Impact of Topology on the Performance of Communication Networks

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***Abstract*—This paper investigates the implications of topologies on the average path length of networks, which is the dominant parameter affecting performance. Based on the available literature, classic network topologies are reviewed and analyzed. Furthermore, a new class of communication networks is introduced, and a topology design algorithm is proposed to improve network performance in terms of average path length.**

I. INTRODUCTION

In recent work [1-3], the impact of the number of hops between the source and the destination on delay and throughput performance of multi-hop communication networks has been investigated. Through analytical and simulation results, the papers show how the number of hops between the source and the destination node affects the throughput of these networks. It is concluded that the negative impact of transiting messages through a large number of hops between the source and the destination node is compelling.

This paper explores the implications of network topologies on performance. The shortest path length equals the least number of hops (or links) between the source and the destination nodes. The average path length (APL) is the average number of hops along the shortest paths for all possible pairs of network nodes. In the following, we briefly discuss how path length (or hop count) impacts the performance of both circuit-switched and packet-switched networks.

A. Circuit-switched networks

In circuit-switched networks, a busy condition of any link in the end-to-end path results in the call being lost. Under heavy load situations, the number of reattempts might result in even lower fraction of the calls materializing resulting in lower throughput [3]. It has been shown that as the incident traffic increases, the carried traffic in a multi-hop circuit switched network reaches a peak and then drops to zero, while the carried single hop traffic goes to an asymptotic limit. Furthermore, when the incident traffic intensity of the network is arbitrarily high, the network can only carry single hop traffic. The reduction in carried traffic or the network throughput as the number of hops increases has been fully captured in the analysis.

B. Packet-switched networks

In packet-switched networks, increasing traffic might result in the end-to-end delay becoming unacceptably high. If delays were bounded to a specified maximum and traffic that exceed this maximum were treated as lost, the actual throughput could be evaluated as a function of the number of hops. This problem has been addressed in [1] where the authors have shown that the throughput declines rather sharply as the number of hops increases. Results presented in [1-3] provide ample motivation for studying how different network topologies would affect network performance.

II. REVIEW OF VARIOUS NETWORK TOPOLOGIES AND ANALYSIS OF THEIR PERFORMANCE

The study of network topologies falls in the area of graph theory. A network is represented as a graph G(V, E), where V is the set of N nodes (or vertices), E is the set of links (or edges). We classify communication network topologies as conventional network topologies and complex network topologies as discussed below.

A. Conventional Network Configurations

Examples of well known conventional network configurations include star, ring, and tree networks. For any networks of N nodes, there are N*(N-1)/2 different pairs of nodes. The degree of a given node is defined as the total number of links that connect that node to other nodes.

For a ring network of N nodes, it can be shown that when N is an odd number, the APL is (N+1)/4; when N is an even number, the APL is $N^2/(4(N-1))$.

When N =7, the APL of a ring network is 2, and it increases with N. Since the APL of a star network is always below 2, when $N \geq 7$, the APL of a star network is always less than that of a ring network of the same size. In addition, in a star topology, all nodes except the central node have the lowest degree 1; under random removal of node or link, a low degree node has high probability of being removed. Therefore it shows high resilience under random link or node failure; however the network is totally disconnected under failure (or targeted removal) of the central node. On the contrary, all nodes or links have the same impact on a ring network structure under node or link failure.

For a tree network, no generalized formula has been found for the APL. References [4, 5] have studied connected trees of size N with a power law degree distribution (explained next). It has been found that the APL scales as $N^{(\alpha-2)/(\alpha-1)}$, where α is the degree exponent (explained in Section II. *B*). The failure or removal of end nodes (or leaves) won't alter the remaining structure of a tree network. However tree networks are vulnerable under the failure of nodes with high degrees.

B. Complex Networks

Complex networks are classified as three basic types: random networks, small-world networks, and scale-free networks [6]. Even though complex networks are constructed under different organizing principles, three basic concepts are defined to describe their generic properties: small-world, clustering, and degree distribution properties.

First, the small-world concept describes the fact that despite the large size of most complex networks, the APL between any two nodes is relatively short. For example, authors in [7] provide a formula relating the APL between any two web pages to the logarithm of the size of the World-Wide Web. The short path length of the web shows that it is just a few clicks away to find any information on the web.

Second, the clustering property of a complex network is quantified by the clustering coefficient. In a complex network, for a selected node *i* of degree k_i, if all the nearest neighboring k_i nodes of node *i* are considered as a small set, there would be $k_i(k_i-1)/2$ edges if these nodes are fully connected. Suppose the actual number of edges among them is E_i, then the clustering coefficient of node *i* is defined as $C_i = \frac{2E_i}{k_i(k_i-1)}$. The clustering coefficient of the whole network is the average of all individual clustering coefficients. Clustering coefficient shows the local connectivity feature of a network.

The third property, degree distribution, has become very important in the research on complex networks since the discovery of a power law degree distribution of many real-life networks such as the Internet. A power law degree distribution means that the number $P(k)$ of nodes with degree k follows a power law $P(k) \propto k^{-\alpha}$, where α, the degree exponent is a constant for the network. In this paper, we consider Poisson degree distribution and power law degree distribution.

A few recent investigations on complex networks can be found in [8-19]. In this section, we are going to review three classic models to build complex networks, and analyze how complex networks formed on different construction principles perform, based on the APL.

1) Random Networks

The classic random network model was proposed by Erdos and Renyi in 1959 [20]. In a graph constructed with the Erdős-Rényi (ER) model, each edge is considered to be present with independent probability *p*. If there are *N* vertices in the graph, and each is connected to an average of *z* edges, then $p=z/(N-1)$, which for large *N* is usually approximated by z/N. The probability p_k that a node has degree *k* is given by

$$p_k = \binom{N-1}{k} p^k (1-p)^{N-1-k} \cong \frac{z^k e^{-z}}{k!},$$

where the second equality shows that p_k follows a Poisson distribution in the limit of large *N* [12].

In a random graph, it is found that APL scales with the logarithm of network size *N* [6]; therefore, random networks have small-world property. It is discussed in [6] that for a sufficiently high *p*, if $pN/\ln(N) \to \infty$, the constructed random graph is homogeneous, and the majority of the nodes have approximately the same number of edges.

Random networks with power law degree distributions are called scale-free random networks. They are constructed under a power law degree constraint, but are random in all the other aspects. Reference [6] shows that the APLs of scale-free random networks are smaller than those of random networks of the same size. The reason is that in a scale-free random network, many nodes have low degree, but a few nodes could have very high degree because of a power law degree distribution. These few high degree nodes are able to provide shortcuts for communication among many low degree nodes. These shortcuts help decrease the APL. This is quite different from the case of random networks with Poisson degree distribution, in which the majority of nodes have approximately the same number of edges.

Scale-free random networks are more resilient to random node or link failures because many nodes have low degree and the removal of low degree nodes does not significantly alter the structure of the remaining nodes. However, under the failure of high degree nodes, network performance would be greatly degraded; it is not so for random networks of Poisson degree distribution.

2) Small-world Networks

Random networks have small-world property, but don't have the high clustering property. To construct networks that not only have small-world property, but also high clustering property, a classic small-world network model (the Watts-Strogatz (WS) model [21]) was proposed in 1998. A WS network is constructed as follows.

1. Start with a ring lattice of N vertices, each node connects to its k nearest neighbors (or k/2 on either side).
2. Randomize: Choose a vertex and the edge that connects it to its nearest neighbor in a clockwise sense. Reconnect this edge with probability *p* to a vertex randomly chosen over the entire ring, with duplicate edges forbidden; otherwise leave the edge in place.
3. Repeat step 2. Randomly rewire each of the edges with probability *p* until all edges have been considered.

Figure 1 illustrates the random rewiring procedure on a ring lattice.

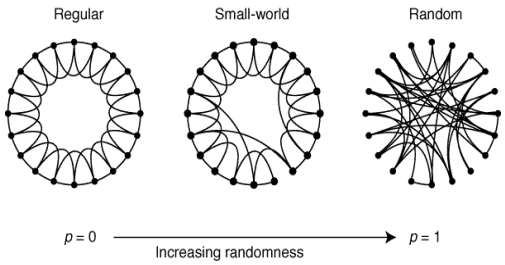

Figure 1. Random Rewiring Process of a Ring Lattice [21]

For a WS network, the average path length $L(p)$ is a function of probability p. It is observed that when p =0, $L(0)$ scales linearly with network size N, and the clustering coefficient is high; when $p \to 1$, $L(p)$ scales logarithmically with N, and the clustering coefficient decreases with N; however, for small values of p, there is a rapid drop of the APL; the graph not only has small APL, but also large clustering coefficient, which implies that at local level, the transition to a small world is almost undetectable. The reason behind the immediate drop in the APL is the introduction of a few long edges (or shortcuts) created by the random rewiring process.

Compared to random networks of the same size, small-world networks do not have shorter APL; however, these networks have better local connectivity feature because of their large clustering property. They are also homogeneous in that all nodes have approximately the same number of links connecting them.

3) Scale-free Networks

The power law degree distribution feature was first found through the observation of the Internet in 1999 [22]. These networks are called power law (or scale-free) networks. The value of the degree exponent α for the Internet is about 2.1 [23].

The classic model to build scale-free networks was proposed by Albert-László Barabási and Réca Albert in 1999 [24] . It is also called the Barabási-Albert (BA) model. A BA network is constructed as follows.

1. Growth: Start with a small set of m_0 vertices, at each time step, add a new node with m ($\leq m_0$) edges connecting it to m different nodes in the set.
2. Preferential attachment: The probability p that the new node will be connected to node i depends on the degree k_i of node i, such that

$$p(k_i) = \frac{k_i}{\sum_j k_j} \quad .$$

After t time steps, this procedure results in a network with $N = t + m_0$ nodes and mt edges. The degree distribution of the network follows a power law with an exponent of 3. Reference [25] has proved that the APL of a BA network approaches $\log(N)/\log(\log(N))$ regardless of the value m. It is shown in [6] that the APL of a BA network is smaller than that of a random network for any N. The reason lies in both the dynamic process in BA model that generates scale-free networks, and a power law degree distribution of these networks. Since most nodes of scale-free networks are low degree nodes, the network is resilient under random node or link failure.

III. DEGREE-CONSTRAINED NETWORKS

Unlike complex networks, the formation of degree-constrained networks is under a nodal degree constraint. The degree constraint comes from a limitation on the number of links that are incident at a node or a limitation on the number of interfaces of a node at physical layer. For example, the nodal degree constraint for a free-space optical (FSO) network comes from a limitation on the number of transceivers that can be carried by each node due to size, power, and weight issues. Similarly, in wireless wide area networks (WWANs), wireless routers can be equipped with directional antennas in order to reduce interference; however the number of antennas that can be installed on each router is limited because of cost. Much research has been going on to improve the performance of degree-constrained communication networks [26-32]. We focus on improving network performance in terms of APL through topology design. The inspiration for this work is from the research on small-world networks [21]. The objective is to achieve shorter APL through topology design without changing the local connectivity feature of a degree-constrained network.

The work relates to the authors' previous research on FSO networks [33] which aims at increasing network reliability through topology design. In [33], we have proposed a network topology design (NTD) algorithm that constructs network topologies of high reliability compared to other network configurations. Here we propose a new degree-constrained topology design (DCTD) algorithm based on modifications of the NTD. In DCTD, a random edge adding process is added in order to improve the average path length performance of degree-constrained networks.

Given a set of randomly distributed network nodes, suppose all nodes are identical with degree constraint Δ. The DCTD algorithm operates as follows.

Step 1: Construct the Delaunay Triangulation (DT) of a given set of nodes

Step 2: Add each unused edge in the graph with probability p.

Step 3: For nodes with degrees higher than Δ in the graph, remove the longer edge of two adjacent edges that form the minimum angle. Repeat until the degree constraint is met.

Step 4: For nodes with degree less than Δ, edges are added in non-descending order of their lengths without violating the degree constraint at any node.

DT is a well-connected graph with most connections being local connections along with a high angular separation between adjacent links. The random edge adding process is run at step 2. It is much simpler compared with the random rewiring process in the classic WS model. The DCTD

algorithm then deletes the longer edge of two adjacent edges that form the minimum angle at each node violating degree constraint. At last, without violating degree constraint, unused edges are added in non-decreasing order of edge length in order to increase network redundancy.

In order to evaluate network performance in terms of APL, three different topologies are constructed. All simulation programs are written in C language. For example, given a set of 100 randomly distributed nodes with degree constraint of 6, Fig. 2 shows the topology constructed with the DCTD algorithm when $p = 0.01$.

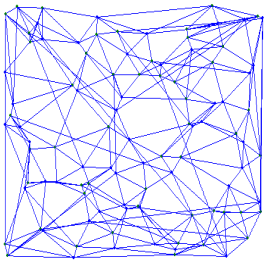

Figure 2. Topology constructed with the DCTD algorithm

For the same set of nodes, Fig. 3 shows the topology constructed with the NTD algorithm. Fig. 4 shows the random graph constructed with the ER model over the same set of nodes, and having the same number of links.

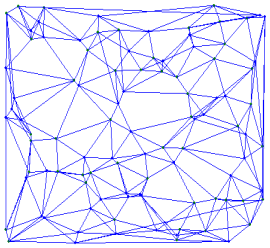

Figure 3. Topology constructed with the NTD algorithm

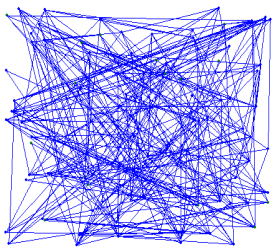

Figure 4. Random graph constructed with the ER model

Comparing the above three graphs, we find that the APL of the random graph is 2.6, which is the shortest; however, it is clearly observed that the random graph is in complete disorder with almost all links being long edges. The APL of the NTD graph is 4.11. Even though the NTD graph has longer APL, it shows nice local connectivity feature in that most links (or edges) are short edges connecting close neighboring nodes. Compared to the NTD graph, only several long edges are added in the DCTD graph, therefore the local connectivity feature of the graph remains unchanged; The APL of the DCTD graph is 3.4, which is about 17% decrease in APL.

In order to know how the random edge adding process affects the average path length $L(p)$ of a degree-constrained network, we have done simulations over the same set of nodes by changing the probability p. The simulation result is shown in Fig. 5.

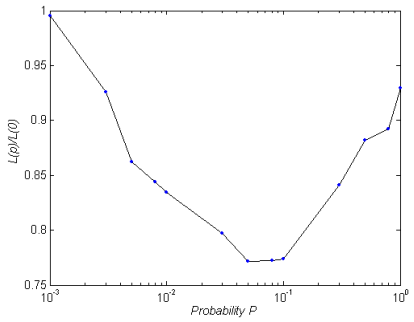

Figure 5. $L(p)/L(0)$ vs. probability p

Fig. 5 shows that for a range of small values of p, the APL decreases rapidly, similar to the work in [21]. However, after reaching a minimum value, $L(p)$ increases back as p becomes larger, which is quite different from what is observed in [21]. The reason is that for small values of p, a small number of long edges are added randomly at step 2. After steps 3 and 4, only a few long edges remain in the graph; however, these edges acting as shortcuts are sufficient to help decrease the average path length. After L(p) reaching a minimum value, as p continues to increase, even though more long edges are added randomly, more of them have to be removed from the graph at step 3 in order to satisfy the nodal degree constraint and keep most local connections unchanged. This leads to the back-increasing of the average path length.

We have also done simulations over ten different sets of nodes, each set is of 100 nodes, and the nodal degree is 6.The probability p is set as 0.05. Table I shows the simulation results.

TABLE I. COMPARISON OF PATH LENGTH

	NTD		DCTD	
topology	max. path	APL	max. path	APL
1	8	4.113	6	3.266
2	8	4.157	6	3.247
3	8	3.977	6	3.296
4	8	4.049	6	3.232
5	8	4.087	6	3.273
6	9	4.129	6	3.287
7	8	4.11	5	3.181
8	8	4.043	6	3.315
9	8	4.035	6	3.265
10	8	4.182	6	3.434

Table I shows that networks constructed with the DCTD algorithm perform better than those constructed with the NTD

algorithm in terms of both maximum path length and APL. During the construction of a graph with the DCTD algorithm, only a small number of edges are added randomly; however, these few randomly added edges are sufficient to help decrease the maximum and the average path length. In addition, each node has approximately the same degree in the constructed graph; therefore no hot spots (nodes or links) of heavy traffic are present as in those networks with a power law degree distribution. The network is resilient to node or link failures, especially targeted removal of nodes or links.

Table II shows how the APL changes with the increase of network size N. Simulations are done over randomly distributed nodal sets of degree 5. The probability p is set as 0.01.

TABLE II.

N	50	100	150	200	250	300	350	400
APL	3.09	4.15	4.62	5.188	5.69	5.668	5.821	6.09

We observe that the APL increases quickly with network size N when N is small, and increases slowly when N becomes larger.

IV. CONCLUSIONS

This paper has studied different network topologies and modeling approaches. Using average path length as the main measure of network performance, it has investigated on how network topologies impact network performance. It has also studied how the shortcuts (high degree nodes or long edges) help decrease the average path length of a network. Further, the paper has introduced a new class of communication networks, and proposed a topology design algorithm to improve network performance through adding shortcuts without changing the local high connectivity of a network.

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