Abstract—Significant throughput gains can be achieved in a multi-user MIMO (MU-MIMO) wireless system by exploiting the combination of multi-user scheduling and multi-user diversity. Open-loop MU-MIMO (OL-MU-MIMO) is a codebook based precoding technique where precoders are fixed a priori at the base station (BS) in a known fashion and the user needs to feedback which precoding vector is to be chosen referred to as precoding vector index (PVI) or stream indicator. This scheme feedbacks the channel quality indicator (CQI) which is used by the BS for allocation of modulation and coding scheme (MCS) to the scheduled users. As the precoders used at all the base stations are known a priori, estimation of co-channel interference (CCI) is accurate and there is negligible mismatch between the CQI fed back by the user in present frame and SINR experienced by the user during next frame for low Doppler, resulting in stable CQI modeling. In this paper, an extensive study is made on OL-MU-MIMO and open loop single-user MIMO (OL-SU-MIMO), with an emphasis on how OL-MU-MIMO exploits multi-user diversity to achieve high spectral efficiencies. We also derive the SINR and CQI expressions for such MU-MIMO systems, and provide simulation results which indicate that OL-MU-MIMO outperforms OL-SU-MIMO only when there are large number of users in the system.

Index Terms—Multi-user MIMO, Open Loop, CQI, Multi-user diversity, Codebook precoding, IEEE 802.16m.

I. INTRODUCTION

Multi-user MIMO is a technique, where the BS schedules multiple users to use the same time-frequency resources. In MU-MIMO, the additional spatial degrees of freedom are shared between multiple users, and individual user throughputs increase due to the fact that users gets scheduled more often to reuse the same time-frequency resources, without consuming extra bandwidth or power. Hence, unlike traditional schemes that rely on good channel conditions for separation of data streams, MU-MIMO system exploits the multi-user diversity [3] and schedules a set of users such that each user causes minimum amount of interference to the remaining set of users.

MU-MIMO precoding schemes will offer tremendous advantages if channel state information (CSI) is available at the transmitter. But it is huge amount of feedback to send complete channel. There are many limited feedback systems proposed in the literature [4][5][6]. Codebook based precoding techniques are also proposed to reduce the amount of feedback involved. The codebook consists of a set of precoding matrices each comprising of one or more precoding vectors depending on the number of streams allocated to the user. These codebooks are pre-designed theoretically based on the sounding criterion [7][8] and is known both at the transmitter and receiver. In a linear precoding system, the transmitted data vector is pre-multiplied by a precoding matrix or precoder for simple.

Based on the feedback mechanism involved and construction of precoders at the BS, MU-MIMO system can be classified into 1) Open-Loop MU-MIMO and 2) Closed-Loop MU-MIMO. OL-MU-MIMO is a codebook based precoding technique in which precoders are fixed a priori at all the BSs and is known to all the users in the system. Precoder is formed by choosing a set of unitary precoding vectors from the codebook. In a closed loop system, the precoders are not fixed and they are formed based on the feedback from users.

In this paper, we focus on OL-MU-MIMO and OL-SU-MIMO as prescribed in the IEEE 802.16m WMAN standard [1][2]. In OL-MU-MIMO, each user feedbacks a) PVI and b) CQI for every subband which is used by the scheduler in the subsequent frame. PVI is used by the BS to decide which of the precoding vectors is to be used for the user to precode its data. CQI is used for link adaptation where the BS varies the MCS allocated to a user to suit its channel conditions. In OL-SU-MIMO, only one user with single stream is scheduled per resource block where single precoding vector is used at the BS and hence, each user feedbacks only CQI. CQI is an estimate of the SINR a user is likely to experience in the next frame. Using channel estimates made through dedicated pilots, a user can estimate its CCI levels in the current frame. Since all the BSs are using same set of precoders which are fixed a priori, interference from the neighboring BSs to a particular user can be estimated accurately even when the precoder is not unitary. This is the major advantage of open loop technique when compared to the closed loop technique where the accurate CCI estimation is impossible when the precoder is not unitary. Hence, the CQI modeling is more stable and reliable in case of open loop system resulting in optimal MCS assignment.

This paper is organized as follows: Section II introduces the OL-MU-MIMO system, signal model and 802.16m frame structure. Section III describes the MMSE receiver and SINR calculations. Section IV presents CQI and PVI computations, feedback mechanism and proportional fair (PF) scheduling algorithm. In Section V OL-MU-MIMO operation is compared with OL-SU-MIMO, with an emphasis on what happens when there are large number of users per sector. Section VI presents the simulation results and Section VII concludes the paper.

1 This work was done by S. Naga Sekhar Kshatriya when he was with Department of Electrical Engineering, IIT Madras, Chennai-600036, India.
II. SYSTEM MODEL

We consider the down link (DL) of a cellular system with hexagonal cell setup and 3 sectors per cell with frequency reuse one. Let us define $N_t$ and $N_r$ as the number of antennas at the BS and mobile-station (MS) respectively, where all the MSs are assumed to have same number of antennas as shown at the BS and mobile-station (MS) respectively, where all the MSs are assumed to have same number of antennas as shown in Fig. 1. Let $N_u$ be the number of active users per sector waiting for scheduling and $K$ out of $N_u$ users are scheduled per resource block with single stream per user. In general MIMO system with $N_t$ antennas at the BS and $N_r$ antennas at the MS is represented in short as $N_t \times N_r$ system.

![Fig. 1. Open Loop Multi-user MIMO System.](image)

The $N_r \times 1$ received signal vector at the $k$th MS can be expressed as

$$ y_k = H_k W x + \sum_{i=1}^{I} G_{ik} W x_i' + n_k, \forall k = 1, \ldots, K. \quad (1) $$

where $x$ is the data-vector and second term in (1) models the co-channel interference (CCI) assuming each user has strong interference from $I$ neighboring BSs. The $N_r \times N_t$ MIMO channel matrix $H_k$ for $k$th user is

$$ H_k = \begin{bmatrix} h_{11}^k & h_{12}^k & \cdots & h_{1N_t}^k \\ h_{21}^k & h_{22}^k & \cdots & h_{2N_t}^k \\ \vdots & \vdots & \ddots & \vdots \\ h_{N_t1}^k & h_{N_t2}^k & \cdots & h_{N_tN_t}^k \end{bmatrix}_{N_r \times N_t} \quad (2) $$

Here $h_{ij}^k$ represents the complex channel coefficient from $i$th antenna at the $k$th BS to $j$th antenna at the base station. Also, $h_{ij}^k \sim \mathcal{CN}(0, 1)$ and its amplitude is $i.i.d$ Rayleigh distributed.

$$ W = \begin{bmatrix} v_1 & v_2 & \cdots & v_K \end{bmatrix}_{N_r \times K} \quad (3) $$

$$ x = \begin{bmatrix} x_1 & x_2 & \cdots & x_K \end{bmatrix}_{K \times 1}^T \quad (4) $$

$$ \mathbb{E}\{|x_k|^2\} = \sqrt{\frac{P}{K}}, \forall k = 1, \ldots, K. \quad (5) $$

where $P$ is total power constraint at the BS which is divided uniformly among $K$ scheduled users i.e., $\mathbb{E}\{|x_k|^2\} = (\frac{P}{K}) \mathbf{I}_K$ (represents the hermitian operation). $G_{ik}$ is $N_r \times N_t$ channel matrix between $k$th user and $i$th interfering BS, and $x_i'$ is $K \times 1$ data vector transmitted by $i$th interfering BS. $n_k \in \mathcal{CN}$ is i.i.d complex circular symmetric AWGN vector at the $k$th MS and $n_k \sim \mathcal{CN}(0, N_\sigma \mathbf{I}_{N_r})$. $W$ is the precoding matrix or precoder formed by set of precoding vectors $\{v_i\}$ each of size $N_t \times 1$ and $v_i \in \mathcal{C}(N_t, 1, B) \forall i = 1, 2, \ldots, K$. $\mathcal{C}(N_t, 1, B)$ is codebook consisting of $2^B$ complex precoding vectors each of size $N_t \times 1$. A subset of $K$ unitary precoding vectors are chosen from the codebook to form the precoder $W$.

A. IEEE 802.16m Down Link Frame Structure

Let us consider an OFDM system with 10 MHz bandwidth and 1024 subcarriers. Though there are 1024 subcarriers only 864 of them are used for fruitful traffic while the remaining are used by control channels and other overheads. Fig. 2 shows the down link OFDM frame structure where the $(i, j)$th entry represents the modulated and coded symbol carried by the $i$th subcarrier in the $j$th OFDM symbol duration.

![Fig. 2. IEEE 802.16m Down Link Frame Structure.](image)

The time duration of the DL frame is 5 ms and it is divided into 5 subframes each of duration 1 ms and 6 OFDM symbols. The smallest time-frequency resource block is a physical resource unit (PRU). A PRU consists of a set of 18 contiguous subcarriers allocated for a duration of 6 OFDM symbols. A data-region is formed by 4 adjacent PRUs along frequency in a subframe. The OFDM frame is divided in frequency into a number of subbands, where a subband is defined as a group of 5 adjacent data-regions along time which is equivalent to 20 PRUs. A data-region is considered as the smallest unit of allocation to any user by the scheduler.

III. MMSE RECEIVER

In general for traditional MIMO detection, a linear receiver is used to detect the transmitted data. Zero forcing (ZF) or minimum mean square error (MMSE) receiver is commonly employed. In order to suppress the interference effectively we used linear MMSE receiver in this paper. The received signal vector at the $k$th MS in (1) can be re-written using (3) and (4) as

$$ y_k = H_k v_k x_k + \sum_{i \neq k, i=1}^{K} H_k v_i x_i + \sum_{i=1}^{I} G_{ik} W x_i' + n_k \quad (6) $$

where $y_k$ is the received signal vector at the $k$th MS, $v_k$ is the precoding vector at the $k$th MS, $x_k$ is the transmitted data vector at the $k$th MS, $H_k$ is the channel matrix between the $k$th MS and the $k$th BS, $G_{ik}$ is the channel matrix between the $i$th interfering BS and the $k$th MS, $v_i$ is the precoding vector at the $i$th interfering BS, $x_i'$ is the transmitted data vector at the $i$th interfering BS, and $n_k$ is the noise vector at the $k$th MS.
where users are aligned so that $v_k$ is the precoding vector used to precode the $k$th user data. The first term in (6) is desired signal, the second term is inter-user interference (IUI) between $K$ scheduled users who uses same time-frequency resources, the third term is co-channel interference (CCI) and the fourth term is circular symmetric complex AWGN vector. The desired and interfering channels $H_k$ and $G_{ik}$, $\forall i = 1, \ldots, I$ and $k = 1, \ldots, K$ are estimated from dedicated pilots using 2D-MMSE estimator. The $1 \times N_b$ linear MMSE filter vector $b_{k,l}$ for $k$th user using $l$th precoding vector is expressed as

$$b_{k,l} = (H_kv_k)^* \left[ H_k^*H_k + \frac{K}{P}K_{CCI_k} + \frac{K}{P}N_oI_N \right]^{-1}$$

(7)

where $H_k = H_kW$ is effective or precoded channel. Desired and interfering data are assumed to be independent which implies $E\{xx^*\} = 0K$, $\forall i = 1, \ldots, I$. Interferers are assumed to be independent i.e., cross-covariance of interfering data vectors is $E\{xx^*\} = \left( \frac{P}{K} \right) I_K$, $\forall i = 1, \ldots, I$. $K_{CCI_k}$ is co-covariance matrix of CCI experienced by $k$th user which is calculated as follows.

$$K_{CCI_k} = E \left\{ \left( \sum_{i=1}^{I} G_{ik}Wx_i^* \right) \left( \sum_{j=1}^{I} G_{jk}Wx_j^* \right)^* \right\}$$

(8)

$$= \sum_{i=1}^{I} \{ (G_{ik}Wx_i^*) (G_{jk}Wx_j^*)^* \}$$

$$= \left( \frac{P}{K} \right) \sum_{i=1}^{I} G_{ik}WW^*G_{ik}^*$$

The estimated symbol of $k$th user after MMSE detection is

$$\hat{x}_k = b_{k,k}y_k$$

$$= b_{k,k}H_kv_k + \sum_{i \neq k}^{K} b_{k,k}H_kv_ix_i$$

$$+ \sum_{i=1}^{I} b_{k,k}G_{ik}Wx_i + b_{k,k}n_k$$

The SINR experienced by the $k$th user can be calculated as

$$SINR_k = \left( \frac{\left( \frac{P}{K} \right) \|b_{k,k}H_kv_k\|^2}{IUI_{k,k} + CCI_{k,k} + N_o\|b_{k,k}\|^2} \right)$$

(10)

$$IUI_{k,l} = \left( \frac{P}{K} \right) \sum_{i,l=1}^{K} \|b_{k,l}H_kv_i\|^2$$

(11)

$$CCI_{k,l} = \sum_{i=1}^{I} E \{ (b_{k,l}G_{ik}Wx_i^*) (b_{k,l}G_{ik}Wx_i^*)^* \}$$

(12)

where $IUI_{k,l}$ and $CCI_{k,l}$ are $IUI$ and $CCI$ experienced by the $k$th user using $l$th precoding vector. As in case of OL-MU-MIMO all the BSs are using same set of precoders which are fixed a priori in a known fashion, CCI estimation is easier and accurate when compared to closed loop system where the precoders are adaptive and it is impossible to estimate the term $WW^*$ in (8) unless the precoder is unitary, which makes the term identity and vanishes from the CCI estimation. Hence stable and reliable CQI modeling is possible in case of OL-MU-MIMO resulting in appropriate MCS selection for the scheduled users based on their CQI feedback.

IV. CQI AND PVI CALCULATION

The SINR experienced by the $k$th user by using $l$th precoding vector to precode its data can be expressed as

$$SINR_{k,l} = \frac{\left( \frac{P}{K} \right) \|b_{k,l}H_kv_k\|^2}{IUI_{k,l} + CCI_{k,l} + N_o\|b_{k,l}\|^2}.$$  

(13)

Now each user has to decide which precoding vector is to be used to precode its data and feedback that index referred to as the precoding vector index (PVI) and CQI calculated with respect to that PVI. CQI and PVI of $k$th user are calculated by maximizing the SINR experienced by the user with respect to the precoding vector used by him as follows.

$$PVI_k = \arg \max_l SINR_{k,l} \ \forall l = 1, 2, \ldots, K.$$  

(14)

$$CQI_k = SINR_{k,PVI_k} \ \forall k = 1, 2, \ldots, N_u.$$  

(15)

A. Feedback Mechanism

Feedback information is sent every frame. Feedback at a subcarrier level is not feasible since it would require a high capacity feedback link. We use subband level feedback, where each user returns 1) PVI and 2) CQI information for every subband. This PVI selection is made such that it represents the best choice of precoding vector from the precoder used for the given subband. The CQI estimate that is fed back is averaged over all subcarriers in subband.

B. PF Scheduling Algorithm for OL-MU-MIMO

Multi-user PF Scheduler is designed to schedule $K$ ($1 < K \leq N_t$) users for every data-region in the frame such that the sum of PF-metrics of the $K$ users is maximized.

Algorithm for the multi-user PF scheduler is given below:

1. subband index: $s = \text{mod}(n, 12)$
2. Maintain count array of length $K$ to count the number of users chosen the same $PVI$ i.e., count[i] represents number of users requesting for $ith$ $PVI$ $\forall i = 1, 2, \ldots, K$.
3. Orthogonal user pairing:

   while ($i \leq K$)
   
   a) If count[i] is zero then change the PVI of the user having highest CQI to $i$ if its PVI count is greater than one.
   b) If such user is not available then pickup the next user and repeat step a).
   c) Reduce the $CQI$ by $\delta$ dB for the users whose PVI was changed to account for the loss in $CQI$ by using other PVI.

end while
Evaluate PF Metrics:
while $(k \leq \text{number of users per sector})$
4) Assign modulation scheme and code rates to the user based on its CQI feedback for subband $s$.
5) Calculate instantaneous rate $R_k(n)$ for the user based on its MCS level.
6) Evaluate PF metric of the user as follows.
$$PF - \text{Metric} = \frac{R_k(n)}{T_k(n)} \quad k = 1, 2, \ldots, N_u \quad (16)$$
end while

Select users for scheduling:
7) Find subsets of $K$ users from $N_u$ users who prefer different PVIs i.e., $\{1, 2, \ldots, K\}$.
8) Select the subset $S$ which has highest sum $PF - \text{Metric}$.
9) Schedule the users in subset $S$ for data-region $n$ such that user $i$ has $i$th $PVI$ $\forall i = 1, 2, \ldots, K$.

Average throughput updation:
while $(k \leq N_u)$
10) Update the average throughput for each user as
$$T_k(n + 1) = \left(1 - \frac{1}{t_c}\right)T_k(n) + \frac{1}{t_c}R_k(n) \quad k \in S$$
$$T_k(n) = \left(1 - \frac{1}{t_c}\right)T_k(n) \quad k \notin S \quad (17)$$
end while

V. DISCUSSION ON OL-MU-MIMO VS OL-SU-MIMO

Single user scheduler does not have orthogonal pairing problem, as it always schedules only one user per data-region. But the multi-user scheduler looks for the orthogonal set of users for scheduling, if it does not find such users, then it forcefully changes the missing PVIs and pairs the users as explained in steps 3(a)-c) in the algorithm in previous section. In multi-user system though there is IUI it achieves higher throughputs due to the fact that users get scheduled more often when compared to a single user system.

Consider a case when there are very few numbers of users per sector then the probability of finding orthogonal users is very less and scheduler goes through steps 3(a)-c) more often which forces orthogonal pairing by reducing CQI and loss in throughput. Hence the gain achieved by scheduling more often is dominated by the loss incurred due to the inability of the scheduler to find orthogonal users. In this case OL-SU-MIMO performs better when compared to the OL-MU-MIMO as it does not rely on the orthogonal pairing of users.

Let us see what happens exactly in a data-region when single user is scheduled and when multiple users are scheduled. In OL-SU-MIMO, the user with highest PF metric is scheduled and let the rate offered per data-region is $R_{SU}$. OL-MU-MIMO with $K$ users per data-region will achieve better performance over OL-SU-MIMO if and only if the sum of the rates of $K$ users is at least $R_{SU}$. Let the rate offered to the $k$th user in OL-MU-MIMO system is represented as $R_{MU,k}$. Then the OL-MU-MIMO outperforms OL-SU-MIMO iff;
$$\sum_{k=1}^{K} R_{MU,k} \geq R_{SU} \quad (18)$$
When there are few number of active users per sector $(18)$ may not satisfy due to the lack of orthogonal set of users and forces the scheduler to go through steps 3(a)-c) in the algorithm which reduces the CQI and hence dip in the rate achieved by the user. But when there are large number of users per sector, many orthogonal sets of users can be found which avoids the scheduler to force the orthogonal pairing and $(18)$ is easily satisfied. Hence, in a system with large number of users interference suppression is possible by choosing the best set of orthogonal users such that each user causes minimum amount of interference to the remaining set of users.

In OL-MU-MIMO, antennas always beamform in the same direction as the precoders are fixed. When there are few number of users, it is difficult to find the users in beamforming direction. As the number of users in the system increase, there is a possibility of users being randomly located in the beamforming direction and hence, the scheduler will find more orthogonal pairs among which it can pick the best set of users thus providing better rates to the users.

Cell-edge user performance of OL-MU-MIMO is always worse when compared to OL-SU-MIMO. As we know that, the cell-edge user experiences high interference from the neighboring BSs and in case of OL-MU-MIMO, IUI also gets added to this which degrades the performance, thus the combined effect of CCI and IUI limits cell-edge performance. In case of OL-SU-MIMO, the cell-edge user experiences only CCI hence better cell-edge performance can be achieved over OL-MU-MIMO. But as the number of users per sector increase both systems will converge to same point in terms of cell-edge user spectral efficiency which can be observed in Fig. 4.

VI. SIMULATION RESULTS

This section presents the system simulation results generated using broadband wireless simulator developed for IEEE 802.16m with 19 cell-setup and 3 sectors per cell and all the BSs are employing OL-MU-MIMO. Modified Ped-B power delay profile is used and the required temporal correlation is achieved using Jakes model with Doppler spread of 7 Hz and the channel across antennas is uncorrelated. Channel and CCI estimation is assumed to be ideal throughout the simulations. There will be 10% degradation due to the channel estimation errors. OL-SU-MIMO and OL-MU-MIMO have pilot overheads of 16.66% and 11.11% respectively. We used $t_c = 3000$ and $I = 8$ in the simulations.
let us look at the performance of $4 \times 2$ system. As there are 4 antennas at the BS we can serve maximum of 4 users in each data-region but as $N_f$ is limited to 2, the MS may not be capable of canceling IUI from more than one user. Hence there is no additional advantage by increasing $N_f$ except some negligible gain obtained by using higher dimensional precoding vectors. This can be observed in Fig. 5, which shows that the throughput distribution of $2 \times 2$ system and $4 \times 2$ system with $K=2$ is almost overlapping.

Consider a $4 \times 4$ system, as $N_r = 4$ each user can suppress IUI from maximum of 3 users. But serving 4 users in a $4 \times 4$ system is not so easy, as the problem of orthogonal pairing becomes more stringent once we want to find 4 orthogonal users. For a given $N_u$ it is always easy to find 2 orthogonal users than 4. As the number of users served per data-region ($K$) increase the orthogonal pairing becomes more and more problematic and IUI limits the performance of MIMO system. Table I shows the variation of sector spectral efficiency with $N_u$. It can be observed that when $N_u = 5$ or $10$, $4 \times 4$ with $K=2$ performs best whereas when $N_u = 50$, $4 \times 4$ with $K=4$ outperforms $K=2$ or 3. When $N_u = 50$ the scheduler has flexibility of choosing set of users who have minimum IUI and achieves high spectral efficiency. Hence the $4 \times 4$ system with $K=4$ outperforms $4 \times 4$ system with $K=2$ or 3 only when there are very large number of users per sector.

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**TABLE I**

**Sector Spectral Efficiency Variation for $4 \times 4$ OL-MU-MIMO.**

**VII. Conclusions**

From the results presented in section VI we conclude that:

- OL-MU-MIMO outperforms OL-SU-MIMO in sector spectral efficiency sense only when there are large number of users per sector. From Fig. 3, there should be atleast 18 users per sector to outperform OL-SU-MIMO though there is slight degradation in cell edge user spectral efficiency. Note that both the systems are good at exploiting multi-user diversity but gains achieved by OL-MU-MIMO is enormous when compared to OL-SU-MIMO when there are large number of users per sector.

**References**


