Closed-Form Error Analysis of Noncoherent FSK for Dual hop Relay Network in Nakagami-\(m\) Fading Channel

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Abstract—We analyze the end-to-end error performance of \(M\)-ary frequency shift keying (MFSK) with noncoherent detection over independent and non-identical flat Nakagami-\(m\) fading channels. A dual-hop relay transmission using amplify-and-forward (AF) protocol with average power scaling (APS) mechanism [1] is considered. Generalized closed-form expressions for the average symbol error probability (ASEP) of noncoherent MFSK has been obtained following the cumulative distribution function (CDF) approach. Moreover, we derive a simplified closed-form expression for the ASEP over high signal-to-noise ratio (SNR) over the source-relay link. Computer simulations are performed to validate our proposed mathematical analysis. The obtained error probability expressions will help the design of two-hop relay networks adopting MFSK in determining various system parameters such as the transmission power at the source and the gain factor at the relay.

Keywords—Amplify-and-forward relay; Nakagami-\(m\) fading; frequency shift keying; noncoherent detection; symbol error probability.

I. INTRODUCTION

Cooperative relaying is emerging as a viable option for energy-efficient wireless networks because of its inherent merits of system coverage extension and capacity enhancement. The idea of cooperative relaying is to substitute a single long-range direct transmission with multiple short-range low-power relay communications. As low power and low cost are the elementary constraints for system designers, it is important to choose a suitable modulation scheme while dealing with relay networks. Significant work has been carried out in [1]-[4] to address the end-to-end error performance with phase shift keying (PSK) and quadrature amplitude modulation (QAM) which are based on full channel state information (CSI). However, for some applications e.g. sensor networks, where hardware structure is a major concern, orthogonal modulation schemes with non-coherent detection might prove to be a good candidate because for non-coherent detection, there is no need to estimate the CSI resulting in simple hardware. Frequency shift keying (FSK) [5]-[8] has been considered as a promising modulation scheme for relay networks. While MFSK can reduce the transmit on-time with increased radio power consumption [9], binary FSK (BFSK) was more energy-efficient than MFSK under start-up power dominant conditions [5]-[7].


In this paper, we analyze the symbol error probability (SEP) performance of MFSK scheme employing noncoherent detection over non-identical Nakagami-\(m\) fading channels. A two-hop relay system with AF-APS mechanism is considered. Following CDF method, a closed-form ASEP expression has been derived. This approach simplifies the calculations since CDF of instantaneous SNR is used to derive the error probability. Also, we analyze the SEP performance considering non-identical Nakagami-\(m\) fading channels (which clearly depicts practical wireless relaying systems) with arbitrary fading parameters and perfect frequency and phase synchronization. Moreover, we compute simplified closed-form ASEP expression for noncoherent...
MFSK under sufficiently large SNR for the source-relay link, valid for arbitrary values of $m$.

II. RELAY ASSISTED CHANNEL MODEL

Consider a relay-assisted transmission scenario as shown in Fig. 1 where a source node $S$ communicates with the destination node $D$ through a relay $R$. In MFSK, the binary data stream is divided into $p$-tuples of $p = \log_2 M$ bits per symbol so as to denote all $M$ possible $p$-tuples as $M$ signals with orthogonal frequency subbands. In order for MFSK signals to be orthogonal, the minimum separation between adjacent frequencies is $\Delta \nu = \frac{1}{T}$ for noncoherent case, where $T$ is the symbol duration which is $p$ times the bit period. Following a baseband-equivalent discrete-time model [7], we can represent the received signals at the relay and destination node. In the first signaling interval, if a symbol is transmitted through the $i$th frequency subband, the output, $r_{ij}$, of the $j$th subband correlator at the relay node is given by

$$r_{ij} = \sqrt{E_{sx}} h_{sx}\delta(i-j) + n_j, \quad j = 1,2,\ldots, M$$

(1)

where $\delta(i-j) = 1$ for $i = j$, and $\delta(i-j) = 0$ for $i \neq j$. Also, $h_{sx}$ denotes the fading coefficient from the source to the relay node and $n_j$ is the zero-mean additive white Gaussian noise (AWGN) of the $j$th subband correlator at the relay node having a two-sided power spectral density $N_0/2$.

In the second signaling interval, the relay node multiplies the incoming signal with a gain factor $G$ and retransmits it to the destination node. Since no CSI is available at the destination node, it is natural to consider a fixed gain model [1], also called “average power scaling” (APS) in [15] with $G = \sqrt{E_r/(E_{sx} + N_s)}$ where $E_r$ is the average transmission power of the relay and $N_s$ is the noise power at the relay. When there is no frequency offset $f_o$, the output $y_{ij}$ of the $j$th subband correlator at the destination node is given by

$$y_{ij} = \sqrt{E_{sx}} h_{sx} h_{rd}\delta(i-j) + G h_{sx} n_j + v_j$$

(2)

where, $h_{sx}$ denotes the fading coefficient from the relay to the destination node and $v_j$ is the AWGN component at the destination node. Also, $E_{sx}$ and $E_{rd}$ represent the average energies available at $R$ and $D$, taking into account for possibly different path loss and shadowing effects in $S \rightarrow R$ and $R \rightarrow D$ links.

Now assuming normalized signal energy, the instantaneous end-to-end SNR of the dual-hop path can be expressed as

$$\gamma_{eq} = \frac{(E_{sx}/N_s)(E_{rd}/N_s) h_{rd}^2 h_{rd}^2}{1 + E_{sx}/N_s + (E_{rd}/N_s) h_{rd}^2}$$

(3)

When $R$ introduces fixed gain to the received signal, $\gamma_{eq}$ can be re-expressed as

$$\gamma_{eq} = \gamma_{eq}(C + \gamma_{rd})$$

(4)

where $C = E_{sx}/G^2 N_s$ and $\gamma_{eq} = h_{rd}^2 E_{sx}/N_s$, $\gamma_{rd} = h_{rd}^2 E_{rd}/N_s$ denote the instantaneous SNRs of $S \rightarrow R$ and $R \rightarrow D$ links respectively. Now, without loss of generality, we can set $E[h_{rd}^2]$ and $E[h_{rd}^2]$ to unity. Since $h_{sx}$ and $h_{rd}$ are mutually-independent and non-identical, they are modelled according to Nakagami-$m$ distribution with fading severity parameters $m_{sx}$ and $m_{rd}$, respectively, and the instantaneous SNR $\gamma$ in both the links, is a gamma distributed random variable (RV) with probability density function (PDF) given by

$$p_\gamma(\gamma) = \frac{1}{\Gamma(m)} \left(\frac{m}{\Omega}\right)^m \gamma^{m-1} \exp\left(-\frac{m \gamma}{\Omega}\right)$$

(5)

where $\Omega = E_{sx}/N_s$ and $\Omega_{rd} = E_{rd}/N_s$ and $\Gamma(\cdot)$ is the gamma function [16].

III. ERROR PERFORMANCE ANALYSIS FOR NONCOHERENT MFSK

The most commonly used technique to compute the ASEP is to average the conditional SEP in AWGN, $P_s(\gamma)$ over the PDF of the end-to-end equivalent SNR as follows

$$P_s(\gamma) = \frac{1}{\Omega} P_s(\gamma) p_{\gamma_{eq}}(\gamma) d\gamma$$

(6)

where, $p_{\gamma_{eq}}(\gamma)$ corresponds to the equivalent PDF of dual hop fading channel.

The alternate representation of $P_s(\gamma)$ in (6), in terms of the derivative of $P_s(\gamma)$ and the cumulative distribution function (CDF) $F_{\gamma_{eq}}(\gamma)$ of SNR $\gamma_{eq}$, can be derived by integrating the integral by parts and using the limiting values $F(0) = 0, P_s(\infty) = 0$, thus resulting in

$$P_s(\gamma) = \frac{-1}{\Omega} F_{\gamma_{eq}}(\gamma) F_{\gamma_{eq}}(\gamma) d\gamma$$

(7)
where $P'_s(\gamma)$ denotes the first derivative of $P_s(\gamma)$.

In order to determine the end-to-end error performance, $F_{\alpha}(\gamma)$ for dual hop relay fading channel, valid for different $m_{sr}$ and $m_{rd}$, can be stated using [17, eq. (9)] as

$$F_{\alpha}(\gamma) = 1 - \sum_{m=0}^{m_{sr}-1} Y(m,n) \exp\left(-\frac{m_{sr} \gamma}{\Omega_{sr}}\right)$$

and

$$Y(m,n) = \frac{2}{\Gamma(m_{sr})m!} \left(\frac{m_{sr} C}{\Omega_{sr}}\right)^{\frac{m_{sr} - 1}{2}} \left(\frac{m_{sr} m_{rd} C}{\Omega_{sr} \Omega_{rd}}\right)^{\frac{m_{sr} - 1}{2}}$$

where, $K_\nu(*)$ is the $\nu$-th order modified Bessel function of the second kind. When CSI is not available at the destination node, noncoherent detection schemes must be used. The optimum receiver selects the signal with the maximum value from M envelope detectors. It is known [18] that the conditional probability of symbol error for noncoherent MFSK is

$$P_s(\gamma) = \frac{1}{M} \sum_{k=1}^{M} (-1)^k \binom{M}{k} \exp[-(1 - 1/k)\gamma]$$

where, $\alpha_k = (-1)^k \binom{M}{k}$ and $\beta_k = (1 - 1/k)$.

After a substitution of (10) and (8) into (7) and making $z^2 = \gamma$, $P_s(\gamma)$ can be expressed as

$$P_s(\gamma) = \frac{1}{M} \sum_{k=1}^{M} \alpha_k \exp[-\beta_k z]$$

where

$$I = \int_0^{m_{sr} m_{rd} - 1} \exp \left(-\frac{m_{sr}}{\Omega_{sr}} z\right) K_\nu(z) dz$$

Now, using [19, eq. (2.16.8.4)] and subsequently utilizing the well known relationship [16, eq. (13.1.33)], $I$ can be derived as

$$I = \frac{R^{\nu+1}}{4} Q(\nu+1) \Gamma(m+1) \Gamma(m+T+1) U\left(m+T+1,1;\frac{R}{Q}\right)$$

where $\theta = \beta_k z$. After a substitution of (10) and (8) into (7) and making $z^2 = \gamma$, $P_s(\gamma)$ can be generalized to the compact form

$$P_s(\gamma) = \frac{1}{M} \sum_{k=1}^{M} \alpha_k \left[1 - \frac{1}{2} \beta_k z \right]$$

In the special case of Rayleigh fading, i.e. $(m_{sr} = 1, m_{rd} = 1)$ a closed form for noncoherent BFSK can be obtained from (14) by setting $M=2$ and after some manipulations as

$$P_s(\gamma) = \frac{1}{2} \left[1 - \frac{1}{\sqrt{2}} \sqrt{\gamma} \right]$$

where, $\ell = C/\Omega_{sr} \Omega_{rd}$ and $K_s(\ell), K_0(\ell)$ are the zero-order and first-order modified Bessel function of the second kind [16, eq. (9.6.21) and eq. (9.6.22)]. It can be observed that (15) can be also derived from (6) and [1, eq. (10)], pointing out the validity and the generality of our approach.

A. Error analysis under Sufficiently Large SNR for S-R Link and Arbitrary $m$

In order to derive the ASEP for arbitrary $m$, we assume sufficiently large SNR for the S-R hop [15], i.e., $E_{sr}/N_0 > E_{rd}/N_0$. Under this assumption, the end-to-end SNR is [15] $\omega_{eq} = E_{rd}/N_0$. Since a squared Nakagami RV is gamma distributed, therefore, utilizing the result due to Malik [20] on the distribution of the product of two Gamma variates, the PDF of $\omega_{eq}$ can be expressed as

$$p_{\omega_{eq}}(\omega) = \frac{2}{\Gamma(m_{sr}) \Gamma(m_{rd})} \left(\frac{m_{sr} m_{rd} C}{\Omega_{sr} \Omega_{rd}}\right)^{-\frac{m_{sr} m_{rd} C}{\Omega_{sr} \Omega_{rd}}} \times \omega^{m_{sr} m_{rd} C - 1} K_{m_{sr} m_{rd} C} \left(2 \frac{m_{sr} m_{rd} C}{\Omega_{sr} \Omega_{rd}} \omega\right)$$

Now, substituting $t^2 = \omega$ in (10) and (16), and thereafter replacing in (6) one obtains ASEP as

$$P_s(\gamma) = \frac{4}{M} \sum_{k=1}^{M} \alpha_k \left[1 - \frac{1}{2} \beta_k \frac{m_{sr} m_{rd} C}{\Omega_{sr} \Omega_{rd}} \right]$$
The involved integral in (17), when evaluated using [19, eq. (2.16.8.4)] and [16, eq. (13.1.33)], reduces to the following simplified closed form for $P_s(e)$ as

$$P_s(e) = \frac{1}{M} \left( \frac{m_{ch}}{\Omega_m} \right)^{\alpha_m} \sum_{m=1}^{M} \alpha_m \beta \sum_{m_{ch}=1}^{m_{ch}} \frac{m_{ch}!}{\alpha_m! \beta!} \frac{m_{ch}^{m_{ch}-1}}{\Omega_m}$$

(18)

IV. NUMERICAL AND SIMULATION RESULTS

In this section, some simulation results along with numerical results are presented to show that the error probability expressions are very effective for ASEP evaluation of two-hop relay networks adopting noncoherent MFSK modulations. Fig. 2 and Fig. 3 show the theoretical error performance of noncoherent BFSK and 4FSK against the average SNR per hop, different values of $m$ and $E_s/N_0 = E_R/N_0 = E_s/N_0$. We have also plotted the ASEP curves for Rayleigh faded case. With $S-R$ link being Rayleigh faded, it is observed that no appreciable improvement in ASEP can be obtained for $m > 3$ in the $R-D$ link. It is observed from Fig. 2 that for an ASEP equal to $10^{-5}$, the SNRs required are 41 dB, 34 dB and 28 dB when fading severity parameters are (a) $\{m_{ch} = 2, m_{so} = 1.5\}$, (b) $\{m_{ch} = 2, m_{so} = 3\}$, and (c) $\{m_{ch} = 3, m_{so} = 4\}$. So, a SNR gain of 7 dB and 13 dB can be achieved when fading parameter changes from set (a) to set (b) and from set (a) to set (c) respectively. Almost same amount of SNR gain can be obtained under same parameter set from Fig. 3, but the ASEP performance degrades in 4FSK case. However, as it is expected, ASEP performance improves significantly with increased fading severity conditions in both links. Fig. 4 depicts the ASEP versus $E_s/N_0$ for two-hop relay network adopting noncoherent 4FSK when the average SNR of the $S-R$ link is 35 dB [15]. It is clear from Fig. 4 that the error performance improves with increased $m$ values in both hops. Also, simulated results have been plotted to check the validity of the analytical ASEP expressions.

V. CONCLUSION

In this paper, we have analyzed the SE performance of two-hop relay networks adopting noncoherent MFSK with the fixed gain amplify-and-forward protocol over flat Nakagami-$m$ fading channels. Firstly, we examine the problem by deriving closed-form ASEP expressions for MFSK using a concise CDF based approach. For a special and practically important case, noncoherent BFSK, we obtain simplified BER expression for Rayleigh channels. Secondly, under sufficiently large SNR for the source-relay link we compute simplified closed-form ASEP expressions for noncoherent MFSK, valid for arbitrary values of $m$.

REFERENCES


Fig. 2 Average SEP versus $E_s/N_o$ for a dual hop relay network adopting noncoherent BFSK with different values of $m$ parameter in $S$–$R$ and $R$–$D$ link

Fig. 3 Average SEP versus $E_s/N_o$ for a dual hop relay network adopting noncoherent 4FSK with different values of $m$ parameter in $S$–$R$ and $R$–$D$ link

Fig. 4 Simulated and theoretical average SEP versus $E_{x0}/N_o$ for a dual hop relay network adopting noncoherent 4FSK considering $\Omega_{sr}=$35 dB