New Approach To Joint MIMO Precoding For 2-way AF Relay Systems

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Abstract—In this paper, we propose methods for joint design of source relay precoders that optimize the performance of two-way amplify and forward (AF) multiple input multiple output (MIMO) relay systems. In particular, we propose three iterative methods to optimize the system performance. The design criteria we consider in this paper are, the conventional Arithmetic sum of average Mean Square Error (AMSE) criteria, and the lesser know Arithmetic sum of average Bit Error Rate (ABER) criteria. The proposed methods are applicable to any number antennas at all three nodes which is advantage compared to the existing joint source relay optimization technique that optimizes the sum rate. Using the extensive simulations we observe that the proposed methods performs better than the existing techniques. By using a judicious combination of both the ABER and AMSE criteria, joint MIMO precoders for two-way relay system are constructed, which clearly outperform existing joint design.

I. INTRODUCTION

Wireless relay based cooperative communication is a promising technology in improving coverage, link reliability and system sum rate. These systems can be broadly classified into one-way(or four phase), three-phase and two-way(of two phase) relay systems, depending on the number of time slots required to exchange the information between the transceiver nodes, $S_1$ and $S_2$ via the relay node $R$.

Recently the two-way relay system has attracted a lot of research interest because of its higher spectral efficiency [1] when compared to the one-way and three phase relay systems. The recent advances in two-way relay systems and its performance analysis can be found in [2] and [3].

Linear precoding is one of the pre-processing techniques used to optimize the communication system performance under the average transmit power constraint. In [4], [5], [6], [7] and [8] the design of optimal relay precoder which maximizes the sum rate of two-way AF relay system has been discussed. The design of relay precoder that maximizes the SINR of two-way relay system has been discussed in [9]. In [10], the design of relay precoder which minimizes the AMSE has been discussed. It can be noted that in [4]-[10] only precoder at the relay node has been optimized where as at the source node equal power have been used. Thus these techniques are called as Relay Only Precoding (ROP) techniques. Recently in [11] the joint design of source relay precoder that maximizes the sum-rate of the two-way AF relay system has been discussed. However, the technique proposed in [11] is applicable only when the number of antenna at both source nodes are equal.

In this paper, we propose three iterative algorithms to jointly design source relay precoders for two-way AF MIMO relay systems. The first and second algorithm uses the AMSE and ABER respectively as design criteria to optimize source relay precoders. The third algorithm uses ABER as design criteria to optimize source precoders and AMSE is used as design criteria to optimize the relay precoder. we assume that all the three nodes have perfect channel state information (CSI) and also that each node has the knowledge of transmit power constraint at all the nodes. With this assumption, it can be noted that the proposed algorithms can be run separately at all the three nodes to get the same set of precoders. We perform intensive simulations to compare the proposed techniques with the exiting technique. These results indicates that the precoders which depends on ABER criteria performs better compared to precoder obtained using AMSE criteria. We also observe that the precoder obtained using a combination of ABER and AMSE criteria performs better than the precoders obtained using ABER and AMSE criteria individually.

The main contributions of the paper are:
1) Three iterative algorithms that use AMSE and ABER criteria to jointly design source relay precoders for two-way AF MIMO systems.
2) The proposed algorithms work for any number of antennas at each node.

II. SYSTEM MODEL

We consider two-way AF MIMO relay based communication system comprising of transceiver nodes $S_1$, $S_2$ and relay node $R$ as shown in figure 1. The nodes $S_1$, $S_2$ and $R$ are equipped with $N_1$, $N_2$ and $N_r$ antennas respectively. During the first phase of transmission nodes $S_1$ and $S_2$ simultaneously transmit information to the relay node $R$, and in next phase of transmission the relay amplifies the received signal and broadcasts it to nodes $S_1$ and $S_2$. The following assumptions are used in this work:

1) The channels are flat fading and reciprocal.
2) All the three nodes have perfect CSI and also that each node has full knowledge of the transmit power constraint at other two nodes.
3) There is no direct path between the source nodes $S_1$ and $S_2$.

Let $s_i \in \mathbb{C}^{L \times 1}$, $\forall i = 1, 2$ be the information symbol vector at node $S_i$, such that $E[s_i s_i^H] = I_{L \times L}$ and $L = min(N_1, N_2)$. Let $F_i \in \mathbb{C}^{N_i \times L}$, $\forall i = 1, 2$ be the linear precoder and let $x_i = F_i s_i$, $\forall i = 1, 2$ be the transmitted symbol vector from...
node $S_i$. Let $p_i$ be the power constraint at node $S_i$, i.e.,

$$tr\left(\mathbb{E}[x_i x_i^H]\right) = tr\left[F_i F_i^H\right] \leq p_i,$$

During the first phase, the received signal at node $R$ is given by,

$$y_r = \sum_{i=1}^{2} H_i x_i + n_r,$$

where $H_i \in \mathbb{C}^{N_r \times N_i}, \forall i = 1, 2$ is the forward channel matrix from node $S_i$ to node $R$. The noise $n_r \in \mathbb{C}^{N_r \times 1}$ is the zero mean circularly symmetric additive complex Gaussian noise having covariance matrix $I_{N_r \times N_r}$. In second phase, the node $R$ broadcasts the signal $x_r$ given by,

$$x_r = G y_r,$$

where $G \in \mathbb{C}^{N_r \times N_r}$ is the relay precoder. Let $p_r$ be the relay power constraint i.e.,

$$tr\left(\mathbb{E}[x_r x_r]\right) = tr\left(G \sum_{i=1}^{2} H_i F_i F_i^H H_i^H + I_n G^H\right) \leq p_r.$$

The signal received at the nodes $S_1$ and $S_2$ are given by,

$$y_1 = H_1^T G H_1 x_1 + H_1^T G H_2 x_2 + H_1^T G n_r + n_1,$$

$$y_2 = H_2^T G H_1 x_1 + H_2^T G H_2 x_2 + H_2^T G n_r + n_2,$$

where $n_i \in \mathbb{C}^{N_i \times 1}, \forall i = 1, 2$ is the additive complex noise vector such that, $n_i \sim \mathcal{CN}(0, I_{N_i \times N_i})$. Let $\tilde{y}_1$ and $\tilde{y}_2$ be the signals obtained after canceling the self interference from the received signals $y_1$ and $y_2$ respectively, i.e.,

$$\tilde{y}_1 = H_1^T G H_2 x_2 + H_1^T G n_r + n_1,$$

$$\tilde{y}_2 = H_2^T G H_1 x_1 + H_2^T G n_r + n_2.$$

The signal $\tilde{y}_1$ and $\tilde{y}_2$ are operated by linear MMSE equalizers to obtain the estimated of $s_2$ and $s_1$ respectively, namely,

$$\hat{s}_1 = D_2 \tilde{y}_2,$$

$$\hat{s}_2 = D_1 \tilde{y}_1,$$

where $\hat{s}_1$ is the estimated symbol vector of $s_i$ and $D_i \in \mathbb{C}^{N_i \times L}, \forall i = 1, 2$ is the equalizer matrix used at node $S_i$.

### III. AMSE and ABER PERFORMANCE MEASURES

In this section we define AMSE and ABER performance measures. Let us define the MSE matrices $E_1$ and $E_2$ at nodes $S_1$ and $S_2$ respectively as,

$$E_1(F_2, G, D_1) = \mathbb{E}\left[(\hat{s}_2 - s_2)(\hat{s}_2 - s_2)^H\right] = I_{L \times L} - D_1^H P_1 - P_1^H D_1 + D_1^H R_1 D_1,$$

$$E_2(F_1, G, D_2) = \mathbb{E}\left[(\hat{s}_1 - s_1)(\hat{s}_1 - s_1)^H\right] = I_{L \times L} - D_2^H P_2 - P_2^H D_2 + D_2^H R_2 D_2,$$

where $P_1, P_2, R_1$, and $R_2$ are defined as, follows:

$$P_1 = H_2^T GH_2 F_2$$

$$P_2 = H_2^T GH_1 F_1$$

$$R_1 = P_1 P_1^H + H_2^T GH_2 H_2^* + I_{N_2 \times N_1}$$

$$R_2 = P_2 P_2^H + H_2^T GH_1 H_1^* + I_{N_1 \times N_2}$$

It is to be noted that for fixed values of $F_1, F_2$ and $G$ the $tr\left(E_i\right), \forall i = 1, 2$ is a convex function with respect to $D_i$. Therefore, the optimal $D_i$ is obtained by solving the equation $\nabla_{D_i} tr\left(E_i\right) = 0$. The solution to this equation is given by,

$$D_{i, opt} = R_i^{-1} P_i, \quad \forall i = 1, 2$$

After substituting the $D_{i, opt}$, in (11) and (12) the MSE matrix $E_1$ and $E_2$ at node $S_1$ and $S_2$ can be written as,

$$\hat{E}_1 = (I + F_2 R_{H,2} F_2^H)^{-1}$$

$$\hat{E}_2 = (I + F_1 R_{H,1} F_1^H)^{-1},$$

respectively, where

$$R_{H,1} = H_1^T G H_1 (I + H_2^T G H_2 H_2^H)^{-1} H_2^T G H_1,$$

$$R_{H,2} = H_2^T G H_2 (I + H_1^T G H_1 H_1^H)^{-1} H_1^T G H_2.$$
B. ABER criteria

Before defining the second objective function, let us first define the relationship between the MSE, SINR and BER of the each individual stream as given in [12]. The MSE of the individual stream $i$ of the $j$-th node and $j$-th parallel stream is defined as

$$MSE_{i,j} = \left(\tilde{E}_i\right)_{j,j}. \quad (24)$$

Assuming that all the sub streams use the same constellation with Gray encoding. The corresponding BER and SINR are given as,

$$BER_{i,j} = \frac{1}{m} \left(\alpha_i Q\left(\beta_i \sqrt{SINR_{i,j}}\right)\right) \quad (25)$$

$$SINR_{i,j} = \frac{1}{MSE_{i,j}} - 1 \forall i, j \in \{1, 2, \ldots, L\} \quad (26)$$

where $m = \log_2 M$ is the number of bits per symbol, and $M$ is the size of the constellation used. The Symbols $\alpha$ and $\beta$ depends on the transmitted signal constellation and $Q$-function defined as $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-\frac{t^2}{2}} dt$.

The ABER performance measure is defined as follows,

$$f_2 = \frac{1}{2L} \left(\frac{1}{m} \sum_{i=1}^{L} \sum_{j=1}^{L} \alpha_i Q\left(\beta_i \sqrt{\frac{1}{E_{i,j}}} - 1\right)\right). \quad (27)$$

IV. JOINT SOURCE RELAY PRECODER DESIGN

The main purpose of this paper is to jointly design source and relay precoder matrices, that optimize the given system performance measure under average transmit power constraints given in (1) and (4). In this regard we propose three iterative algorithms,

1. JAM: In this algorithm precoders at all the nodes jointly designed to optimize AMSE criteria.
2. JAB: In this algorithm precoders at all the nodes jointly designed to optimize ABER criteria.
3. JABM: In this algorithm source precoders are used to optimize the ABER criteria and the relay precoder is used to optimize AMSE criteria.

All the three algorithms iterate over the two main steps to obtain the optimum precoders. Let us elaborate the first and second step in the algorithm.

A. STEP 1: Design of $F_1$ and $F_2$ for a given value of $G$

In JAM algorithm during this step $F_1$ and $F_2$ are obtained to optimize the AMSE criteria ($f_1$), where as in case of JAB and JABM during this step $F_1$ and $F_2$ are obtained to optimize ABER criteria ($f_2$). It can be noted that the $G$ acts as the coupling variable between the $\text{tr}\left(\tilde{E}_1(F_1, G)\right)$ and $\text{tr}\left(\tilde{E}_2(F_1, G)\right)$. Thus for given value of $G$ the optimization of $f_1$ with respect to $F_1$ and $F_2$, under the power constraint given in (1) decomposes into two independent optimization problems $P1$ and $P2$.

$$P1: \min_{F_2} \text{tr}\left(\tilde{E}_1(F_2, G)\right) \quad \text{subject to} \quad \text{tr}\left(F_2F_2^H\right) \leq p_2$$

The optimization problems $P1$ and $P2$ are similar to the optimization of precoder for point to point MIMO channels. The optimal solution $\hat{F}_1$ of at most rank $\hat{L}_i = \min(L, \text{rank}(R_{H,i}))$, $\forall i \in \{1, 2, \ldots, L\}$ is given by [13],

$$\hat{F}_1 = \hat{U}_{H,i} \Sigma_i \forall i = 1, 2, \quad (28)$$

where $\hat{U}_{H,i} \in \mathbb{C}^{N_i \times \hat{L}_i}$, $\forall i \in \{1, 2\}$ is the matrix of first $\hat{L}_i$ columns of $U_{H,i}$ given in (22) and $\Sigma_i = [\text{diag}\left(\{\lambda_{i,j}\}\right) \cdot 0] \in \mathbb{C}^{L_i \times L_i}$, $\lambda_{i,j} \geq 0$, $\forall i = 1, 2, \forall j = 1, 2, \ldots, L_i$. The diagonal value $\lambda_{i,j}$ is given by [13, sec.V.A],

$$\lambda_{i,j} = \sqrt{\left(\mu_i^{-1/2} \lambda_{H_{i,j}}^{1/2} - \lambda_{H_{i,j}}^{-1}\right)^2}, \quad (29)$$

where $\mu_i$ is the Lagrangian multiplier chosen to satisfy the power constraint $p_i$ given in (1).

Similarly, for a given value of $G$, the optimization of $f_2$ with respect to $F_1$ and $F_2$ under the power constraint (1) can also be decoupled into two independent optimization problems $P3$ and $P4$, namely.

$$P3: \min_{F_2} \frac{1}{L} \sum_{j=1}^{L} \alpha_2 Q\left(\beta_2 \sqrt{\frac{1}{E_{1,j}} - 1}\right) \quad \text{subject to} \quad \text{tr}\left(F_2F_2^H\right) \leq p_2$$

$$P4: \min_{F_1} \frac{1}{L} \sum_{j=1}^{L} \alpha_1 Q\left(\beta_1 \sqrt{\frac{1}{E_{2,j}} - 1}\right) \quad \text{subject to} \quad \text{tr}\left(F_1F_1^H\right) \leq p_1$$

The optimization problems $P3$ and $P4$ are similar to the optimization of precoders for point to point MIMO channels and the optimal solution $\hat{F}_1$ of at most rank $\hat{L}_i$, as given in [13], can be expressed as

$$\hat{F}_1 = \hat{U}'_{H,i} \Sigma_i V_i', \forall i = 1, 2, \quad (30)$$

where $\hat{U}'_{H,i}$ and $\Sigma_i$ are defined as earlier and $V_i \in \mathbb{C}^{L_i \times L_i}$ is a unitary matrix such that it results in equal diagonal values in MSE matrix.

B. STEP 2: Design of $G$ for given values of $F_1$ and $F_2$

In JAM and JABM, during this step the relay precoder matrix $G$ is designed to optimize the AMSE criteria ($f_1$), where as in case of JAB $G$ is designed to optimize ABER criteria ($f_2$). Thus, for given values of $F_1$ and $F_2$, the optimization of $f_1$ with respect to $G$ under the power constraint given in (4) can be written as,

$$P5: \min_G \text{tr}\left(\tilde{E}_1 + \tilde{E}_2\right) \quad \text{subject to} \quad \text{tr}\left(G\sum_{i=1}^{2} H_i F_i F_i^H H_i^H + I\right) G^H \leq p_r$$

and it can be noted that, when the values $F_1$ and $F_2$ are given the objective function in $P5$ is a nonlinear function of $G$ and the constraint is quadratic function in $G$. The Sequential Quadratic Programming (SQP) is one of the numerical techniques that can be used to solve this kind of problem.
Similarly, for given values of \( F_1 \) and \( F_2 \), the optimization of \( f_2 \) with respect to \( G \) under the power constraint given in (4) can be written as,

\[
P_6:\quad \min_G \sum_{i=1}^{2} \sum_{j=1}^{L} \alpha_i Q \left( \beta_i \sqrt{\frac{1}{|F_{i,j}|}} - 1 \right)
\]
subject to \( \text{tr} \left( G \sum_{i=1}^{2} |H_i F_i F_i^H H_i^H + I| G^H \right) \leq p_r \)

The objective function in \( P_6 \) is also a nonlinear function of \( G \) and constraint is quadratic function in \( G \). We use SQP to solve \( P_6 \).

C. JAM

In this algorithm, we find the values of \( F_1 \), \( F_2 \) and \( G \) that optimize the function \( f_1 \) under the constraints (1) and (4). Let \( k \) be the iteration index and \( \epsilon \) be the tolerance for termination of the algorithm. The steps followed to find the matrices \( F_1 \), \( F_2 \) and \( G \) are as follows,

Input: Channel matrices \( H_1 \) and \( H_2 \), power constraints \( p_1 \), \( p_2 \) and \( p_r \), and termination tolerance \( \epsilon \).
Initialize: \( G_0 = \sqrt{\gamma} I \) where \( \gamma = \frac{\sum_i p_i/n_i}{\text{tr}(H_i H_i^H) + n_r} \), and \( k = 1 \).

Step 1: Substitute \( G_{k-1} \) for \( G \) in (20) and (21), find \( F_{i,k} \) using (28), \( \forall i = 1, 2 \).

Step 2: Substitute \( F_{i,k} \) for \( F \) in \( P_5 \) and Find \( G_k \) that optimize \( P_5 \) using SQP. Let the optimum value of \( P_5 \) obtained in this step be \( \text{AMSE}_k \).

Step 3: If \( \text{AMSE}_k - \text{AMSE}_{k-1} \leq \epsilon \) then go to step 1, else increment \( k \) by 1 i.e., \( k = k + 1 \) and got step 1.

Step 4: Set \( F_{i,JAM} = F_{i,k} \), \( G_{JAM} = G_k \) using this find \( D_{i,JAM} \) from (17).

Output: Source precoder matrices \( F_{i,JAM} \), decoder matrices \( D_{i,JAM} \), \( \forall i = 1, 2 \) and the relay precoder matrix \( G_{JAM} \).

D. JAB

In this algorithm we find the values of \( F_1 \), \( F_2 \) and \( G \) that optimize the function \( f_2 \) under the constraints (1) and (4). The \( k \) and \( \epsilon \) are defined as in JAM. The steps followed to find the matrices \( F_1 \), \( F_2 \) and \( G \) are as follows,

Input: Channel matrices \( H_1 \) and \( H_2 \), power constraints \( p_1 \), \( p_2 \) and \( p_r \), and termination tolerance \( \epsilon \).
Initialize: Set \( G_0 \) as described in JAM.

Step 1: Substitute \( G_{k-1} \) for \( G \) in (20) and (21), find \( F_{i,k} \) using (30), \( \forall i = 1, 2 \).

Step 2: Substitute \( F_{i,k} \) for \( F \) in P5 and Find \( G_k \) that optimize P6 using SQP. Let the optimum value of P6 obtained in this step be ABER_k.

Step 3: If \( ABER_k - ABER_{k-1} \leq \epsilon \) then go to step 4, else increment \( k \) by 1 i.e., \( k = k + 1 \) and got step 1.

Step 4: Set \( F_{i,JAB} = F_{i,k} \), \( G_{JAB} = G_k \) using this find \( D_{i,JAB} \) from (17).

Output: Source precoder matrices \( F_{i,JAB} \), decoder matrices \( D_{i,JAB} \), \( \forall i = 1, 2 \) and the relay precoder matrix \( G_{JAB} \).

E. JABM

In this algorithm we find the values of \( F_1 \) and \( F_2 \) that optimize the function \( f_1 \) under the power constraint (1) and \( G \) that optimize the function \( f_2 \) under the power constraint (4). The \( k \) and \( \epsilon \) are as defined in JAM. The steps followed to find the matrices \( F_1 \), \( F_2 \) and \( G \) are as follows,

1) Run the JAM algorithm.
2) After the algorithm converges, the optimum precoders \( F_i, \forall i = 1, 2 \) and \( G \) are obtained as,

\[
F_{i,JABM} = F_{i,JAM} V^H, \quad (31)
\]

\[
G_{JABM} = G_{JAM}. \quad (32)
\]

where \( V \) is a normalized DFT matrix of size \( L \).

V. PERFORMANCE ANALYSIS OF PROPOSED ALGORITHMS

In this section, we analyze the performance of the proposed algorithms, using simulation results. All the simulation have been performed for \( p_1 = p_2 = p_r \). We assume that the number of antennas at nodes \( S_1, S_2 \) and \( R \) to be 2, 4 and 3, respectively. Entries of channel matrices \( H_1, H_2 \) are assumed to be independent and distributed as \( CN(0,1) \). The entries of \( s_1 \) and \( s_2 \) take one of the possible value from gray coded QPSK constellation with equal probability. All the results shown in this section have been averaged over 10000 channel realizations. The BER is calculated using (26) for \( \alpha = 1 \) and \( \beta = 2 \) which corresponds to QPSK constellation [12]. The existing technique used in this paper for comparison are 1. Uniform power allocation (UP) in which \( F_i = \sqrt{\frac{p_i}{P}} [I_0]_{N_i \times L} \) and \( G = G_0 \) defined in JAM 2). Relay only precoding with AMSE criteria (ROPM)[10] and 3). Relay only precoding with ABER criteria (ROPB).

The AMSE performance comparison of the proposed techniques and the existing technique are shown in figure 2. From this following observations can be made:
The following observations can be made:

- The performance of JAM and JABM is same for all the values of input power.
- The JAM performs 2dB better than JAB and ROPM at AMSE of $10^{-2}$, where as JAM performs 5dB better than ROPB at AMSE of $10^{-2}$ and 7dB better than UP at AMSE of $10^{-1}$.

The ABER performance comparison of the proposed techniques and the existing technique are shown in figure 3. From this following observations can be made:

- The performance of JAB and JABM is same for all the values of input power.
- The JAB performs 2dB better than the JAM at ABER of $10^{-5}$. It also gives 4dB better performance than ROPM and ROPB at ABER of $10^{-4}$.
- The JAB performs 7dB better than UP at ABER of $10^{-2}$.
- The ROPB and ROPM gives same performance in terms ABER for all values of input power. Thus performance in terms ABER is not changed when the relay precoder is designed to optimize AMSE criteria or ABER criteria.
- The JAM gives same performance as ROPM and ROPB in terms ABER for low values of input power and it performs 3dB better than ROPM and ROPB at ABER of $10^{-4}$.
- The performance of JAM can be improved in terms of ABER for low values of input power by doing rate adaptation i.e., by setting the total number of sub stream equal to rank $F_i, \forall i = 1, 2$. We refer to this as JAM_RA.

Let us now summarize the advantages of proposed algorithms:

- The JAM performs better than the ROPM, ROPB and UP in terms of AMSE because the joint design of source relay extract the mutual dependence between the source and relay precoder when optimizing the performance measure.
- The JABM performs better than the JAB in terms of AMSE and it performs better than the JAM in terms of ABER.

- The JAB performs better than JAM in terms of ABER this because JAB consider the knowledge constellation used when optimizing the performance measure which is lagging in JAM.
- The proposed algorithms work for any value $N_1, N_2$ and $N_r$.

VI. Conclusion

In this paper we have formulated and jointly designed the source relay precoder matrices for two-way MIMO AF relay system based on the criteria AMSE and ABER criteria. We have proposed three iterative algorithms namely, JAM, JAB and JABM, to obtain source and relay precoder. It was shown using simulation that the proposed algorithms outperform all the existing methods. It was observed that JABM gives the same performance as JAB in terms of ABER and it also gives the same performance as JAM in terms of AMSE. This work can be extended to a realistic case where the relay transmit power is fixed. How do multiple antennas on the relay help now?.

REFERENCES