Cooperative Spectrum Sensing with an Improved Energy Detector in Cognitive Radio Network

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Abstract—In this paper, performance of cooperative spectrum sensing with an improved energy detector is discussed. The Cognitive radios (CRs) utilize an improved energy detector for taking a decision of the presence of the primary user (PU). The improved energy detector uses an arbitrary positive power of the amplitude of the received samples of the primary user’s signals. The decision of each CR is orthogonally forwarded to a fusion center (FC), which takes final decision of the presence of the PU. We minimize sum of the probability of false alarm and missed detection in cooperative spectrum sensing to obtain an optimized number of CRs for detecting a spectrum hole. An optimized value of $p$ and sensing threshold of each CR is also obtained by minimizing the total probability of error.

I. INTRODUCTION

Detecting an unused spectrum and sharing it without interfering the primary users (PUs) is a key requirement of the cognitive radio network. For detection of a spectrum hole, the cognitive communication network relies upon the cognitive radios (CRs) or secondary users [1]. Spectrum sensing is a procedure in which a CR captures the information of a band available for transmission and then shares the frequency band without interfering the primary users. Cooperation among the CRs provides improvement in detection of the primary signals [2]. Due to the random nature of the wireless channel, the signals sensed by the CRs are also random in nature. Therefore, sometimes a CR can falsely decide the presence or absence of the PU [3]. In [3], a cooperation based spectrum sensing scheme is proposed to detect the PU with an optimal linear combination of the received energies from the cooperating CRs in a fusion center (FC). The cooperative spectrum sensing scheme provides better immunity to fading, noise uncertainty, and shadowing as compared to a cognitive spectrum sensing scheme provides better immunity to fading, cooperation among the CRs in a fusion center (FC), which takes final decision of the presence of the PU. We minimize sum of the probability of false alarm and missed detection in cooperative spectrum sensing to obtain an optimized number of CRs for detecting a spectrum hole. An optimized value of $p$ and sensing threshold of each CR is also obtained by minimizing the total probability of error.

II. SYSTEM MODEL

We consider a system consisting of $N$ number of CRs, one primary user, and a fusion center. There are two hypothesis $H_0$ and $H_1$ in the $i$-th CR for the detection of the spectrum hole

$$H_0 : y_i(t) = v_i(t), \quad \text{if PU is absent}$$

$$H_1 : y_i(t) = s(t) + v_i(t), \quad \text{if PU is present}$$

where $i = 1, 2, ..., N$, $s(t) \sim \mathcal{N}(0, \sigma_s^2)$, where $\sigma_s^2$ is the average transmitted power of the PU, denotes a zero mean Gaussian signal transmitted by the PU, and $v_i(t) \sim \mathcal{N}(0, \sigma_n^2)$ is additive white Gaussian noise (AWGN) with zero mean and $\sigma_n^2$ variance. The variance of the signal received at each secondary user under $H_1$ will be $\sigma_s^2 + \sigma_n^2$. It is assumed that each CR contains an improved energy detector [8]. The $i$-th CR utilizes the following statistic for deciding of the presence of the PU [8]

$$W = |y_i|_p, \quad p > 0.$$  

It can be seen from (3) that for $p = 2$, $W$ reduces to statistics corresponding to the conventional energy detector [7]. In a cooperative sensing scheme, multiple CRs exists in a cognitive radio network such that each CR makes independent decision regarding the presence or absence of PU. We consider a...
cooperative scheme in which each secondary user sends its binary decision \(d_i\) (0 or 1) to the fusion center by using an improved energy detector over error free orthogonal channels. The fusion center combines these binary decisions to find the presence or absence of the PU as follows:

\[
D = \sum_{i=1}^{N} d_i,
\]

where \(D\) is the sum of the all 1-bit decisions from the CRs. Let \(n, n \leq N\) corresponds to a number of cooperating CRs out of \(N\) CRs. The FC uses a majority rule for deciding the presence or absence of the PU. As per the majority decision rule if \(D\) is greater than \(n\) then hypothesis \(H_1\) holds and if \(D\) is smaller than \(n\) then hypothesis \(H_0\) will be true. The hypothesis \(H_0\) and \(H_1\) can be written as

\[
H_0 : \; D < n, \; \text{if PU is absent},
\]

\[
H_1 : \; D \geq n, \; \text{if PU is present}.
\]

III. PROBABILITY OF FALSE ALARM AND MISSED DETECTION IN THE CRs

The cumulative distribution function (c.d.f.) of the improved energy detector can be written as

\[
\Pr(|y_i|^p \leq y) = \Pr(|y_i| \leq y^{\frac{1}{p}}) = \Pr(-y^{\frac{1}{p}} \leq y_i \leq y^{\frac{1}{p}}) = \Pr(y_i \leq y^{\frac{1}{p}}) - \Pr(y_i < -y^{\frac{1}{p}}),
\]

where \(\Pr(\cdot)\) denotes the probability. For finding the probability density function (p.d.f.) of the decision statistics \(W_i\), we need to differentiate (8) with respect to (w.r.t.) \(y\) to get

\[
f_W(y) = \frac{1}{p} y^{\frac{1-p}{p}} f_{y_i}(y^{\frac{1}{p}}) + \frac{1}{p} y^{\frac{1-p}{p}} f_{y_i}(-y^{\frac{1}{p}}),
\]

where \(f_{y_i}(\cdot)\) is the p.d.f. of received signal at the FC under hypothesis \(H_0\) or \(H_1\) depending on the presence or absence of the primary signal. Let \(f_{y_i|H_0}(\cdot)\) and \(f_{y_i|H_1}(\cdot)\) be the p.d.f.s of the received signal under hypothesis \(H_0\) and \(H_1\), respectively. From (5) and (6), it can be deduced that

\[
f_{y_i|H_0}(x) = \frac{1}{\sqrt{2\pi}\sigma_n} \exp\left(-\frac{x^2}{2\sigma_n^2}\right),
\]

\[
f_{y_i|H_1}(x) = \frac{1}{\sqrt{2\pi}(\sigma_n^2 + \sigma_\nu^2)} \exp\left(-\frac{x^2}{2(\sigma_n^2 + \sigma_\nu^2)}\right).
\]

The p.d.f. of \(W\) under hypothesis \(H_0\) is given from (8) and (9) as follows:

\[
f_{W|H_0}(y) = \frac{\sqrt{2}y^{\frac{1-p}{p}}}{p\sqrt{\pi}\sigma_n} \exp\left(-\frac{y^{\frac{2}{p}}}{2\sigma_n^2}\right).
\]

From (8) and (10), the p.d.f. of \(W\) under hypothesis \(H_1\) can be obtained as

\[
f_{W|H_1}(y) = \frac{\sqrt{2}y^{\frac{1-p}{p}}}{p\sqrt{\pi}(\sigma_n^2 + \sigma_\nu^2)} \exp\left(-\frac{y^{\frac{2}{p}}}{2(\sigma_n^2 + \sigma_\nu^2)}\right).
\]

The probability of false alarm \(P_f\) is defined as [10, Eq. (41), Chapter 2]

\[
P_f \triangleq \int_{Z_1} f_{W|H_0}(y) dy,
\]

where \(Z_1\) is the decision region corresponding to hypothesis \(H_1\). Let us assume that the decision threshold in each CR is given as \(\lambda\). The probability of false alarm \(P_f\) in each CR is derived in Appendix I as

\[
P_f = \frac{1}{\sqrt{\pi}} \Gamma\left(\frac{3}{2}, \frac{\lambda^2}{2}\right),
\]

where \(\Gamma(a)\) is the gamma function [9, Eq. (6.1.1)] and \(\Gamma(a, x)\) is the incomplete gamma function \(\Gamma(a, x) = \int_0^x t^{a-1} \exp(-t) dt\) [9, Eq. (6.5.3)]. The probability of miss \(P_m\) is defined by [10, Eq. (41), Chapter 2]

\[
P_m \triangleq \int_{Z_0} f_{W|H_1}(y) dy,
\]

where \(Z_0\) is the decision region corresponding to the hypothesis \(H_0\). In Appendix I, it is shown that the probability of miss \(P_m\) in each CR will be

\[
P_m = \frac{1}{\sqrt{\pi}} \gamma\left(\frac{3}{2}, \frac{\lambda^2}{2(\sigma_n^2 + \sigma_\nu^2)}\right),
\]

where \(\gamma(a, x)\) is the incomplete gamma function \(\gamma(a, x) = \int_0^x t^{a-1} \exp(-t) dt\) [9, Eq. (6.5.2)].

IV. OPTIMIZATION OF COOPERATIVE SPECTRUM SENSING SCHEME

The probability of false alarm \(P_f\) of the FC for cooperative sensing will be

\[
P_f = \Pr(H_1|H_0),
\]

From (5), (6), and (17), the probability of false alarm of the FC can be written as

\[
P_f = \sum_{l=n}^{N} \left(\begin{array}{c} N \\ l \end{array}\right) P_f^l (1 - P_f)^{N-l}.
\]

In the FC, the probability of missed detection \(P_M\) will be

\[
P_M = \Pr(H_0|H_1).\]

The probability of missed detection of the FC can be obtained from (5), (6), and (19) as follows:

\[
P_M = 1 - \sum_{i=n}^{N} \left(\begin{array}{c} N \\ l \end{array}\right) (1 - P_m)^l (P_m)^{N-l}.
\]

Let us define \(Z\), which denotes the total error rate in the cooperative scheme as

\[
Z \triangleq P_F + P_M
\]

\[
= 1 + \sum_{l=n}^{N} \left(\begin{array}{c} N \\ l \end{array}\right) P_f^l (1 - P_f)^{N-l} - \sum_{i=n}^{N} \left(\begin{array}{c} N \\ l \end{array}\right) (1 - P_m)^l (P_m)^{N-l}.
\]
It can be seen from (21) that \( Z \) denotes the total error rate in the case when \( P_Z \) and \( P_M \) are equiprobable.

**A. Optimization of Number of Cooperating CRs**

In a cognitive network with very large number of cooperative CRs, large delays are incurred in deciding the presence of the spectrum hole. Therefore, it is expedient to find an optimized number of CRs which significantly contribute in deciding the presence of the PU.

**Theorem 1:** Optimal number of CRs \((n^*)\) for a fixed value of the sensing threshold \( \lambda \), the positive power \( p \) of the amplitude of received sample of PU, and the signal-to-noise ratio \( \text{SNR} \) between the PU-CR link will be

\[
(n^*) = \min \left( N, \left[ \frac{N}{1 + \alpha} \right] \right),
\]

where

\[
\alpha = \frac{\ln \frac{1}{\lambda^* \Gamma \left( \frac{3}{2}, \frac{\lambda^*}{2(\sigma_f^2 + \sigma_s^2)} \right)}}{\ln \frac{1}{\lambda^* \Gamma \left( \frac{3}{2}, \frac{\lambda^*}{2(\sigma_f^2 + \sigma_s^2)} \right)} - 1},
\]

and \( \left[ . \right] \) denotes the ceiling function.

**Proof:** In order to obtain an optimized number of the CRs, we need to differentiate (21) w.r.t. \( n \) and set the result equal to zero, to get

\[
\left( \begin{array}{c} N \\ l \end{array} \right) [P_m^{N-l}(1 - P_m)^{n-P_f^n(1 - P_f)^{N-n}}] = 0. \]  

(24)

After some algebra and with the help of results given in [6] we get (22) from (24).

**B. Optimization of the Improved Energy Detector**

In order to get an optimized value of \( p \) we need to differentiate (21) w.r.t. \( p \) keeping \( n, \lambda \), and \( \text{SNR} \) fixed and set the result to zero. After some algebra we get

\[
\frac{\partial Z}{\partial p} = -\sum_{l=n}^N \left( \frac{N}{l} \right) (N - l) P_f^l (1 - P_f)^{N-l-1} \frac{\partial P_f}{\partial p} + \sum_{l=n}^N \left( \frac{N}{l} \right) l P_f^l (1 - P_f)^{N-l} \frac{\partial P_f}{\partial p} - \sum_{l=n}^N \left( \frac{N}{l} \right) (N - l) P_m^{N-l-1} (1 - P_m) \frac{\partial P_m}{\partial p} + \sum_{l=n}^N \left( \frac{N}{l} \right) l P_m^{N-l} (1 - P_m)^{l-1} \frac{\partial P_m}{\partial p},
\]

(25)

where \( \frac{\partial P_f}{\partial p} \) and \( \frac{\partial P_m}{\partial p} \) are given as

\[
\frac{\partial P_f}{\partial p} = \frac{\lambda^2 \log \lambda \exp(-\frac{\lambda^2}{2\sigma_f^2})}{\sqrt{2\pi}p^2}\sigma_n^2,
\]

(26)

\[
\frac{\partial P_m}{\partial p} = -\frac{\lambda^2 \log \lambda \exp(-\frac{\lambda^2}{2(\sigma_f^2 + \sigma_s^2)})}{\sqrt{2\pi}p^2(\sigma_n^2 + \sigma_s^2)^2}.
\]

(27)

It is difficult to obtain a closed form solution of \( p \), however, we can obtain an optimized value of \( p \) numerically by using (25).

**C. Optimization of Sensing Threshold**

An optimal value of \( \lambda \) for given \( n, p \), and \( \text{SNR} \) is obtained by differentiating (21) w.r.t. \( \lambda \) and setting the derivative equal...
to zero. After some algebra we get

\[
\frac{\partial Z}{\partial \lambda} = - \sum_{l=0}^{N} \binom{N}{l} (N-l) P_f^l (1-P_f)^{N-l-1} \frac{\partial P_f}{\partial \lambda} + \sum_{l=0}^{N} \binom{N}{l} l P_f^l (1-P_f)^{N-l} \frac{\partial P_f}{\partial \lambda} - \sum_{l=0}^{N} \binom{N}{l} (N-l) P_m^{N-l-1} (1-P_m) \frac{\partial P_m}{\partial \lambda} + \sum_{l=0}^{N} \binom{N}{l} l P_m^{N-l} (1-P_m)^{l-1} \frac{\partial P_m}{\partial \lambda},
\]

(28)

where \(\frac{\partial P_f}{\partial \lambda}\) and \(\frac{\partial P_m}{\partial \lambda}\) can be obtained as follow:

\[
\frac{\partial P_f}{\partial \lambda} = - \frac{\lambda^{\frac{2}{p}} - 1}{\sqrt{2\pi \sigma^2_n}} \exp\left(-\frac{\lambda^{\frac{2}{p}}}{2\sigma^2_n}\right),
\]

(29)

\[
\frac{\partial P_m}{\partial \lambda} = - \frac{\lambda^{\frac{2}{p}} - 1}{\sqrt{2\pi \rho (\sigma^2_n + \sigma^2_m)}} \exp\left(-\frac{\lambda^{\frac{2}{p}}}{2(\sigma^2_n + \sigma^2_m)}\right).
\]

(30)

The optimized value of \(\lambda\) can be found numerically from (28).

### D. Probability of Detection of Spectrum Hole

Probability of detection of spectrum hole \(P_D\) in the FC can be obtain from (20) as follows:

\[
P_D = 1 - P_M = \sum_{l=0}^{N} \binom{N}{l} \left[1 - \frac{1}{\sqrt{\pi}} \gamma \left(\frac{3}{2} \frac{\lambda^{\frac{2}{p}}}{2(\sigma^2_n + \sigma^2_m)}\right)\right]^l \times \left[\frac{1}{\sqrt{\pi}} \gamma \left(\frac{3}{2} \frac{\lambda^{\frac{2}{p}}}{2(\sigma^2_n + \sigma^2_m)}\right)\right]^{N-l}.
\]

(31)

### V. Numerical Results

In Fig. 1, we have plotted the total error rate versus \(p\) plots for different number of cooperative CRs \(n = 1, 2, 3, 4, 5, 6, \lambda = 10, \) and \(\text{SNR}=10\) dB. It can be seen from Fig. 1 that the total error rate is a convex function of \(n, \lambda, \) and SNR. It can be seen from Fig. 1 that for \(\lambda = 10\) and \(\text{SNR}=10\) dB, the total error rate is minimum for \(n = 2\).

It implies that for \(\lambda = 10\) and \(\text{SNR}=10\) dB only two CRs are required for deciding the presence of the PU by using an improved energy detector. We have shown plots of the optimal number of CRs required for cooperation versus \(p\) for \(\lambda = 10, 20, 50, \) and \(\text{SNR}=10\) dB in Fig. 2. It can be seen from Fig. 2 that the optimal number of the cooperative CRs varies with the value of \(p\) and \(\lambda\) at \(\text{SNR}=10\) dB. For example, for \(p = 3\) the optimal number of the CRs is 2, 3, and 4 for \(\lambda = 50, 20,\) and 10, respectively. Fig. 3 shows that the total error rate versus SNR plot with \(p = 1.75, 2, 2.25, 2.50, \) \(n = 1, \) and \(\lambda = 50.\) It can be seen from Fig. 3 that the total error rate decreases by increasing the SNR and the value of \(p.\) Further, it can be seen from Fig. 3 that the improved energy detector with \(p = 2.50\) significantly outperforms the conventional energy detector, i.e., when \(p = 2.\) For \(p = 1.75, 2, 2.25, 2.50, \) \(n = 1, \) and \(\lambda = 50,\) the probability of detection of a spectrum hole versus SNR curve is plotted in Fig. 4. It is shown in Fig. 4 that the probability of detection of a spectrum hole increases with increase in the value of \(p.\)

Fig. 5 illustrates that for a fixed range of \(\lambda\) and \(p\) there exists an optimal number of CRs which minimizes the total error rate. In Fig. 6, we have plotted the total error rate versus \(p\) and \(\lambda\) plots for different number of cooperative CRs at 10 dB SNR. It can be seen from Fig. 6 that there exists an optimal value of \(p\) and \(\lambda\) for which the total error rate is minimized for a fixed number of CRs. The plots of Figs. 5 and 6 suggest that in order to optimize the overall performance of the cooperative cognitive system utilizing the improved energy detector, we need to jointly optimize the number of cooperative CRs, the energy detector, and the sensing threshold in the CR.
VI. CONCLUSIONS

We have discussed how to choose an optimal number of cooperating CRs in order to minimize the total error rate of a cooperative cognitive communication system by utilizing an improved energy detector. It is shown that the performance of the cooperative spectrum sensing scheme depends upon the choice of the power of the absolute value of the received data sample and the sensing threshold of the cooperating CRs as well. We have obtained an analytical expression of the optimal number of CRs required for taking the decision of the presence of the primary user when the CRs utilize the improved energy detector. It is shown by simulations that an optimal value of the total error rate can be obtained by utilizing a joint optimization of the number of cooperating CRs, threshold in each CR, and the energy detector.

APPENDIX I

PROOF OF (14) AND (16)

From (11) and (13), the probability of false alarm \( P_f \) in each CR can be obtained as

\[
P_f = \int_{\lambda}^{\infty} f_{W/H_0}(y) dy
= \int_{\lambda}^{\infty} \frac{\sqrt{2} y^{-\frac{3}{2}}}{p \sqrt{\pi} \sigma_n} \exp\left(-\frac{y^2}{2 \sigma_n^2}\right) dy.
\]

After applying some algebra in (32) we get

\[
P_f = \frac{1}{\sqrt{\pi}} \int_{\lambda}^{\infty} \frac{\sqrt{2}}{p \sqrt{\pi} \sigma_n} \exp\left(-\frac{y^2}{2 \sigma_n^2}\right) dy.
\]

By comparing (33) with the expression of the incomplete gamma function [9, Eq. (6.5.3)] we get (14).

From (12) and (15), the probability of miss \( P_m \) in each CR will be

\[
P_m = \int_{\lambda}^{\infty} f_{W/H_1}(y) dy
= \int_{\lambda}^{\infty} \frac{\sqrt{2} y^{-\frac{3}{2}}}{p \sqrt{\pi} \sqrt{\left(\sigma_n^2 + \sigma_s^2\right)}} \exp\left(-\frac{y^2}{2 \left(\sigma_n^2 + \sigma_s^2\right)}\right) dy.
\]

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