Abstract— Recently, it is shown in [1] that by interference cancellation through zero-forcing a diversity order of $2(M-1)$ can be achieved in a multiple access channels (MAC) based uplink two-user MIMO system with two transmit antennas at each user and $M > 2$ receive antennas at the base-station. In this paper, we show that by using improved array processing technique a diversity order of $2M$ can be achieved in a two-user interference cancellation system with two transmit and more than two receive antennas. We derive pairwise error probability (PEP) bounds for different number of receive antennas and prove the diversity order for different cases theoretically.

I. INTRODUCTION

Single user multiple-input multiple-output (MIMO) communications system provides many benefits like array gain, diversity gain, coding gain, and improved capacity as compared to the single-input single-output systems [2]. Orthogonal space-time block codes (OSTBCs) are proposed to exploit the diversity with symbol wise decoding, when the transmitter does not have channel knowledge. However, in the case of multiple-users transmitting over multiple access channels (MAC) the OSTBCs does not provide low complexity decoding. Zero-forcing is an attractive and simple method of removing interference of signals of non-favorable users from favorable user. An array processing method based on zero-forcing for the receiver is explained in [3], [4] for inter-user cancellation in multiple users MIMO system using Alamouti code [5]. The method of [3], [4] is applicable when the number of receive antennas is same as number of users. Later on an optimal decoder based on maximum a posteriori probability (MAP) rule is obtained in [6] for the interference cancellation method of [3], [4]. In [1], it is shown that by using zero forcing method it is possible to remove multiuser interference in the MIMO system where the number of receive antennas exceeds the number of MAC users. It is proved in [1] that the maximum diversity order obtained for these systems is $N(M-1)$, where $N$ and $M$ are the number of transmit and receive antennas, respectively.

In this paper, our main contributions are as follows: 1) We propose an improved interference cancellation method for two-user MIMO system with two transmit and more than two receive antennas which provides better diversity than the previously proposed interference cancellation method. 2) We have theoretically analyzed the diversity order and it is proved that the proposed system can provide diversity of the order of $2M$. 3) Analytical pairwise error probability (PEP) bounds are also obtained in order to characterize the behavior of the multiuser system.

II. SYSTEM MODEL

Consider a uplink two-user MAC based MIMO system, where each user is equipped with 2 transmit antennas and the destination or base station (BS) has $M$ receive antennas. Both users transmit Alamouti space-time block code (STBC) [5] simultaneously to the BS. It is assumed that both users transmit in the same frequency band and their transmissions are perfectly synchronized in time. Let $s_1$ and $s_2$ are the symbols of User 1 and $z_1$ and $z_2$ are the symbols of User 2, $H_1$ and $H_2$ be $2 \times M$ channel gain matrices of User 1 and User 2, respectively, consisting of zero mean circularly Gaussian distributed random values, $S_k$ and $Z_k$ be the $2 \times 2$ OSTBC matrix transmitted at block $k$ by User 1 and User 2, respectively. The received data at block $k$ of size $2M$ is

$$Y_k = S_k H_1 + Z_k H_2 + Q_k,$$  

(1)

where $Q_k$ is an $2 \times M$ matrix containing additive white complex-valued Gaussian noise (AWGN), whose elements are i.i.d. Gaussian random variables with zero mean and variance $\sigma^2$. By using the property of Alamouti STBC we can rewrite the received data $Y_k$ as [7, Eq. (11.3.6)]

$$[y_1 \ y_2] = [A_1 \ A_2 \ B_1 \ B_2] [s \ z] + [q_1 \ q_2] ,$$  

(2)

where $A_i$ and $B_i$, $i = 1, 2$ have the following form

$$A_i = \begin{bmatrix} h_{1,i,1} & h_{1,i,2} \\ -h_{1,i,2} & h_{1,i,1} \end{bmatrix}, \quad B_i = \begin{bmatrix} h_{2,i,1} & h_{2,i,2} \\ -h_{2,i,2} & h_{2,i,1} \end{bmatrix},$$  

(3)

where $h_{p,i,k} \in H_i$, $p,k \in \{1, 2\}$, $s = [s_1 \ s_2]^T$, and $z = [z_1 \ z_2]^T$. It can be further noticed that we have dropped block index $k$ in (2) for simplicity. It can be seen from (3) that $A_i$ and $B_i$ are proportional to the unitary matrices:

$$A_i^H A_i = (|h_{1,i,1}|^2 + |h_{1,i,2}|^2) I_2 = |a_i|^2 I_2$$

$$B_i^H B_i = (|h_{2,i,1}|^2 + |h_{2,i,2}|^2) I_2 = |b_i|^2 I_2,$$  

(4)

where $|| \cdot ||$ represents vector norm.

III. INTERFERENCE CANCELLATION AND DIVERSITY ANALYSIS FOR TWO RECEIVE ANTENNAS

In [3], [4], it is proposed that in the case of $N = M = 2$, the interference of the signals of User 2 can be zero forced by multiplying the received data of (2) with the following beam forming matrix:

$$D = \begin{bmatrix} I_2 \ -B_1 B_2^H \end{bmatrix}.$$

(5)

By multiplying $D$ with (2) we obtain

$$[y_1 - B_1 B_2^H y_2] = [A_1 - B_1 B_2^H A_2] s + [q_1 - B_1 B_2^H q_2].$$

(6)
By multiplying both side of (6) with unitary matrix \( B_1^H / \|b_1\| \) we get
\[
\begin{align*}
\left[ B_1^H / \|b_1\| \right] y_1 - \left[ \|b_1\| B_2^H / \|b_2\|^2 \right] y_2 &= \left[ B_1^H / \|b_1\| A_1 - \|b_1\| B_2^H / \|b_2\|^2 A_2 \right] s \\
&+ \left[ B_1^H / \|b_1\| q_1 - \|b_1\| B_2^H / \|b_2\|^2 q_2 \right].
\end{align*}
\]
(7)

We can represent (7) as
\[
\begin{bmatrix}
  r_1 \\ r_2
\end{bmatrix} = \mathcal{H} s + \begin{bmatrix}
  n_1 \\ n_2
\end{bmatrix},
\]
(8)
where \( \mathcal{H} = \left[ B_1^H / \|b_1\| A_1 - \|b_1\| B_2^H / \|b_2\|^2 A_2 \right] \). Rearranging (8) by using the property of Alamouti code [7, Eq. (6.3.9)] we get
\[
\begin{bmatrix}
  r_1 \\ r_2
\end{bmatrix} = S \begin{bmatrix}
  h_1 \\ h_2
\end{bmatrix} + \begin{bmatrix}
  n_1 \\ n_2
\end{bmatrix},
\]
(9)
where \( h_1 = a_1 + j\alpha_2 \) and \( h_2 = a_3 + j\alpha_4 \) are effective channel coefficients.

**Lemma 1:** The additive noise vector after zero-forcing is uncorrelated zero-mean additive white Gaussian noise with variance \( \sigma_n^2 \left( 1 + \|b_1\|^2 / \|b_2\|^2 \right) \).

**Proof:** Lemma 1 can be proved by calculating 
\[
E \left[ \begin{bmatrix}
  n_1 \\ n_2
\end{bmatrix} \right] \text{ and } E \left[ \begin{bmatrix}
  n_1^* \\ n_2^*
\end{bmatrix} \right]^H.
\]

Let
\[
A_i = \begin{bmatrix}
  a_{i,1} + j\alpha_{i,2} & a_{i,3} + j\alpha_{i,4} \\
a_{i,3} - j\alpha_{i,4} & a_{i,1} - j\alpha_{i,2}
\end{bmatrix},
\]
\[
B_i = \begin{bmatrix}
b_{i,1} + j\beta_{i,2} & b_{i,3} + j\beta_{i,4} \\
b_{i,3} - j\beta_{i,4} & b_{i,1} - j\beta_{i,2}
\end{bmatrix}.
\]
(10)

After some algebra, it can be shown that
\[
\begin{align*}
\alpha_1 &= \frac{1}{\|b_1\|^2} (a_{1,1} a_{1,1} b_{1,1} + a_{1,2} a_{1,2} b_{1,2} - a_{1,3} a_{1,3} b_{1,3} - a_{1,4} a_{1,4} b_{1,4}) \\
&+ \frac{\|b_1\|^2}{\|b_2\|^2} (-a_{2,1} a_{2,1} b_{2,1} + a_{2,2} a_{2,2} b_{2,2} - a_{2,3} a_{2,3} b_{2,3} + a_{2,4} a_{2,4} b_{2,4}) \\
\alpha_2 &= \frac{1}{\|b_1\|^2} (a_{1,1} a_{1,2} b_{1,2} + a_{1,3} a_{1,4} b_{1,4} + a_{1,1} a_{1,2} b_{1,1} + a_{1,3} a_{1,4} b_{1,3}) \\
&+ \frac{\|b_1\|^2}{\|b_2\|^2} (-a_{2,1} a_{2,2} b_{2,1} + a_{2,3} a_{2,4} b_{2,3} - a_{2,2} a_{2,1} b_{2,2} + a_{2,4} a_{2,3} b_{2,4}) \\
\alpha_3 &= \frac{1}{\|b_1\|^2} (a_{1,1} a_{1,3} b_{1,1} - a_{1,2} a_{1,4} b_{1,1} + a_{1,3} a_{1,4} b_{1,4} + a_{1,1} a_{1,2} b_{1,3}) \\
&+ \frac{\|b_1\|^2}{\|b_2\|^2} (-a_{2,1} a_{2,3} b_{2,1} + a_{2,2} a_{2,4} b_{2,3} - a_{2,3} a_{2,4} b_{2,2} - a_{2,1} a_{2,2} b_{2,3}) \\
\alpha_4 &= \frac{1}{\|b_1\|^2} (a_{1,1} a_{1,1} b_{1,1} + a_{1,2} a_{1,2} b_{1,2} + a_{1,3} a_{1,3} b_{1,3} - a_{1,4} a_{1,4} b_{1,4}) \\
&+ \frac{\|b_1\|^2}{\|b_2\|^2} (-a_{2,1} a_{2,1} b_{2,1} - a_{2,2} a_{2,2} b_{2,2} + a_{2,3} a_{2,3} b_{2,3} - a_{2,4} a_{2,4} b_{2,4}).
\end{align*}
\]

From (11), the channel correlation matrix can be obtained as
\[
R_h = \mathbb{E} \left[ \begin{bmatrix}
  h_1 \\ h_2
\end{bmatrix} \right] \left[ \begin{bmatrix}
  h_1 \\ h_2
\end{bmatrix} \right]^H
= \mathbb{E} \left[ \begin{bmatrix}
  \alpha_1 + j\alpha_2 \\ \alpha_3 + j\alpha_4
\end{bmatrix} \left[ \begin{bmatrix}
  \alpha_1 + j\alpha_2 \\ \alpha_3 + j\alpha_4
\end{bmatrix} \right]^H
\right]
= \left( 1 + \|b_1\|^2 / \|b_2\|^2 \right) \sigma_n^2 I_2,
\]
(12)
where \( \sigma_n^2 \) is the variance of \( \alpha_1 + j\alpha_2 \) and \( \alpha_3 + j\alpha_4 \). The effective channel gains obtained after zero-forcing are uncorrelated similar to the effective additive noise. Therefore, the diversity of the zero-forced system is not effected by the channel correlation matrix [8].

**A. PEP Analysis for Two Receive Antennas**

From Lemma 1 and (8) the probability of error in transmitting a codeword \( S_0 \) and decoding it as \( S \) can be written as
\[
\Pr (S_0 \rightarrow S | A_i, B_i) = Q \left( \frac{-\|Sh\|}{\sqrt{2 \left( 1 + \|b_1\|^2 / \|b_2\|^2 \right) \sigma_n^2}} \right),
\]
(13)
where \( h = [h_1, h_2]^T \) and \( S = S_0 - S \). By applying Chernov bound [9] on (13) we get
\[
\Pr (S_0 \rightarrow S | A_i, B_i) \leq \exp \left( \frac{-\|Sh\|^2}{\|b_1\|^2 + \|b_2\|^2} \sigma_n^2 \right),
\]
(14)
We can average (14) over \( A_i \) by using [7, Theorem 4.2] as
\[
\mathbb{E}_{A_i} [\Pr (S_0 \rightarrow S | A_i, B_i)] \leq I_2 + \frac{\|S_0\| \bar{S} S^H}{4 \left( 1 + \|b_1\|^2 / \|b_2\|^2 \right) \sigma_n^2}.
\]
(15)
Substituting the value of \( R_h \) from (12) into (14) we get
\[
\mathbb{E}_{A_i} [\Pr (S_0 \rightarrow S | A_i, B_i)] \leq I_2 + \frac{\sigma_n^2 \|S_0\| \bar{S} S^H}{4 \|b_1\|^2 \sigma_n^2} \left( \frac{\sigma_n^2}{4 \sigma_n^2} \right)^{-1}
\leq \frac{\sigma_n^2 \|S_0\| \bar{S} S^H}{4 \|b_1\|^2 \sigma_n^2} \left( \frac{\sigma_n^2}{4 \sigma_n^2} \right)^{-1}.
\]
(16)
It can be seen from (16) that the zero-forced MIMO system with two receive antennas achieve diversity of the order of 2.

**IV. INTERFERENCE CANCELLATION AND DIVERSITY ANALYSIS FOR MORE THAN TWO RECEIVE ANTENNAS**

Let us now consider the case of three receive antennas, i.e., \( M = 3 \). The received data after rearrangement can be represented as
\[
\begin{bmatrix}
y_1 \\ y_2 \\ y_3
\end{bmatrix} = \begin{bmatrix}
A_1 & B_1 & A_2 & B_2 & A_3 & B_3
\end{bmatrix} \begin{bmatrix}
s \\ z
\end{bmatrix} + \begin{bmatrix}
q_1 \\ q_2 \\ q_3
\end{bmatrix},
\]
(17)
We propose the following zero-forcing matrix which is used by the receiver to suppress the interference of User 2:

\[
D = \begin{bmatrix}
    I_2 & -\frac{B_2 B_2^H}{\|b_2\|^2} & 0_2 \\
    0_2 & I_2 & -\frac{B_2 B_2^H}{\|b_2\|^2} \\
    -\frac{B_2 B_2^H}{\|b_2\|^2} & 0_2 \\
\end{bmatrix},
\]
(18)

where \(0_2\) represents an all zero matrix of size \(2 \times 2\). From (17) and (18) we get

\[
\begin{bmatrix}
    y_1 - \frac{B_2 B_2^H}{\|b_2\|^2} y_2 \\
    y_2 - \frac{B_2 B_2^H}{\|b_2\|^2} y_3 \\
    y_3 - \frac{B_2 B_2^H}{\|b_2\|^2} y_1
\end{bmatrix} = \begin{bmatrix}
    A_1 - \frac{B_2 B_2^H}{\|b_2\|^2} A_2 \\
    A_2 - \frac{B_2 B_2^H}{\|b_2\|^2} A_3 \\
    A_3 - \frac{B_2 B_2^H}{\|b_2\|^2} A_1
\end{bmatrix} s + \begin{bmatrix}
    q_1 - \frac{B_2 B_2^H}{\|b_2\|^2} q_2 \\
    q_2 - \frac{B_2 B_2^H}{\|b_2\|^2} q_3 \\
    q_3 - \frac{B_2 B_2^H}{\|b_2\|^2} q_1
\end{bmatrix}.
\]
(19)

By multiplying (19) with a unitary matrix \(U = \text{diag} \left\{ \frac{B_2}{\|b_2\|}, \frac{B_2}{\|b_2\|}, \frac{B_2}{\|b_2\|} \right\}\), where \(\text{diag} \{ X \}\) represents a diagonal matrix with \(X\) in the main diagonal and the off-diagonal elements as zero, we obtain

\[
\begin{bmatrix}
    B_2^H y_1 \\
    B_2^H y_2 \\
    B_2^H y_3
\end{bmatrix} = \begin{bmatrix}
    \frac{B_2}{\|b_2\|} A_1 - \frac{B_2 B_2^H}{\|b_2\|^2} A_2 \\
    \frac{B_2}{\|b_2\|} A_2 - \frac{B_2 B_2^H}{\|b_2\|^2} A_3 \\
    \frac{B_2}{\|b_2\|} A_3 - \frac{B_2 B_2^H}{\|b_2\|^2} A_1
\end{bmatrix} s + \begin{bmatrix}
    \frac{B_2}{\|b_2\|} q_1 - \frac{B_2 B_2^H}{\|b_2\|^2} q_2 \\
    \frac{B_2}{\|b_2\|} q_2 - \frac{B_2 B_2^H}{\|b_2\|^2} q_3 \\
    \frac{B_2}{\|b_2\|} q_3 - \frac{B_2 B_2^H}{\|b_2\|^2} q_1
\end{bmatrix}.
\]
(20)

We can rewrite (20) as

\[
[r_1, r_2, r_3] = S [h_1, h_2, h_3] + [q_1, q_2, q_3],
\]
(21)
where \(h_1 = [\alpha_1 + j\alpha_2, \alpha_3 + j\alpha_4]^T\), \(h_2 = [\beta_1 + j\beta_2, \beta_3 + j\beta_4]^T\), and \(h_3 = [\gamma_1 + j\gamma_2, \gamma_3 + j\gamma_4]^T\), are effective channel coefficients after zero-forcing, and \([q_1', q_2', q_3'] \in \mathbb{C}^{2 \times 3}\) is effective additive noise matrix. The definitions of \(A_i\) and \(B_2\) given in (10) are valid in this case as well. Therefore, \(\alpha_1, \alpha_2, \alpha_3\), and \(\alpha_4\) are given in (11) and the values of \(\beta_1, \beta_2, \beta_3, \beta_4, \gamma_1, \gamma_2, \gamma_3, \gamma_4\) can be obtained after some algebra.

\[
\begin{aligned}
\beta_1 &= \frac{1}{\|b_2\|^2} (a_{21} b_{21} - a_{23} b_{22} - a_{22} b_{23} - a_{24} b_{24}) \\
&+ \frac{1}{\|b_2\|^2} (-a_{31} b_{31} + a_{32} b_{32} + a_{33} b_{33} + a_{34} b_{34}) \\
\beta_2 &= \frac{1}{\|b_2\|^2} (a_{11} b_{21} + a_{12} b_{22} - a_{13} b_{23} - a_{14} b_{24}) \\
&+ \frac{1}{\|b_2\|^2} (-a_{41} b_{31} + a_{42} b_{32} + a_{43} b_{33} - a_{44} b_{34}) \\
\beta_3 &= \frac{1}{\|b_2\|^2} (a_{21} b_{23} - a_{23} b_{21} + a_{22} b_{24} + a_{24} b_{22}) \\
&+ \frac{1}{\|b_2\|^2} (-a_{31} b_{33} + a_{32} b_{31} - a_{33} b_{32} - a_{34} b_{34})
\end{aligned}
\]

\[
\gamma_1 = \frac{1}{\|b_3\|^2} (a_{11} b_{31} - a_{12} b_{32} - a_{13} b_{33} - a_{14} b_{34}) \\
&+ \frac{1}{\|b_3\|^2} (-a_{11} b_{11} + a_{12} b_{12} + a_{13} b_{13} + a_{14} b_{14}) \\
\gamma_2 = \frac{1}{\|b_3\|^2} (a_{31} b_{32} + a_{32} b_{31} - a_{33} b_{33} + a_{34} b_{34}) \\
&+ \frac{1}{\|b_3\|^2} (-a_{11} b_{12} - a_{12} b_{11} + a_{13} b_{13} - a_{14} b_{14}) \\
\gamma_3 = \frac{1}{\|b_3\|^2} (a_{11} b_{33} - a_{12} b_{34} + a_{13} b_{31} + a_{14} b_{32}) \\
&+ \frac{1}{\|b_3\|^2} (-a_{11} b_{13} + a_{12} b_{14} - a_{13} b_{11} - a_{14} b_{12}) \\
\gamma_4 = \frac{1}{\|b_3\|^2} (a_{31} b_{33} + a_{32} b_{34} - a_{33} b_{32} + a_{34} b_{31}) \\
&+ \frac{1}{\|b_3\|^2} (-a_{11} b_{13} - a_{12} b_{14} + a_{13} b_{12} - a_{14} b_{11}).
\]

Next, we can calculate the correlation matrix of the effective channel as

\[
R_R = \mathbb{E}_{A_1} \left[ \left[ h_1, h_2, h_3 \right] \left[ h_1, h_2, h_3 \right]^T \right],
\]
(25)
where \(\mathbb{E} \{ \cdot \}\) stands for vectorization operator. From (11), (22), (23), and (25) we get

\[
R_R = \sigma_R^2 \begin{bmatrix}
    (1 + \frac{1}{\|b_3\|^2}) I_2 \\
    \|b_3\|^2 I_2 \\
    \|b_3\|^2 I_2
\end{bmatrix} \begin{bmatrix}
    \|b_3\|^2 I_2 \\
    \|b_3\|^2 I_2 \\
    \|b_3\|^2 I_2
\end{bmatrix} \left( 1 + \frac{1}{\|b_3\|^2} \right) I_2
\]
(26)

**Lemma 2:** The correlation matrix of the effective channel of User 1 obtained after zero-forcing the interference of the User 2 is a full rank matrix.

**Proof:** The channel correlation matrix \(R_R\) can be expressed as \(\sigma_R^2 \cdot R'_R \otimes I_2\). For a non-zero \(3 \times 1\) vector \(w = [w_1, w_2, w_3]^T\) it can be shown after some algebra that

\[
w^T R'_R w = \left( w_1 + \frac{\|b_3\|^2}{\|b_3\|^2} w_3 \right)^2 + \left( w_2 + \frac{\|b_3\|^2}{\|b_3\|^2} w_1 \right)^2 + \left( w_3 + \frac{\|b_3\|^2}{\|b_3\|^2} w_2 \right)^2.
\]
is a real symmetric matrix, it can be deduced that $R_h$ of the zero-forced MIMO system \[8\]. After some algebra we can show that $R_h$ of Kronecker product \[10\] is positive definite matrix and hence, full rank. From the property of full rank matrix, it does not effect the diversity of the zero-forced MIMO system \[8\].

Probability of error in decoding is a full rank matrix, it does not effect the diversity. By applying Chernov bound \[9\] in (31) and averaging over $i$, we get

$$\text{Pr}(S_0 \rightarrow S | A_i, B_i) = Q\left( \frac{\left\| R_q^{-1/2} (I_3 \otimes S) \text{vec} \{[h_1, h_2, h_3]\} \right\|^2}{2} \right).$$ \hspace{1cm} (31)

Substituting the values of $R_h$ and $R_q$ from (26) and (28), respectively, in (32), and using the property of Kronecker product \[10\] we get

$$\mathbb{E}_{A_i} [\text{Pr}(S_0 \rightarrow S | A_i, B_i)] \leq \left| I_2 + \frac{\sigma_h^2}{4\sigma^2} S S^H \right|^{-3}$$

$$= \left( |s_1|^2 + |s_2|^2 \right)^{-6} \left( \frac{\sigma_h^2}{4\sigma^2} \right)^{-6}. \hspace{1cm} (33)$$

It can be seen from (33) that the proposed zero-forcing matrix achieves diversity of $NM = 6$ in the case of two transmit and three receive antennas. This is significant improvement over the interference cancellation proposed in [1] where it is shown that in a two-user MIMO system with two transmit and three receive antennas a diversity order of only $N(M - 1) = 4$ can be achieved.

V. IMPROVED ARRAY PROCESSING FOR $M$ RECEIVE ANTENNAS

In the case of $M$ receive antennas, we can write the rearranged received data as

$$\begin{bmatrix}
y_1 \\
y_2 \\
\vdots \\
y_M
\end{bmatrix} = \begin{bmatrix}
A_1 & B_1 \\
A_2 & B_2 \\
\vdots & \vdots \\
A_M & B_M
\end{bmatrix} \begin{bmatrix}
s \\
z
\end{bmatrix} + \begin{bmatrix}
q_1 \\
q_2 \\
\vdots \\
q_M
\end{bmatrix}. \hspace{1cm} (34)$$

For $M$ receive antennas we propose to zero-force the interference of User 2 by using the following $2M \times 2M$ array processing matrix:

$$D = \begin{bmatrix}
I_2 & \frac{-B_1B_H^T}{\|b_2\|^2} & 0_2 & \cdots & 0_2 \\
0_2 & I_2 & \frac{-B_1B_H^T}{\|b_3\|^2} & \cdots & 0_2 \\
0_2 & 0_2 & I_2 & \cdots & 0_2 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
\frac{-B_1B_H^T}{\|b_M\|^2} & 0_2 & \cdots & 0_2 & I_2
\end{bmatrix}. \hspace{1cm} (35)$$

Multiplying $D$ with (34) and then multiplying the resultant zero-forced data with a unitary matrix $U = \cdots$
Subsection IV-A we can obtain the PEP bound for $R_T$. We can rewrite (36) as follows:

$$w^T R_i w = \begin{pmatrix} b_1^T & b_2^T & \cdots & b_M^T \end{pmatrix} \begin{pmatrix} b_1 & b_2 & \cdots & b_M \end{pmatrix} w$$

for $i = 1, 2, \cdots, M$.

By using the procedure followed in Section IV we can obtain the correlation matrix of the effective channel shown in (24). The correlation matrix of noise can also be obtained after some algebra and written as $R_b = \frac{1}{T} R_h$. $R_h$ in (24) can expressed as $R_h = \sigma_h^2 R_b$. For a non zero $M \times 1$ real vector $w = [w_1, w_2, \cdots, w_M]^T$, it can be shown after some algebra that

$$w^T R_i w = w_1^2 + \frac{\|b_1\|^2}{\|b_1\|^2} w_2^2 + \cdots + \frac{\|b_{M-1}\|^2}{\|b_{M-1}\|^2} w_M^2.$$  

(39)

It is possible to construct example to show that $w^T R_i w$ is zero only when $w$ is an all zero vector and due to the square terms in (39) $w^T R_i w$ is always greater than zero. Therefore, the symmetric matrix $R_i$ is a full rank matrix (10) and from the property of the Kronecker product (10), $R_i$ is also a full rank matrix. Therefore, $R_i$ is also a full rank matrix and hence, invertible. By following the procedure of Subsection IV-A we can obtain the PEP bound for $M$ receive antennas as follows:

$$\mathbb{E}_{A_i} \left[ \Pr \left( S_0 - S | A_i, B_i \right) \right] \leq \left( |s_1|^2 + |s_2|^2 \right)^{-2M} \left( \frac{\sigma_h^2}{\text{SNR}} \right)^{-2M}.$$  

(39)

It can be seen from (39) that the proposed zero-forcing scheme achieves the diversity of $2M$ for two transmit and $M > 2$ receive antennas.

**VI. SIMULATION RESULTS**

We have plotted the performance of the proposed and existing [1], [3], [4] zero-forcing matrices for $N = 2, M = 2, 3, 4$, BPSK constellation, and Alamouti code in Fig. 1. The performances of the scheme of [3], [4] for two receive antennas and the scheme of [1, Eq. (17)] for three receive antennas are plotted in Fig. 1. It can be seen from Fig. 1 that the proposed improved array processing scheme is able to achieve higher diversity order than the scheme of [1, Eq. (17)] for three receive antennas. The proposed scheme achieves diversity of six as compared to the diversity of four achieved by the scheme of [1, Eq. (17)] for three receive antennas. Moreover, for four receive antennas case the proposed interference cancellation scheme achieves the eighth order diversity.

**VII. CONCLUSIONS**

In this paper, we have shown that by using an appropriate interference cancellation method it is possible to achieve a diversity gain of $2M$ in a two-user multiple access based MIMO system. The diversity gain is theoretically proved by analytical PEP upper bound. The theoretical results are also corroborated by the simulations.

**REFERENCES**


