Robust, Low Complexity LLRs using the Generalised Likelihood Principle

Arun Ayyar K. Giridhar
Telecommunication and Computer Networks (TeNeT) Group
Department of Electrical Engineering, Indian Institute of Technology Madras
Chennai-600036, India
Email: arun_ayyar, giri@tenet.res.in

***Abstract*—In this work we consider the performance of turbo coded OFDM systems in the presence of co-channel interference (CCI) in reuse-1 OFDM systems such as 802.16d/e and LTE. Because of frequency selectivity and diversity mapping, CCI does not affect all the subcarriers of the desired user. Hence CCI behaves like Narrow Band Interference(NBI) with the number of affected subcarriers ranging from 10% to 30%. The NBI in frequency domain is similar to impulsive noise in the time domain. Three different ϵ-mixture models have been considered for modelling impulsive noise. In this work, we propose a low complexity LLR(Log-likelihood Ratio) for turbo codes which is robust under all the three model assumptions. The proposed LLR does not assume the knowledge NBI power, NBI pdf, fraction of subcarriers affected by NBI and the Signal to Noise Ratio(SNR). Simulation results indicate that the proposed method performs within 0.8-1dB from the optimal LLR which would have complete knowledge of the pdf parameters and it is computaionally less complex.**

I. Introduction

OFDM is a potential candidate for the physical layer of the fourth generation mobile systems. This technology has been applied in many wideband applications especially in broadcast systems such as Asymmetric Digital Subscriber Line (ADSL),Digital Audio Broadcasting (DAB), and Digital Video Broadcasting TV (DVB-TV) as well as being the proposed technique for standards such as IEEE 802.11a/g ,IEEE 802.16d/e and IEEE 802.20 and LTE(Long Term Evolution). However, wireless OFDM system on its own does not yield low bit error rates (BERs), therefore, some form of forward error correction (FEC) must be used to lower the bit error rates (BER).

Turbo codes [1] have drawn particular interest because they come close to achieving the Shannon limit . Turbo coded OFDM systems attain much lower bit error rates compared to uncoded OFDM systems within a few iterations [2].

In reuse-1 cellular systems , the main source of interference is the use of same sub-carriers at the same time in the neighboring cells/sectors. The main aim of the work is to handle symbol detection in presence of Co-channel Interference(CCI) in reuse-1 cellular systems. In these systems frequency selectivity and diversity mapping make CCI behave as Narrow Band Interference(NBI). In 802.16d/e, the base stations are frequency synchronised and hence the subcarriers of the interfering user will fall exactly on the subcarrier of the desired user. The fraction of subcarriers ϵ affected by NBI can range from $0 \leq \epsilon < 0.5$ with values greater than 0.3 occuring with very low probability. The fraction of affected sub carriers depends on number of users, the geographical distance between the desired user and the interfering user, the channel between them, channel between deisred user and the base station. Since all the factors are time varying, ϵ is also time varying.

NBI is also common in communication systems using unlicensed bands, such as the 2.4 GHz spectrum available for Bluetooth and 802.11 a/b/g. The effect of NBI in frequency domain will be similar to that of impulsive noise in time domain. The pdf of impulsive noise have tails that are thicker than the Gaussian pdf..It is well known that turbo decoding based on Gaussian pdf fails if the pdf deviates a little from being gaussian [3][4]. One approach that does not require any major modification of the existing turbo decoder can be to prevent the NBI affected bits from playing any significant in decoding. Assuming that the NBI affected subcarriers are known [4] suggests the erasure of those bits to improve the performance of the turbo decoder. However, assumption that the NBI affected subcarriers are known is not realistic in most cases, especially when the subcarriers positions that are affected by NBI may randomly vary from one OFDM symbol to another.In [5] the authors used nonlinear pre-processors to filter the received measurements without changing the internal decoder operation for decoding in presence of a class of heavy tailed noise with Cauchy, Gaussian mixture and double exponential distributions. [3] discusses the use of Hubers cost function for log-likelihood ratio (LLR) computation in the presence of alpha-stable noise. [6] proposes a Weighted-LLR(W-LLR) method to mitigate the effect of NBI on the turbo decoding. This solution does not require any major modification of the algorithm or architecture of the turbo decoder, and nor does it need any a priori knowledge about the CG pdf. In [7], author proposes an improvement on W-LLR, called the Parametric -Cauchy LLR (PC-LLR) was based on the Cauchy-Gaussian mixture model. This LLR was designed for the worst case scenario.i.e, ϵ and SIR were set to their maximum expected values, which are 0.3 and -6 dB respectively.However, the PC-LLR computation is complex.

The contributions of this paper may be summarized as follows:

- Proposing two new metrics by the use of Generalised Likelihood Principle
- We show through computer simulations that GLR-2 performs within 0.8-1 dB from the optimal performance.

The rest of the paper is organised as follows. In Section II the System model i.e. the OFDM system in presence of NBI modelled as ϵ-mixture pdf, is defined, Section III describes the decision metrics, finally Section IV explains the simulation results obtained for the proposed scheme.

II. System Model

A conventional turbo coded OFDM system with single transmitter and single receiver is considered. At the transmitter, the input bit stream is grouped into blocks of M bits.Each block of data is encoded by turbo encoder that generates N coded bits. This sequence of N code bits is then M-PSK or M-QAM modulated to produce sequence of modulated symbols $\mathbf{X_n}$ where n denotes the time index.The modulated symbols $\mathbf{X}_n$ are then transformed by IFFT to produce the OFDM symbol. A cyclic prefix is added and the OFDM symbol is transmitted through the channel. At the receiver, the cyclic prefix is discarded and received OFDM symbol is transformed by FFT to produce the sequence of received symbols $\mathbf{Y_n}$.

$$\mathbf{Y}_n = \mathbf{X}_n + \mathbf{N}_n \tag{1}$$

where $\mathbf{Y}_n = [Y_n(0),\ Y_n(1),\ \ldots\ Y_n(N)]$ is the output of the FFT block at the n^{th} instant of time. $\mathbf{X}_n$= $[X_n(0),\ X_n(1),\ ..\ X_n(N)]$ are turbo coded M-PSK or M-QAM modulated symbols, $\mathbf{N}_n$ is a vector of noise samples from the ϵ-mixture noise model.

Noise model

Several models have been proposed for the impulsive noise in the literature.

Contaminated Gaussian PDF

The most commonly used one is the Middleton class A model [8] which is composed of mixture of Rayleigh distribution for the impulse amplitude a Poisson distribution for occurence of the impulses. However this model can be approximated by a simpler ϵ-Gaussian mixture model as mentioned in [9].

$$f_g(x) = \frac{1-\epsilon}{\pi\sigma_1^2} e^{-\frac{x^2}{\sigma_1^2}} + \frac{\epsilon}{\pi\sigma_{nb}^2} e^{-\frac{x^2}{\sigma_{nb}^2}} \tag{2}$$

Contaminated Cauchy PDF

Another candidate for modelling impulsive noise are the symmetric alpha stable noise models. It has been shown they can be represented as scale mixture of Gaussian pdfs(Gaussian Mixture models(GMMs)). However GMMs cannot capture the algebraic tails of the pdf. [10] proposes to use Cauchy-Gaussian(CGM) mixture model, which inspite of being simple can capture the algebraic tail as well as the mode. CGMs can be used to fit Middleton Class B impulsive noise model which includes alpha-stable noise as a special case [11] .

$$f_c(x) = \frac{1-\epsilon}{\pi\sigma_1^2} e^{-\frac{x^2}{\sigma_1^2}} + \frac{\epsilon\gamma}{2\pi(\gamma^2+x^2)^{\frac{3}{2}}} \tag{3}$$

Since the variance is not defined for the Cauchy pdf. As an alternate estimate of noise power,we chose Geometric-Signal-to-Noise Ratio (GSNR). GSNR is defined as [3]

$$GSNR = \frac{1}{2C_g}\left(\frac{A}{S_0}\right)^2 \tag{4}$$

where $C_g = e^{C_e} = 1.78$ is the exponential of the Euler constant $C_e = 0.5772$. Here, S_0 is the geometric power of symmetric α-stable pdf. To keep the GSNR of the Cauchy pdf and the Gaussian pdf, we choose γ=$\frac{\sigma_{nb}}{\sqrt{C_g}}$ [7].

Contaminated Laplacian PDF

The ϵ-mixture model with the contaminating pdf being double exponential is a limitng case of the Mertz model which is used to model impulsive noise in a telephone plant [12] Hence the considered noise pdf is :

$$f_l(x) = \frac{1-\epsilon}{\pi\sigma_1^2} e^{-\frac{x^2}{\sigma_1^2}} + \frac{\epsilon}{\sigma_{nb}^2} e^{-\frac{2|x_R|+|x_I|}{\sigma_{nb}}} \tag{5}$$

As seen from the above equations ϵ-mixture model has four unknowns

- The contaminating pdf
- σ_1 := Signal-to-Noise Ratio(SNR) of the underlying Gaussian pdf
- σ_{nb}:= Signal-to Interference Ratio (SIR)(variance or a measure of the variance of contaminating pdf
- ϵ :=The number of affected sub-carriers

Out of these four parameters, σ_1 can be estimated, while estimating the remaining parameters is a non-trivial task.

Each received symbol represents $log_2 M$ coded bits. A soft-decision metric in the form of a log-likelihood ratio is computed at the receiver for each coded bit and is fed to a soft-decision decoder. The log-likelihood ratio of the j^{th} coded bit in the k^{th} symbol at the n^{th} time instant is defined as

$$LLR(b_j) = log\left(\frac{\sum_{b_j=1} p(\mathbf{Y_n(k)}|\mathbf{X_n(k)})}{\sum_{b_j=1} p(\mathbf{Y_n(k)}|\mathbf{X_n(k)})}\right) \tag{6}$$

Writing in terms of the density of noise

$$LLR(b_j) = log\left(\frac{\sum_{b_j=1} f_n(\mathbf{Y_n(k)} - \mathbf{X_n(k)})}{\sum_{b_j=1} f_n(\mathbf{Y_n(k)} - \mathbf{X_n(k)})}\right) \tag{7}$$

where $f_n(x)$ can be any of the above mentioned noise models. Let $\mathbf{Y_n(k)} - \mathbf{X_n(k)}) = \mathbf{Z_n(k)}$

III. Robust Decision Metrics

We are interested in looking at LLR functions that are robust to mis-specification of the noise model. Parametric Cauchy LLR (PC-LLR) [7] had the following form

$$LLR(b_j) = ln \left[\frac{\sum_{b_j=1} \frac{0.7}{\pi\sigma_1^2} e^{-\frac{|Z_n(k)|^2}{\sigma_1^2}} + \frac{0.3*0.7496}{2\pi((0.7496)^2+|Z_n(k)|^2)^{\frac{3}{2}}}}{\sum_{b_j=0} \frac{0.7}{\pi\sigma_1^2} e^{-\frac{|Z_n(k)|^2}{\sigma_1^2}} + \frac{0.3*0.7496}{2\pi((0.7496)^2+|Z_n(k)|^2)^{\frac{3}{2}}}} \right] \tag{8}$$

Generalised Likelihood Principle

The likelihood ratio test statistic obtained by replacing the unknown parameters under each hypothesis with their maximum-likelihood estimates (MLEs) is known as the generalized likelihood ratio test (GLRT). Hence it can be written as

$$L(x) = \frac{max_{\theta\in\Omega 1} p(x;\theta)}{max_{\theta\in\Omega 0} p(x;\theta)} \tag{9}$$

[13] had used GLR to propose metrics for the alpha-stable noise. It is a known fact that estimators designed for heavy tailed noise work well even if the noise is not actually heavy tailed, but the reverse is not true. Out of the three pdf considered, Contaminated Cauchy has a heavier tail than the other two. Since the SIR estimation is a non trivial task, let us reduce the dependence of the Contaminated Cauchy density on γ.

$$f_c(x;\sigma_1,\gamma,\epsilon) = \frac{1-\epsilon}{\pi\sigma_1^2} e^{-\frac{x^2}{\sigma_1^2}} + \frac{\epsilon\gamma}{2\pi(\gamma^2+x^2)^{\frac{3}{2}}}$$

Maximising the above equation w.r.t γ

$$f_c(x;\sigma_1,\epsilon) = \frac{1-\epsilon}{\pi\sigma^2} e^{-\frac{\|x\|^2}{\sigma_1^2}} + \frac{\epsilon}{\pi(3)^{\frac{3}{2}}\|x\|^2} \tag{10}$$

The resulting LLR can be written as
$\mathbf{GLR-1}$:

$$LLR(b) = ln \left[\frac{\sum_{b_j=1} \frac{1-\epsilon}{\pi\sigma_1^2} e^{-\frac{\|Z_n(k)\|^2}{\sigma_1^2}} + \frac{\epsilon}{\pi(3)^{\frac{3}{2}}\|Z_n(k)\|^2}}{\sum_{b_j=0} \frac{1-\epsilon}{\pi\sigma_1^2} e^{-\frac{\|Z_n(k)\|^2}{\sigma_1^2}} + \frac{\epsilon}{\pi(3)^{\frac{3}{2}}\|Z_n(k)\|^2}} \right] \tag{11}$$

Maximising (10) further w.r.t σ_1

$$f_c(x;\epsilon) = \frac{1}{\pi\|x\|^2} \cdot \left\{ \frac{1-\epsilon}{e} + \frac{\epsilon}{3^{\frac{3}{2}}} \right\} \tag{12}$$

The resulting bit-metric after simplification can be written as $\mathbf{GLR-2}$:

$$LLR(b_j) = ln \left[\frac{\sum_{b_j=1} \frac{1}{\pi|Z_n(k)|^2}}{\sum_{b_j=0} \frac{1}{\pi|Z_n(k)|^2}} \right] \tag{13}$$

As is clear from the above equations , GLR-1 does not assume the knowledge of SIR, while GLR-2 does not depend on any of the afore mentioned parameters and it is computationally simplest compared to GLR-1 and PC-LLR.

IV. Simulation Results

Turbo coded OFDM system with QPSK modulation is considered. The blocklength of the turbo codes is chosen to be 1024. The parameters of the turbo codes are taken from the LTE standard [14].Number of turbo iterations is set to 6. Since cauchy pdf is a candidate of the alpha stable family of pdfs, it can be generated according to [15].We have considered NBI with σ_{nb} = 1.0 (0dB)and σ_{nb} = 4.0 (6 dB), ϵ=0.1 and 0.25. We compare GLR-2 scheme with the GLR-1,PC-LLR and optimum LLR. We assume that for the optimal LLR scheme we know the NBI pdf, NBI power and the fraction of subcarriers contaminated by NBI.

Fig.(3) and Fig.(4) depict the BER curves for Contaminated Cauchy Noise with ϵ=0.1 and 0.25 respectively.Fig.(5) and Fig.(6) show the same for Contaminated Gaussain noise. The minimum expected SIR =0dB and the maximum expected SIR =-6dB. Out of the three noise models, Contaminated Gaussian has less thicker tail compared to other two, while Contaminated Cauchy has the thickest tail. In all the conditions, we observe that the proposed GLR-2 and is poorer to Opitmal-LLR by about 0.8-1dB in most cases.

V. Summary

Generalised Likelihood principle was used to propose two new soft-decision metrics. The metrics are robust under all three considered noise models. GLR-2 is computaionally less complex than GLR-1 and PC-LLR. Its assumes knowledge of none of the parameters.However, it is 0.8-1dB poorer than the optimal performance which assumes the knowledge of all the parameters.

VI. Figures

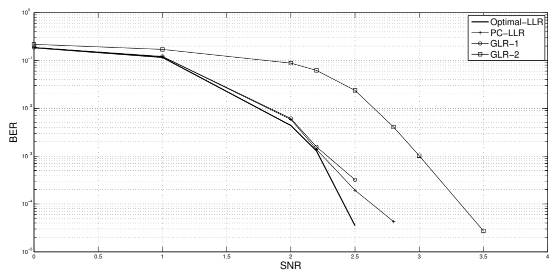

Fig. 1. BER curve for Contaminated Cauchy noise,ϵ=0.1,SIR = 0dB

REFERENCES

[1] C. Berrou, A. Glavieux, and P. Thitimajshima, "Near shannon limit error-correcting coding and decoding: Turbo codes," in *International Conference on Communications*, May 1993, pp. 1064–1070.

[2] K. K. Loo, K. K. Chai, S. A. Jimaa, T. Alukaidey, and K. Salman, "Performance of turbo coded ofdm system," 15-17,July 2002, pp. 181–184.

[3] T.C.Chuah, "Robust iterative decoding of turbo codes in heavy-tailed noise," vol. 152. IEE, Feb 2005, pp. 29–38.

[4] L. Zhang and A. Yongacoglu, "Turbo decoding with erasures for high-speed transmission in the presence of impulse noise," in *International seminar on Broadband Commmunications Access*, Feb 2002.

[5] T. Summers and S. Wilson, "Turbo code performance in heavy-tailed noise," *Proc. Conf. on Information Sciences and Systems (CISS98), Princeton, NJ, USA*, 1998.

[6] S. Kalyani, K.Giridhar, and V. Raj, "Narrow band interference mitigation in turbo coded ofdm systems," in *IEEE International Conference on Communications*, June 2007.

[7] S. Kalyani and K.Giridhar, "Interference mitigation in turbo-coded ofdm systems using robust llrs." IEEE, May 2008, pp. 646–651.

[8] D. Middleton, "Statistical-physical models of electromagnetic interference," *Electromagnetic Compatibility, IEEE Transactions on*, pp. 106–127, 1977.

[9] K. Vastola, "Threshold detection in narrow-band non-gaussian noise," *IEEE Transactions on Communications*, vol. 32, no. 2, pp. 134–139, 1984.

[10] A. Swami, "Non-gaussian mixture models for detection and estimation in heavy-tailed noise," *Acoustics, Speech, and Signal Processing, 2000. ICASSP'00. Proceedings. 2000 IEEE International Conference on*, vol. 6, 2000.

[11] D. Middleton, "Non-gaussian noise models in signal processing for telecommunications: New methods an results for class A and class B noise models," *Information Theory, IEEE Transactions on*, vol. 45, no. 4, pp. 1129–1149, 1999.

[12] J. Miller and J. Thomas, "Detectors for discrete-time signals in non-gaussian noise," *Information Theory, IEEE Transactions on*, vol. 18, no. 2, pp. 241–250, 1972.

[13] B. Peric, M. Souryal, E. Larsson, and B. Vojcic, "Soft Decision Metrics for Turbo-coded FH M-FSK Ad Hoc Packet Radio Networks," *Vehicular Technology Conference, 2005. VTC 2005-Spring. 2005 IEEE 61st*, vol. 2, pp. 724–727, June 2005.

[14] G. T. . V. (2008-05), "Technical specification group radio access network; evolved universal terrestrial radio access (e-utra); multiplexing and channel coding (release 8)," Tech. Rep., May 2008.

[15] J. M. Chambers, C. L. Mallows, and B. W. Stuck, "A method for simulating stable random variables," *Journal of the American Statistical Association*, vol. 71, no. 354, pp. 340–344, 1976. [Online]. Available: http://www.jstor.org/stable/2285309

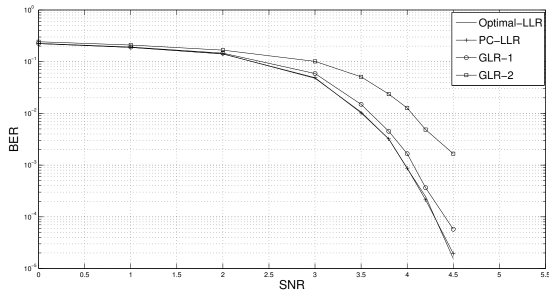

Fig. 2. BER curve for Contaminated Gaussian noise,ϵ=0.1,SIR = 0dB

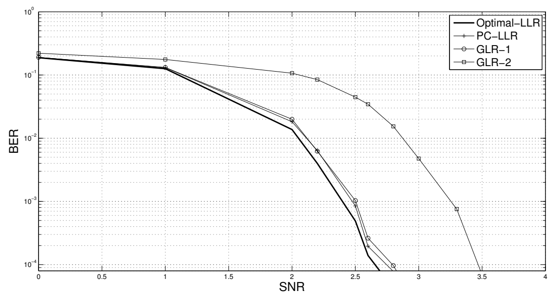

Fig. 3. BER curve for Contaminated Cauchy noise,ϵ=0.25,SIR = -6dB

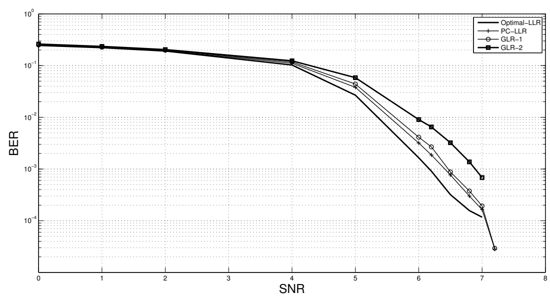

Fig. 4. BER curve for Contaminated Gaussian noise,ϵ=0.25,SIR = -6dB