Abstract—This paper addresses the problem of target detection in the presence of a non-Gaussian clutter modeled in compound-Gaussian form which realizes the clutter process as a product of two independent random processes ‘texture’ and ‘speckle’. The likelihood ratio test (LRT) detector applied to this detection problem reduces the detector to a matched filter (MF) when the texture is considered as completely correlated during a coherent processing interval (CPI). However, in practical applications, the textural component exhibits a correlation which is less than unity. The conventional form of MF based detectors existing in the literature yield a significant fall in the detection performance in such clutter scenarios. In this paper, we propose a generalized likelihood ratio test (GLRT) detector which can effectively detect a fluctuating target in the presence of a compound-Gaussian clutter with partially correlated texture. The results are presented to show the performance superiority of the proposed detector over the existing MF detector in such varying texture scenarios.

Index Terms—Non-Gaussian clutter, GLRT detection, Matched Filter detector, Varying texture.

I. INTRODUCTION

The LRT [1], [2] detectors are shown to yield excellent performance and it is attractive for radar detection in the presence of correlated non-Gaussian clutter modeled as multivariate compound-Gaussian form. In many of the contributions in the existing literature, the compound-Gaussian clutter model has been well accepted not only for its suitability to the formulation of the LRT detection schemes [3] but also for its consistency with the K-distribution which gives a deep insight into the scattering mechanism of high resolution and/or low-grazing angle land and sea clutter [4], [5].

An extensive research has been done by various researchers in modeling K-distributed clutter in compound-Gaussian form and it has been suggested that the complex envelope of the n'th pulse return can be written as the product of two independent random processes as given below [6].

\[ d(n) = \sqrt{\tau(n)} x(n) \]  

(1)

Here the fast fluctuating ‘speckle’ component \( x(n) \) is a stationary complex Gaussian process with \( E[|x(n)|^2] = 1 \) and the comparatively slow varying ‘texture’ component \( \tau(n) \) is a non-negative real random process which describes the underlying mean power level of the resultant clutter process. In some special scenarios it is acceptable to assume that the texture is completely correlated during a particular coherent processing interval (CPI) and different for different CPI’s. Therefore, the corresponding clutter model is simplified as

\[ d(n) = \sqrt{\tau} x(n) \]  

(2)

where \( n = 1, 2, \ldots, L \) and \( L \) is the number of pulses integrated in one CPI. The above clutter model (2) is useful in formulating log-likelihood ratio test (LLRT) and GLRT detection schemes [7]-[9].

A comparative study on the detection performance of GLRT and LLRT detectors shows that the GLRT detector performs better compared to the LLRT detector [10]. The performance of the GLRT detector is effective even for lower speckle correlation values and hence it is attractive for practical applications [11]. Furthermore, the GLRT detector can be formulated as a matched filter (MF) form with data dependent threshold and it promises for optimum target detection [12], [13].

In the existing literature, the MF detectors are formulated by using an inaccurate clutter model (2). Because, in practice, the texture of a compound-Gaussian clutter is varying in nature. Therefore, the existing MF detectors must incur a performance degradation while detecting target in such varying texture scenarios and severity increases with fluctuation of the scattering background [14]. This motivated us to propose a detector for detecting targets in such clutter with varying texture which may arise for sea surface undulation or due to the continuous movement of an antenna beam in scanning radar application [15], [16].

In this paper, we propose a GLRT detection scheme which can detect a fluctuating target in the presence of compound-Gaussian clutter with a varying texture as modeled in [17]. The detector turns out to a simpler in structure compared to the detector proposed by Lombardo et al. in [18]. A detailed study on the detection performance of the proposed GLRT detector is done. The Experimental results are presented in this paper to compare the detection performance of the proposed detector to the same of the existing MF detector.

This paper is organized as follows. Section II gives a brief discussion on the system model followed by the description of the proposed GLRT detection scheme in Sections III. The Experimental results and the corresponding discussions are given in section IV. Finally the work is concluded in section V.
II. SYSTEM MODEL

A. Model of the Target Signal

The fluctuating signal from the target is assumed to be following the Swerling-I model expressed as

\[ s = \alpha p = \gamma e^{j\psi}p \]  

where the Rayleigh distributed amplitude \( \gamma \) is constant for the entire scan and \( \psi \) is uniformly distributed in the range \([-\pi, +\pi]\). That is, \( \alpha \) is a complex circular Gaussian random variable, with zero mean and variance \( \sigma_c^2 = E[|\alpha|^2] \). \( \alpha \) is considered as unknown as it is so in practice. Here \( p = e^{j2\pi f_{\Delta t}nT_p} \) is the \( L \times 1 \) dimensional Doppler steering vector of the target, where \( f_{\Delta t} \) represents the Doppler frequency of the target and \( T_p \) the radar pulse repetition period. The time response of the MF for the signal turns out to be same as the desired signal replica \( p \) where the center frequency of the MF matches with the Doppler frequency of the target and it can be considered as known to the detector.

B. The Proposed Clutter Model in [17]

In our proposed clutter model [17] where we consider that a range cell is made up of finite number clusters and signal returns from each cluster modeled in compound-Gaussian form as given in (1) but modulated by the antenna scanning motion. Now if there are \( K \) number of clusters within one range cell, the clutter from a particular range cell can be considered as a sum of the signals from each cluster and hence it can be written as

\[ c(n) = \sum_{a=1}^{K} \sqrt{\tau_a(n)} x_a(n)e^{j2\pi f_a n} = \sum_{a=1}^{K} c_a(n) \]  

where \( c_a(n) = \sqrt{\tau_a(n)} x_a(n)e^{j2\pi f_a n} \). Here the integer number ‘a’ is used to represent the clusters within the range cell under test (CUT) which is made up of similar kind of scatterers and scatterer characteristics are different for different values of ‘a’. The Doppler frequency corresponding to the \( a^{th} \) cluster is \( f_a \) which can be considered as zero for static radar applications. Therefore, the non-Gaussian clutter compound-Gaussian form can be used for individual clusters and the probability density function (pdf) of \( \tau_a(n) \) can be considered as

\[ p_{\tau_a}(\tau_a) = \frac{\nu_a/\eta_a}{\Gamma(\nu_a)} \tau_a^{\nu_a-1} \exp\left(-\frac{\nu_a}{\eta_a}\tau_a\right) \]  

where \( \nu_a \) and \( \eta_a \) respectively are the shape parameter and mean of the Gamma random variable, and \( \Gamma(\cdot) \) is the Gamma function.

C. Binary Hypothesis

For radar target detection the binary hypothesis can be considered as given by

\[ H_0 : z = c \]
\[ H_1 : z = s + c = \alpha p + c \]

where \( z \in \mathbb{C}^L \) is the complex observation vector of dimension \( L \times 1 \). \( s \) and \( c \) are \( L \times 1 \) dimensional signal vectors representing the target and clutter respectively. Then, the clutter covariance matrix of order \( L \times L \) can be expressed as \( M_c = E[cc^H] \) where \( H \) represents the Hermitian operation including a transpose operation and complex conjugate operation. Considering the case of a very high clutter-to-thermal noise power ratio, the effect of thermal noise is neglected in (6).

D. The GLRT Detection

Using the conditional probability density functions of the signal, the generalized likelihood ratio test can be represented as

\[ \Gamma(z) = \max_{\beta_1} \frac{f_{z|\beta_1}(z|\beta_1)}{f_{z|\beta_0}(z|\beta_0)} > \Lambda \]  

where if \( \beta_1 \) and \( \beta_0 \) range over the subspaces \( H_1 \) and \( H_0 \) respectively [19]. Therefore, if the maximum likelihood (ML) estimates of \( \beta_1 \) and \( \beta_0 \) are available, it is possible to evaluate both \( f_{z|\beta_1}(z|\beta_1) \) and \( f_{z|\beta_0}(z|\beta_0) \) and hence, the ratio can be compared with the threshold value \( \Lambda \) to decide the presence of the radar target in a particular range cell under test (CUT).

When the clutter in (6) is modeled in multivariate compound-Gaussian form and if it is possible to ML estimate the fluctuating unknown target amplitude \( \alpha \) and also the clutter covariance matrix \( M_c \), the GLRT detection statistic can be written as

\[ \max_{\alpha,M_c;\beta_1} p_{z|\beta_1}(z|\alpha, M_c; H_1) > \max_{\alpha,M_c;\beta_0} p_{z|\beta_0}(z|\alpha, M_c; H_0) \]  

where \( \Lambda \) can be kept constant for a particular probability of false alarm \( (P_{fa}) \).

Interestingly, when the compound-Gaussian clutter texture assumed to be completely correlated, the detection structure that can be obtained from (8) reduces to a MF detector which decides the presence or absence of the target by comparing the MF output with a data dependent threshold as given in [7]. However, the formulation of the detector is not so straightforward when the pulse to pulse texture correlation value is less than unity. It is not available any useful detection scheme which can be used for detecting targets in such varying texture scenarios. Hence, we propose a GLRT based detection scheme as discussed next.

III. THE PROPOSED GLRT DETECTION SCHEME

Here we discuss in detail about the proposed GLRT detector which can detect a fluctuating target in the presence of clutter modeled as in (4) where it is considered that texture is varying from pulse to pulse within one CPI unlike the clutter model as considered by Sangston et al. in [7] which can be claimed as a crude assumption of our proposed clutter model.

If it is possible to separate out the textural part from the speckle component of the pulse returns obtained from the neighboring cells of the CUT, it is easy to find the average speckle covariance matrix which can be considered as the speckle covariance matrix of the CUT and hence can be used...
for target detection. This idea is used in our proposed scheme for GLRT detection in the presence of the clutter model (4) to detect a fluctuating target.

For mathematical simplifications let us rewrite the clutter model (4) as

\[ c(n) = \sum_{a=1}^{K} \sqrt{\tau_a(n)} y_a(n) \]  

where \( y_a(n) = x_a(n)e^{2\pi f_a n} \) and the corresponding covariance matrix is \( E[Y_a Y_a^H] \). Let the value of \( K \) is considered as known to the detector and the covariance matrix of the clutter is expressed as

\[ M_c = \sum_{a=1}^{K} T_a M_y T_a^H \]  

(10)

where \( T_a = \text{diag}\{\sqrt{\tau_a(1)}, \sqrt{\tau_a(2)}, \ldots, \sqrt{\tau_a(L)}\} \) and \( M_y = E[Y_a Y_a^H] \) for all values of \( a \). Now, if \( T_a \) is estimated accurately, the speckle covariance matrix, \( M_y \) can easily be estimated from the clutter total covariance matrix, \( M_c \). This \( M_y \) can then be used in the detector. Therefore, for GLRT detection the binary hypothesis (6) can be used to form the generalized likelihood ratio (GLR) as

\[ \Gamma = \frac{\max_{\alpha, \text{all} T_a} f(z; \alpha, \text{all} T_a; H_1)}{\max_{\alpha, \text{all} T_a} f(z; \alpha, \text{all} T_a; H_0)} \]  

(11)

where the target amplitude, \( \alpha \) is considered as unknown at the detector. Therefore, (11) can further be expressed as

\[ \Gamma = \frac{\max_{\alpha, \text{all} T_a} \frac{1}{|M_{c_1}|} \exp[-(z - \alpha p)^H \hat{M}_{c_1}^{-1}(z - \alpha p)]}{\max_{\text{all} T_a} \frac{1}{|M_y|} \exp[-z^H \hat{M}_{c_0}^{-1} z]} \]  

(12)

Here the covariance matrix under the hypotheses \( H_0 \) and \( H_1 \) are different and can be written as \( M_{c_0} \) and \( M_{c_1} \) respectively.

In order to derive the detector in a matched filter form the GLRT can now be written as

\[ \frac{1}{|M_{c_1}|} \exp[-(z - \alpha p)^H \hat{M}_{c_1}^{-1}(z - \alpha p)] \]  

\[ \frac{1}{|M_y|} \exp[-z^H \hat{M}_{c_0}^{-1} z] = e^{\lambda_1} \]  

(13)

where \( \lambda_1 \) is a constant for a particular false alarm rate. Here the unknown fluctuating target \( \alpha \) can be estimated by using an ML estimator expression, \( \hat{\alpha} = \frac{p^H \hat{M}_{c_1}^{-1} \hat{M}_{c_1}}{p^H \hat{M}_{c_0}^{-1} p} \). By substituting the estimate of \( \alpha \) and proceeding with some mathematical simplifications we get the detection structure (13) as given by

\[ \frac{p^H \hat{M}_{c_0}^{-1} z}{p^H \hat{M}_{c_0}^{-1} p} = \frac{e^{-\lambda_1/L}}{(L-1)\lambda_1} e^{-\lambda_1/L} + z^H \hat{M}_{c_1}^{-1} z(1 - e^{-\lambda_1/L}) \]  

(14)

Interestingly, it can be noticed from the left hand side of (14) that the detector is a matched filter detector with Doppler steering vector of the target which can be considered as known at the detector. The important point to be noticed here is that the detection performance is highly sensitive to the total covariance matrix estimate of the clutter under \( H_0 \) hypothesis in view of its involvement in both left and right hand sides of (14). Here the right hand side of (14) is representing the threshold expression of the detector and it is data dependent due to the presence of the second term. The total threshold expression also depends on the targeted false alarm rate through \( \lambda_1 \).

It should be noted here that this detection scheme can be considered as a generalized form of the detector for the target in the presence of compound-Gaussian clutter. The major advantage of our proposed GLRT detector is that it can reliably detect fluctuating target in the presence of clutter with varying texture.

**Application of the Detection Scheme to Clutter with Completely Correlated Texture**

The proposed generalized detector reduces to a simple form when the texture of compound-Gaussian clutter is completely correlated over the entire CPI time. Here, let us consider that the texture part of the clutter given in (4) is fixed or completely correlated in one CPI. That means \( \sqrt{\tau_a(1)} = \sqrt{\tau_a(2)} = \cdots = \sqrt{\tau_a(L)} = \sqrt{\tau_a} \). Then, (9) can be written as

\[ c_f(n) = \sum_{a=1}^{K} \sqrt{\tau_a} y_a(n) \]  

(15)

This clutter model becomes almost similar to the clutter model as considered in the problem given in [20] where a suitable detection scheme is given for such clutter. However, this detection structure is not much attractive for implementation because of the necessity of computing the individual cluster texture power and it is difficult to compute the data dependent threshold also.

For the sake of further simplification of the detection structure we may consider the total range cell as a single cluster that means \( K = 1 \) and hence the clutter model becomes same as given in (2) which can be written as

\[ c_f(n) = \sqrt{T} y(n) \]  

(16)

where \( \tau_a = \tau \). Then, the total covariance matrix of the clutter becomes as \( M_{f0} = \tau M_y \) which is exactly the same as considered in [7]. In this case the GLRT detector for detecting a fluctuating target reduces to

\[ \frac{p^H M_{y_0}^{-1} z}{p^H M_{y_0}^{-1} p} = (z^H M_{y_0}^{-1} z)(1 - \exp(-\lambda_2/L)) \]  

(17)

where \( \lambda_2 \) is a constant for a particular false alarm rate. This simple detection structure (17) shows that it is a matched filter (MF) detector with data dependent threshold \( (z^H M_{y_0}^{-1} z)(1 - \exp(-\lambda_2/L)) \) which is data independent for Gaussian clutter case and promises the optimum target detection in such scenarios. Here the detector needs to know the value of speckle covariance matrix \( M_y \) only which can easily be computed online from the data of the range cells around the CUT.

To study the detection performance of the existing MF detector the probability of detection \( (P_d) \) can be computed
from the expression as given by [7]

\[
P_d = \int_0^\infty d\tau p_r(\tau) \left[ \frac{1}{1 + \frac{\tau}{\tau + SCR}} \right]^{L-1} \]

However, to include the effect of texture fluctuation we needed to modify the expression of \(SCR\) by incorporating the texture and speckle correlation coefficients \(\rho_r\) and \(\rho_y\), respectively. The corresponding \(SCR\) can be given by [14]

\[
SCR = (1 + \rho_c)\sigma^2_p \Phi_y \Phi_y \]

where

\[
\Phi_y = \begin{bmatrix}
1 & -\rho_y & 0 & 0 \\
-\rho_y & 1 + \rho_y^2 & -\rho_y & 0 \\
0 & -\rho_y & 1 + \rho_y^2 & -\rho_y \\
0 & 0 & -\rho_y & 1
\end{bmatrix}
\]

and \(\rho_c = \rho_r \cdot \rho_y\).

It is clear from (18) that the \(P_d\) is highly dependent on the \(SCR\) and hence on the texture and speckle correlation values. Therefore, if the existing MF detector is used to detect target in the presence of clutter model (4), the detection performance will degrade with the reduction of the texture correlation value. It can also be noticed from (19) and (18) that the improvement of the detection performance with the increase of \(\rho_y\) is not much attractive because the performance improvement is same as it could be achieved for the case of completely correlated texture.

Though the detection structure (17) is little simpler compared to the proposed detection scheme (14), a severe detection loss must incur while detecting a target in partially correlated texture environments. This fact is more clearly be substantiated by using the experimental results given in Section IV. This loss is due to the shortfall in the clutter model (2) which is used for formulating the existing MF detector. Hence, we preferred to use a more accurate model (4) to formulate the proposed detector, the performance of which is presented here in the next Section.

IV. NUMERICAL AND SIMULATION RESULTS

The detection performance of the proposed GLRT detector (14) in the presence of clutter model (4) is discussed here by using the numerical results obtained in our experiment. The results of the existing MF detector in the presence of clutter model (4) is also presented to show the effect of texture fluctuation on the performance of existing MF detector. The corresponding analytical and experimental results obtained through simulation are given for studying the detection performance in detail. The similar experiment was conducted with our proposed detector (14) for the same clutter environments of various texture and speckle conditions. A few results are given here to compare the performances of the proposed and existing MF detector.

For performance study, the probability of detection \(P_d\) is plotted against the signal to clutter ratio \((SCR)\) for a fixed false alarm rate \((P_{fa})\) of 0.001. Since presently no closed form expressions of \(P_{fa}\) are available which carry the relationship between \(P_{fa}\) and \(\lambda_1\) or \(\lambda_2\) we have computed both from simulation results. The results are obtained for \(L = 4\), target Doppler frequency \(f_{dt} = 0.5/T_p\), compound-Gaussian clutter texture mean value \(\eta = 1000\) and gamma shape parameter \(\nu = 1.5\). The clutter Doppler frequencies corresponding to \(K\) different clusters are considered as \(f_a = [0.02, 0.01, 0, -0.01, -0.02] \) where \(K=5\).

It is found in literature that the existing MF detector (17) gives an attractive detection performance for completely correlated texture environments. The performance improves further with the increase of speckle correlation \((\rho_y)\) value. However, it was needed to study the performance of the existing MF detector (17) in varying texture scenarios as given in Fig. 1. Both the analytical and simulation results are presented in Fig. 1 to show that the existing MF detector performance degrades significantly with the reduction of texture correlation value. Here, the results are presented for the values of \(\rho_r = 1\) and \(\rho_r = 0.7\) where the speckle correlation value is considered as \(\rho_y = 0.9\). It can be noticed from Fig. 1 that for \(P_d = 0.8\), the curve obtained for the texture correlation value \(\rho_r = 0.7\) is almost 3dB down to the curve obtained for \(\rho_r = 1\). Hence, the existing MF detector (17) does not perform effectively in varying texture scenarios. We also found from the experimental results that the performance does not improve much even for the higher value of \(\rho_r = 0.995\).

Therefore, it was necessary to find a suitable detector which can reliably perform in such partially correlated texture scenarios. We proposed a GLRT detector of structure as given in (14) which seems to be effective in target detection in the presence of varying texture scenarios. Therefore, the obvious interest is to find the detection characteristics of the proposed detector in various clutter scenarios and compare it with the existing MF detector (17).

The Experiment is conducted for the combination of various values of \(\rho_r\) and \(\rho_y\). A few representative results are given in Fig. 2. It is found from the results given in Fig. 2 that for a particular speckle correlation value of \(\rho_y = 0.9\), both the detectors are performing same for \(\rho_r = 1\). However, for the fractional texture correlation value of \(\rho_r = 0.7\) the proposed detector performs significantly better compared to the existing MF detector. We observed from the Experimental results obtained for other values of \(\rho_r\) and \(\rho_y\), that for a fractional \(\rho_r\) value which is more appropriate to consider in practice while modeling clutter, the proposed detector always performs better compared to the existing MF detector.

After analyzing the extensive results on the detection performance we found that our proposed detector is an effective detection scheme which can be used for both the completely or partially correlated texture environments. However, it is to be noticed from the above discussed results that the proposed detector does not always perform as good as it performs for \(\rho_r = 1\). This shortfall can obviously be overcome by estimating the varying texture more accurately.
MF detector and hence, is superior for detecting a target in partially correlated texture environments. The performance of the proposed detector can further be improved by using more accurate texture estimator.

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