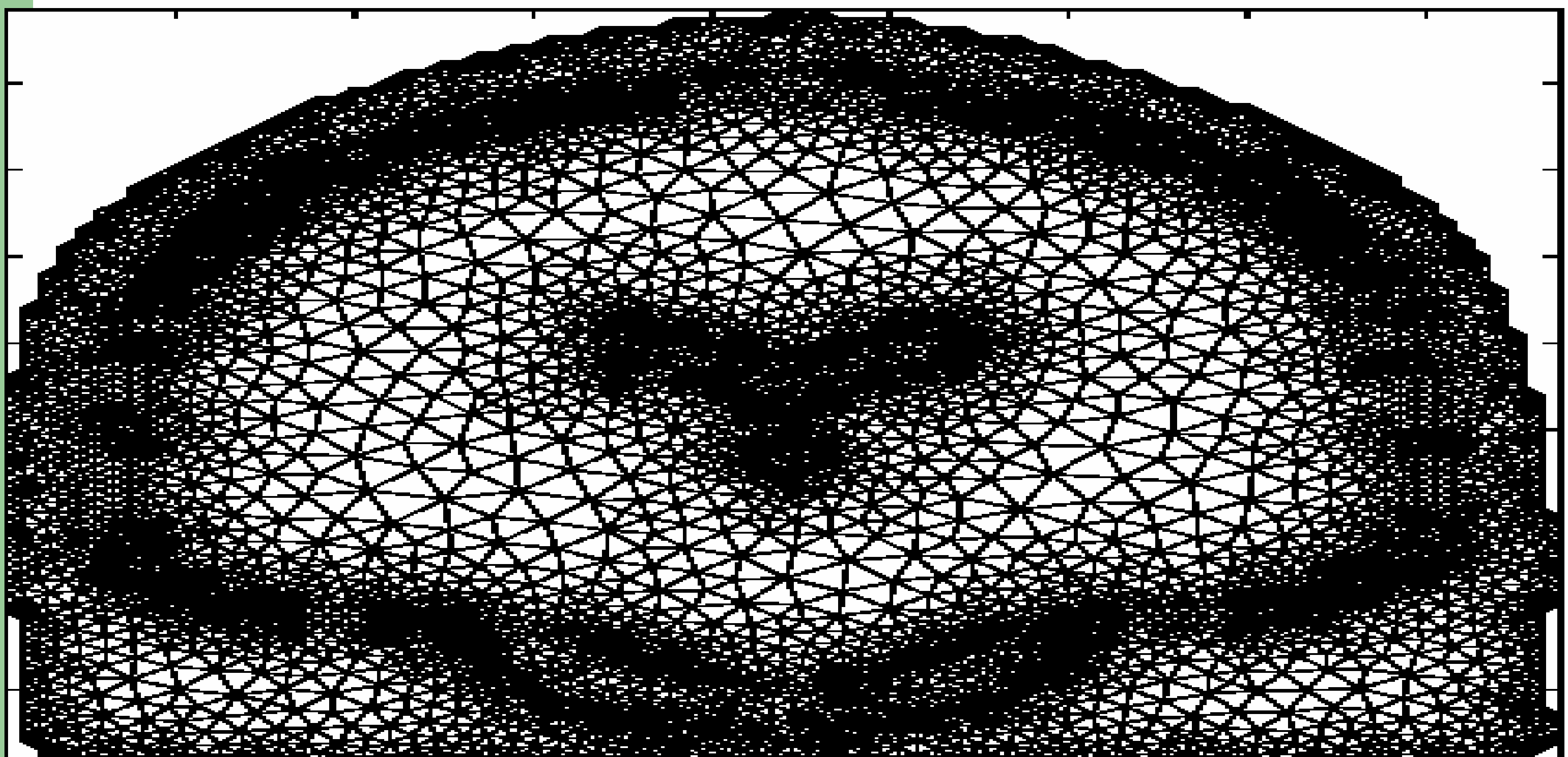
$$p = \sum_{l=1}^{N_t} y_l \psi_l$$

Gain matrix is to be obtained by multiplying several matrices A, one for each layer in the single layer formulation.

Hallez et al., J. NeuroEng. Rehab., 2007, open access.

http://www.jneuroengrehab.com/content/4/1/46

Finite Elements Method



Further Reading on FEM

- Awada et al., “Computational aspects of finite element modeling in EEG source localization,” *IEEE Trans. Biomed. Eng.*, 44(8), pp. 736 – 752, Aug 1997.
- Zhang et al., “A second-order finite element algorithm for solving the three-dimensional EEG forward problem,” *Phys. Med. Biol.*, vol. 49, pp. 2975 – 2987, 2004.

Inverse Problem : Peculiarities

- Inverse problem is ill-posed, because the number of sensors is less than the number of possible sources.

- Solution is unstable, i.e., susceptible to small changes in the input values. Scalp EEG is full of artifacts and noise, so identified sources are likely to be spurious.

Mattout et al., *NeuroImage*, vol. 30(3), p. 753 – 767, Apr 2006

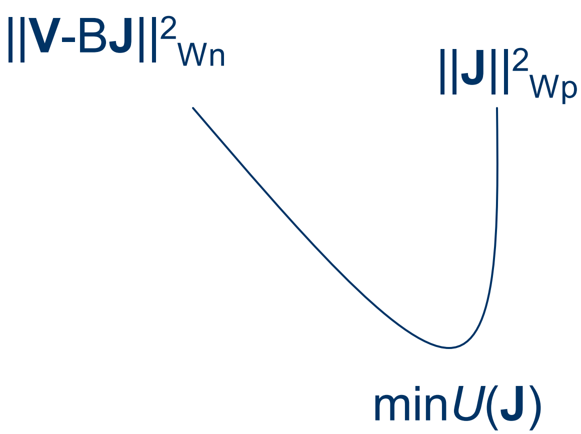
Weighted Minimum Norm Inverse

$\mathbf{V} = B\mathbf{J} + E$, $\mathbf{V}$ is scalp potential, B is gain matrix and E is additive noise.

Minimize $U(\mathbf{J}) = ||\mathbf{V} - B\mathbf{J}||^2_{Wn} + \lambda||\mathbf{J}||^2_{Wp}$

λ is a regularization positive constant between 0 and 1.

Geometric Interpretation



Convex combination

of the two terms with

λ very small.

Derivation

$U(\mathbf{J}) = ||\mathbf{V} - B\mathbf{J}||^2_{Wn} + \lambda||\mathbf{J}||^2_{Wp}$

$= <W_n(\mathbf{V} - B\mathbf{J}), W_n(\mathbf{V} - B\mathbf{J})> + \lambda<W_p\mathbf{J}, W_p\mathbf{J}>$

$\Delta_J U(\mathbf{J}) = 0$ implies (using $<AB,C> = <B,A^TC>$)

$\mathbf{J} = C_p B^T[BC_pB^T + C_n]^{-1}\mathbf{V}$

where $C_p = (W^T{}_p W_p)^{-1}$ and $C_n = \lambda(W^T{}_n W_n)^{-1}$.

Different Types

- When $C_p = I_p$ (the p x p identity matrix) it reduces to classical minimum norm inverse solution.
- In terms of Bayesian notation we can write $E(\mathbf{J}|B) = C_p B^T[BC_p B^T + C_n]^{-1}\mathbf{V}$. On this expectation maximization algorithm can be readily applied.

Types (cont)

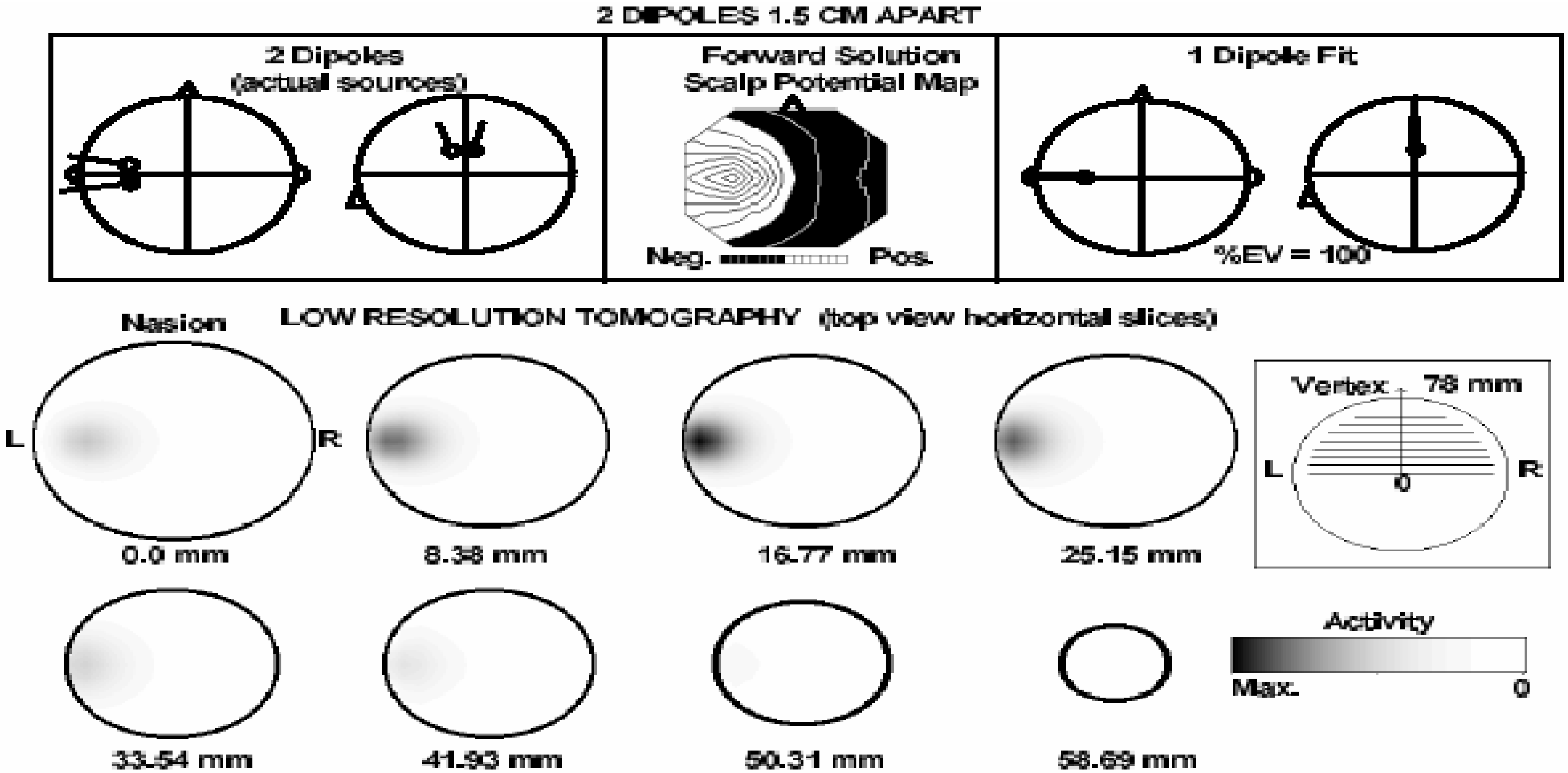
- If we take W_p as a spatial Laplacian operator we get the LORETA inverse formulation.
- If we derive the current density estimate by the minimum norm inverse and then standardize it using its variance, which is hypothesized to be due to the source variance, then that is called sLORETA.

Types (cont)

- Recursive – MUSIC

Mosher & Leahy, *IEEE Trans. Biomed. Eng.*, vol. 45(11), p. 1342 – 1354, Nov 1998.

Low Resolution Brain Electromagnetic Tomography (LORETA)



Pascual-Marqui et al., *Int. J. Psychophysiol.*, vol. 18, p. 49 – 65, 1994.

Standardized Low Resolution Brain Electromagnetic Tomography (sLORETA)

$U(\mathbf{J}) = ||\mathbf{V} - \mathrm{B}\mathbf{J}||^2 + \lambda||\mathbf{J}||^2$

$\hat{\mathbf{J}} = \mathrm{T}\mathbf{V}$, where

$\mathrm{T} = \mathrm{B}^{\mathrm{T}}[\mathrm{B}\mathrm{B}^{\mathrm{T}} + \lambda \mathrm{H}]^{+}$, where

$\mathrm{H} = \mathrm{I} - \mathrm{L}\mathrm{L}^{\mathrm{T}}/\mathrm{L}^{\mathrm{T}}\mathrm{L}$ is the centering matrix.

Pascual-Marqui at http://www.uzh.ch/keyinst/NewLORETA/sLORETA/sLORETA-Math02.htm

sLORETA (cont)

$\hat{\mathbf{J}}$ is estimate of **J**, A^+ denotes the Moore-Penrose inverse of the matrix A, I is n x n identity matrix where n is the number of scalp electrodes, L is a n dimensional vector of 1's.

Hypothesis : Variance in $\hat{\mathbf{J}}$ is due to the variance of the actual source vector **J**.

$\hat{\mathbf{J}} = B^T[BB^T + \lambda H]^+ B\mathbf{J}$.

Form of H

$$\begin{pmatrix} 1-\frac{1}{n} & -\frac{1}{n} & -\frac{1}{n} & . & . & . & . & . & . & . & . & . & . & . & -\frac{1}{n} \\ -\frac{1}{n} & 1-\frac{1}{n} & -\frac{1}{n} & . & . & . & . & . & . & . & . & . & . & . & -\frac{1}{n} \\ . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\ -\frac{1}{n} & -\frac{1}{n} & -\frac{1}{n} & . & . & . & . & . & . & . & . & . & . & . & 1-\frac{1}{n} \end{pmatrix}$$

If # Source = # Electrodes = n

- B and B^T both will be n x n identity matrix.

$$T = \begin{pmatrix} 1-\lambda+\frac{\lambda}{n} & \frac{\lambda}{n} & \frac{\lambda}{n} & . & . & . & . & . & . & . & . & . & . & . & \frac{\lambda}{n} \\ \frac{\lambda}{n} & 1-\lambda+\frac{\lambda}{n} & \frac{\lambda}{n} & . & . & . & . & . & . & . & . & . & . & . & \frac{\lambda}{n} \\ . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\ \frac{\lambda}{n} & \frac{\lambda}{n} & \frac{\lambda}{n} & . & . & . & . & . & . & . & . & . & . & . & 1-\lambda+\frac{\lambda}{n} \end{pmatrix}^{+}$$

with $\lambda = 0$

http://www.uzh.ch/keyinst/loreta.htm

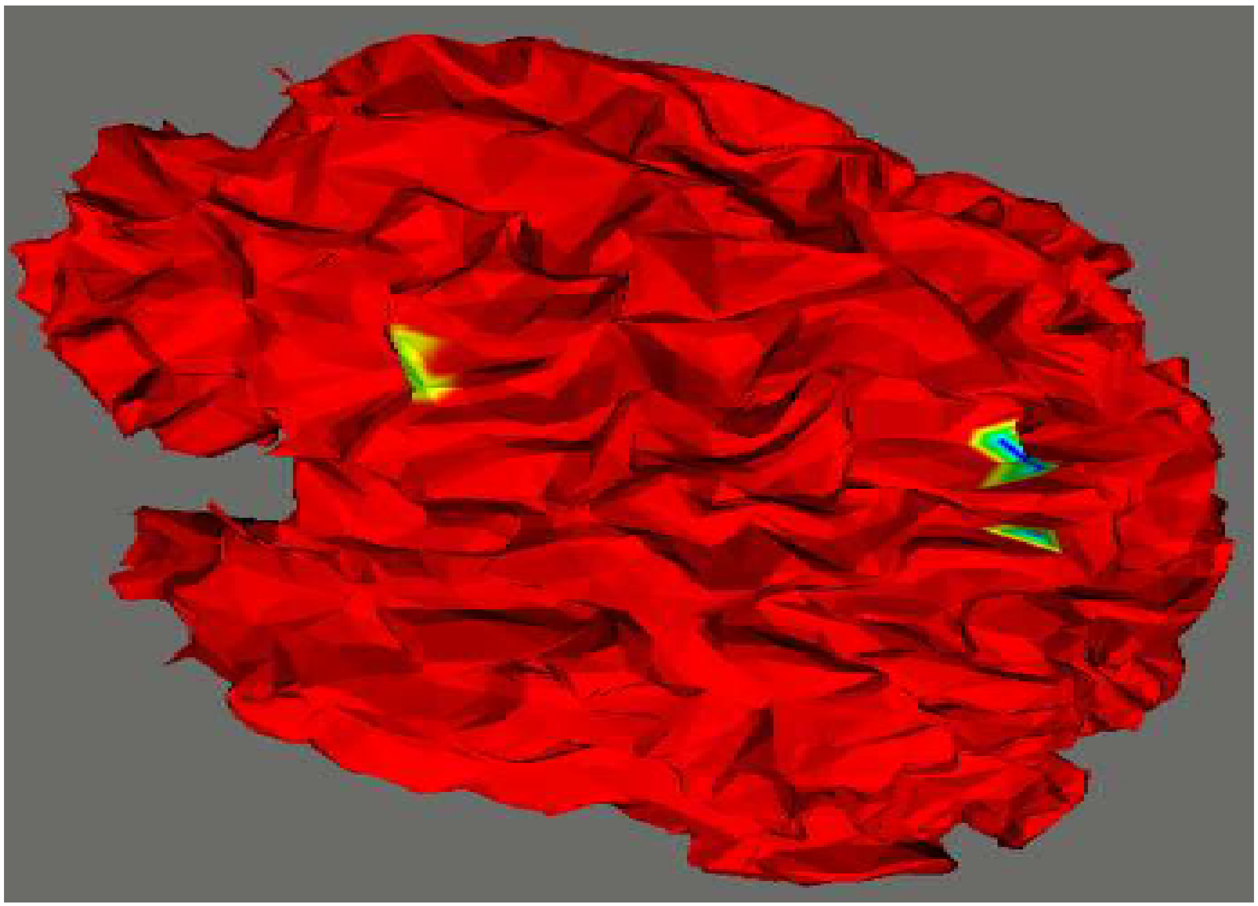
sLORETA Result

Single Trial Source Localization

- Averaging signals across the trials to increase the SNR cannot be done.

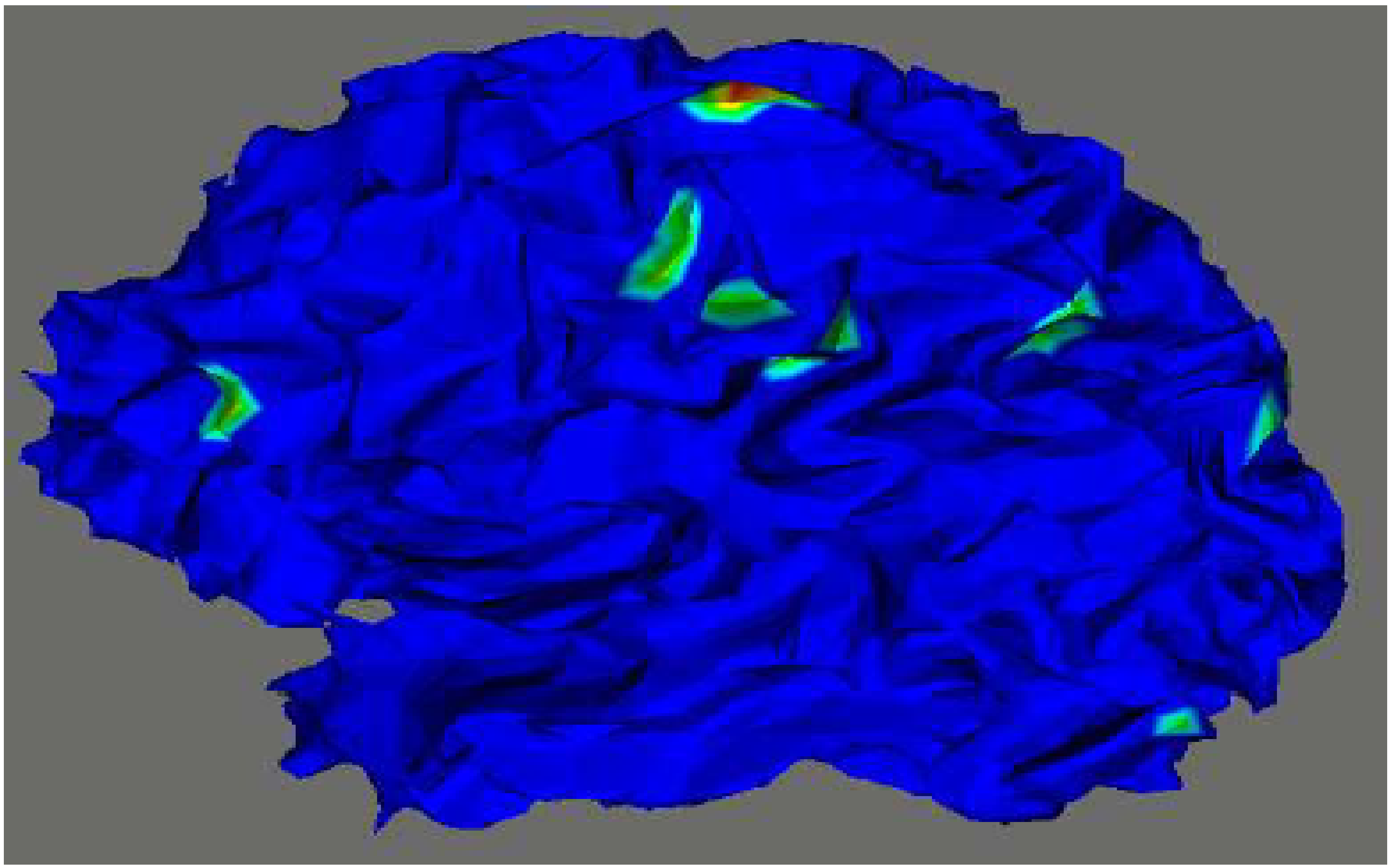
Generated using OpenMeeg in INRIA Sophia Antipolis in 2008.

Cortical Sources



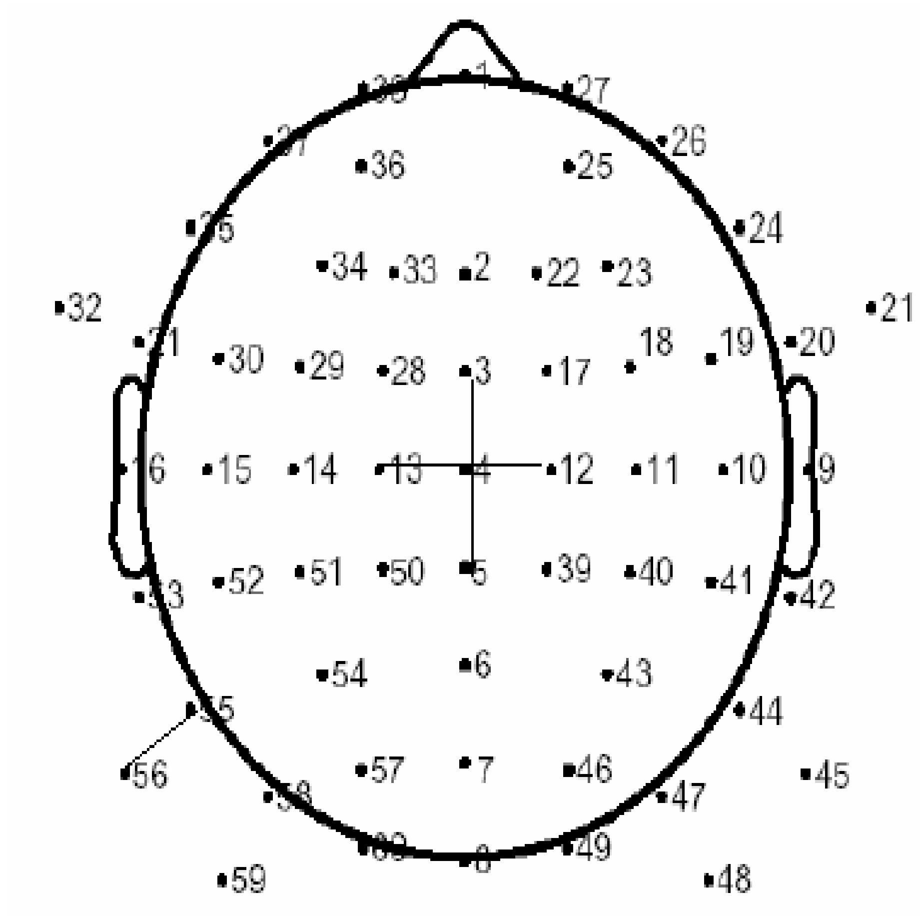
Generated using OpenMeeg in INRIA Sophia Antipolis in 2008.

Localization by MN



Majumdar, IEEE Trans. Biomed. Eng., vol. 56(4), p. 1228 – 1235, 2009.

Clustering



Trial Selection

- Identify the time interval.
- Identify channels.
- Calculate cumulative signal amplitude in those channels.
- Sort trials according to the decreasing amplitude.

Phase Synchronization

$$m\alpha - n\beta = C$$

$$m = n = 1$$

Synchronization (cont.)

$$x_j(t) = \frac{a_{j0}}{2} + \sum_{n=1}^{\infty} (a_{jn}^2 + b_{jn}^2)^{1/2} \sin\left(\frac{2\pi nt}{p} + \alpha_{jn} \right)$$

$$\alpha_{jn} = \tan^{-1}\left(\frac{a_{jn}}{b_{jn}} \right)$$

$$x_k(t) = \frac{a_{k0}}{2} + \sum_{n=1}^{\infty} (a_{kn}^2 + b_{kn}^2)^{1/2} \sin\left(\frac{2\pi nt}{p} + \alpha_{kn} \right)$$

$$\alpha_{kn} = \tan^{-1}\left(\frac{a_{kn}}{b_{kn}} \right)$$

Synchronization (cont.)

$$\alpha_{j1} - \alpha_{k1} \approx \ldots\ldots\ldots \approx \alpha_{jn} - \alpha_{kn} \approx \ldots\ldots,$$

which implies

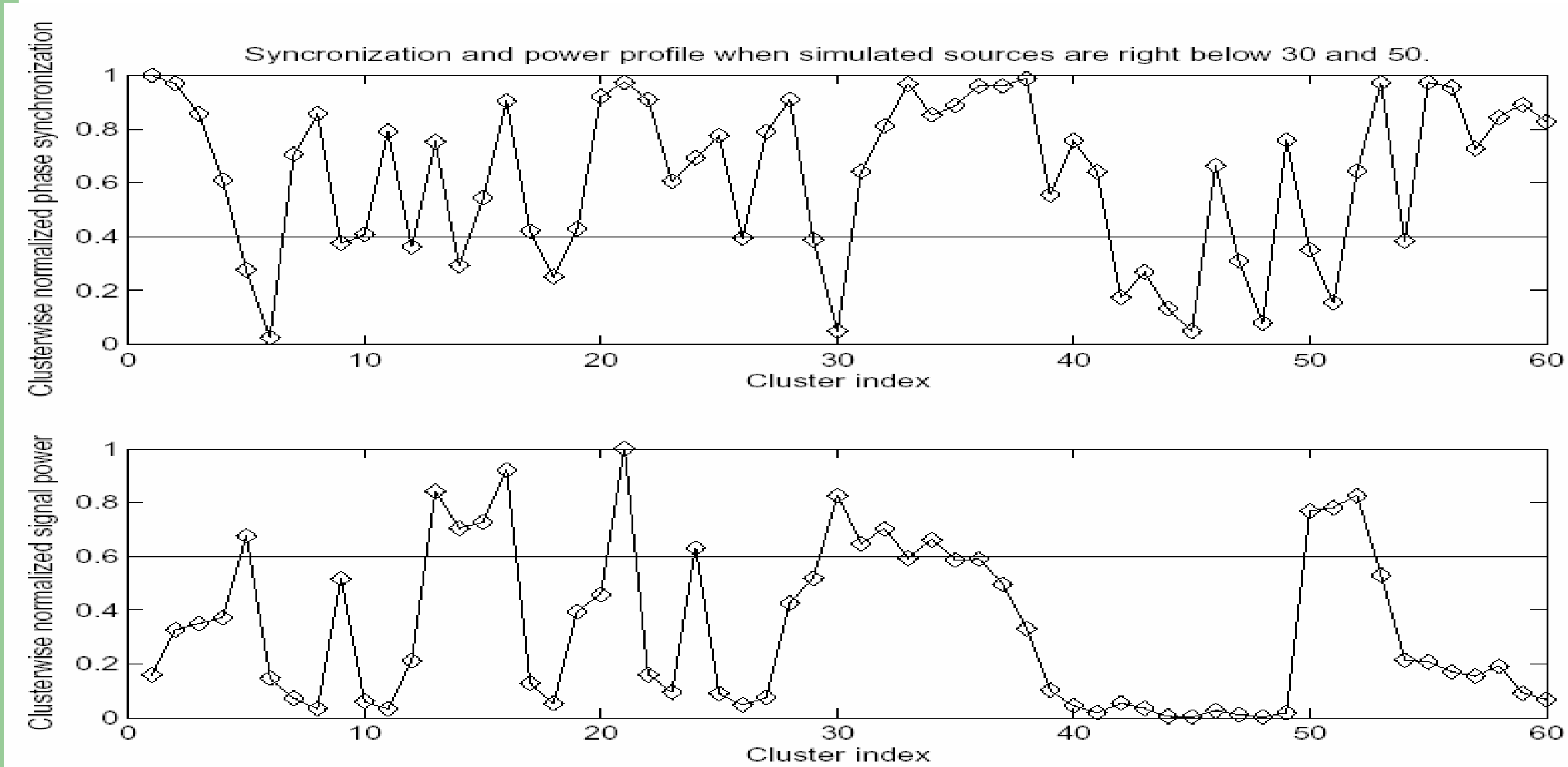
$$\frac{a_{j1}b_{k1} - a_{k1}b_{j1}}{a_{j1}a_{k1} + b_{j1}b_{k1}} \approx \ldots\ldots\ldots \approx \frac{a_{jn}b_{kn} - a_{kn}b_{jn}}{a_{jn}a_{kn} + b_{jn}b_{kn}} \approx \ldots\ldots$$

Synchronization (cont.)

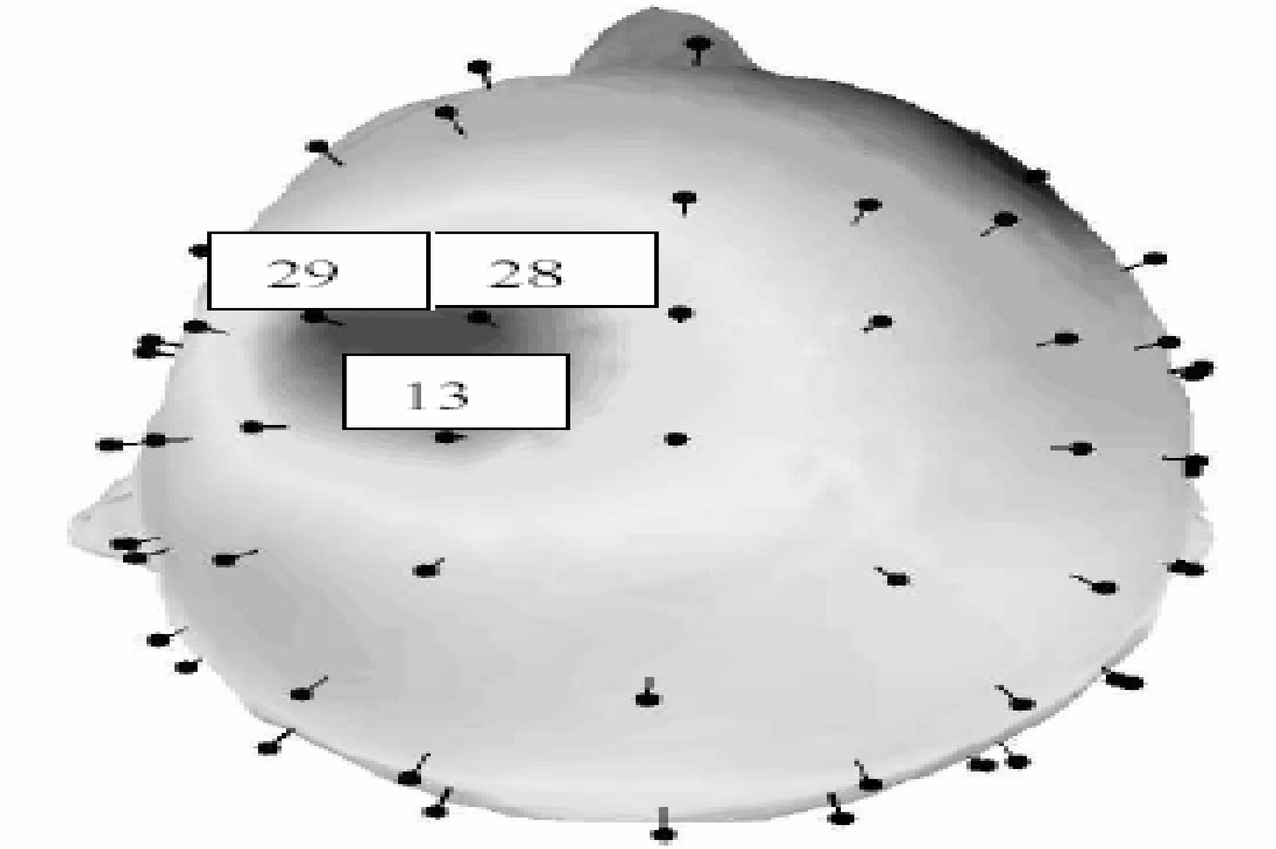
$$E[n] = \frac{a_{jn}b_{kn} - a_{kn}b_{jn}}{a_{jn}a_{kn} + b_{jn}b_{kn}} - \frac{a_{jn+1}b_{kn+1} - a_{kn+1}b_{jn+1}}{a_{jn+1}a_{kn+1} + b_{jn+1}b_{kn+1}}$$

$$syn(x_j(t), x_k(t)) = mean(|E(n)|) + std(|E(n)|)$$

Phase Synchronization and Power

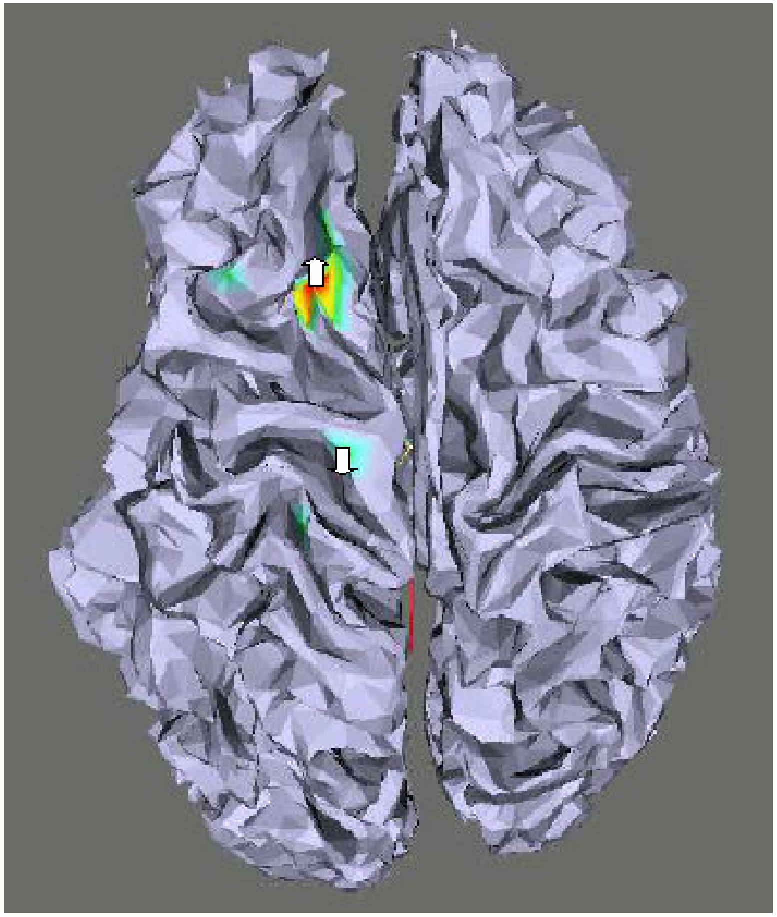


The Best Time Epoch



Majumdar, IEEE Trans. Biomed. Eng., vol. 56(4), p. 1228 – 1235, 2009.

Source Localization



Must Reading

- Baillet, Mosher & Leahy, "Electromagnetic brain mapping," *IEEE Sig. Proc. Mag.*, p. 14 – 30, Nov 2001.
- Hallez et al., "Review on solving the forward problem in EEG source analysis," *J. Neuroeng. Rehab.*, open access, available at http://www.jneuroengrehab.com/content/4/1/46

Must Reading (cont)

- Grech et al., “Review on solving the inverse problem in EEG source analysis,” *J. Neuroeng. Rehab.*, open access, available at http://www.jneuroengrehab.com/content/5/1/25

THANK YOU

This presentation is available at http://www.isibang.ac.in/~kaushik