COMPARATIVE STUDY OF CHANNEL TRUNCATION METHODS FOR LARGE DELAY SPREAD OFDM SYSTEMS

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Abstract: Orthogonal frequency division multiplexing (OFDM) systems use a cyclic prefix (CP) to mitigate inter-block interference (IBI) induced by the delay spread of the wireless channel. In broadcast systems such as DVB-T which use large cells and where different cells will be transmitting the same information, the effective delay spread (or channel length) can exceed the CP duration, especially for a user in the cell boundary. Since increasing the CP will result in throughput loss, many researchers have considered a form of time-domain channel truncation or pre-equalization to reduce the effective channel length when the channel length is exceeds (or is likely to exceed) the CP length. For this pre-processing of the received signal, a channel shortening pre-filter (CSP) is used just before the frequency domain processing block (FFT block) of the receiver. The coefficients of the CSP are determined from considering the characteristics of the channel, and these could also be time-varying as the channel fades. Four of the methods of implementation of CSP are described in this paper, namely: (1) MMSE channel shortening method (2) Minimization of the energy of the effective channel response outside the CP length, (3) Maximization of SIR, and (4) Maximization of SINR. This is the first work that presents a comparative performance study of these algorithms, and some insightful conclusions are drawn from this simulation study.

1. Introduction

The Cyclic Prefix (CP) is added in front of each symbol frame, to mitigate the effect of IBI (Inter-Block Interference), which might otherwise be experienced during the demodulation of the transmitted signal. The variation in channel and the effect of multipath, places limitations on the performance of an OFDM system, especially when the order of channel is larger than the cyclic prefix. We experience IBI, which would affect the SINR (Signal to Information+Noise Ratio) of the received signal. The time domain equalization appears to be an attractive option to boost the SINR of the received signal as well as mitigate IBI. We have considered four methods of building the CSP (Channel Shortening Pre-filter), are aimed at reducing the effective channel length, such that the effect of IBI is reduced. The desired effect of this CSP is illustrated in figure 1.

Figure 1: necessity of CSP to mitigate IBI

OFDM system generally suffers from severe degradation due to IBI in multipath fading environments that the delay spread is longer than the guard interval. We require a CP length of atleast 128 (when we consider Vehicular B channel conditions, as shown in the figure 2), as opposed to the actual CP length of 64. However, in practice, this IBI cannot be avoided, as the CP length is rarely long enough to mitigate it. For this, we introduce the Channel Shortening Pre-filter, which 'shortens' the channel which is perceived by the receiver. Hence, the effect of IBI is reduced as compared to the condition of without a CSP.

Figure 2: PDP of a vehicular B channel

1.1 Description of OFDM System used

A baseband OFDM model is shown in the figure below.

Figure 3: OFDM System model

After the input data is made the parallel to series conversion, N point IFFT operation is performed. A CP length $L_c$ is added to the frame. After leaving the transmitter block, the signal goes through a 'slow fading channel', which is represented by the block 'Channel'. We multiply the signal with the impulse response of a channel model. Some of the prominent channel models are described in [6]. After the channel impulse response is taken into consideration, an AWGN noise is added. At the receiver, we would consider two situations of without the CSP and the introduction of CSP. As discussed earlier, the necessity of CSP depends on the interference level in the system. After that we would remove the CP, and perform the FFT operation, followed by series to parallel conversion of the received data. Please note that this model
does not include the modulation and demodulation steps and is assumed to be done prior to the P/S and after the S/P conversions respectively.

Mobile WiMAX uses OFDMA (Orthogonal Frequency Division Multiple Access), a multi-user version of OFDM. Here, we model the wireless system of our consideration as closely as possible to the WiMAX system. IEEE 802.16e supports the FFT size of 2048, 1024, 512 and 128, so that it can support data transmission at various bandwidths [7]. Here, we assume the FFT size to be 512, and bandwidth of 5MHz, with a CP length of 64. This means that the bandwidth efficiency of the wireless system is 88.89. We simulate with the channel conditions of Vehicular B [6], where we expect the maximum effect of interference.

2. Methods used for CSP:
As mentioned earlier, the four methods of CSP are discussed here, which broadly represents two ways of approach to the problem. Four algorithms are discussed in this paper, where the algorithm discussed in section 2.2, is an extension of the algorithm in the section 2.1, and algorithm in 2.4 is an extension of the algorithm in 2.3.

2.1 MMSE Channel shortening
Originally proposed by Falconer and Magee in 1973 [1], and as discussed in Daly et al.[8], this algorithm aims at reducing the MMSE corresponding to the error signal. The algorithm proposed in 2.2 is an extension of this method.

The effective channel impulse response is given by,

\[ h_{\text{EFF}} = (L_{\text{CSP}} + 1) \]

Where, \( L_{\text{CSP}} \) represents the length of the effective channel impulse response, the coefficients of CSP are calculated.

2.2 Minimization of the Energy outside the Guard length
This algorithm, as described in [2], involves the modeling of a CSP, which reduces the energy outside the CP length in the effective channel. This involves minimization of error signal which is the difference between the effective channel response and desired channel response, same as the previous algorithm proposed in section 2.1, and could be thought as an extension of it. Given an initial estimate of the channel impulse response, the coefficients of CSP are calculated.

The channel h is represented by,

\[ h = [h(0), h(1),..., h(L_{\text{CSP}})]^T \]

The Channel shortening pre-filter is represented by

\[ f = [f(0), f(1),..., f(L_{\text{CSP}})]^T \]

Where the length of the CSP is ‘L_{\text{CSP}} + 1’.

The effective channel impulse response is given by,

\[ h_{\text{EFF}} = h \ast f = [h_{\text{EFF}}(0), h_{\text{EFF}}(1),..., h_{\text{EFF}}(L_{\text{CSP}})]^T \]

We can construct a convolution matrix \( F \) of size \((L_{\text{g}} + L_{\text{CSP}} + 1) \times (L_{\text{g}} + L_{\text{CSP}} + 1)\). The length of this effective channel is given by \( L_{\text{g}} + L_{\text{CSP}} + 1 \). As we observe, the length of the effective channel is more than the channel, but the significant taps outside the CP length and F is denoted as,

\[ F \ast f = h_{\text{EFF}} \]

For the effective channel impulse response, we assume,
1. The first condition of \( h_{\text{EFF}}(0) = 1 \), prevents a zero solution. 2. Since, all the elements with in \{1,..., N_g\} are non-relevant, hence we can reduce the scope of our consideration, which is represented by \( h_{\text{NEW}} \).

\[ h_{\text{NEW}} = \begin{cases} 1 & \text{for } k = 0 \\ 0 & \text{for } k > 0 \end{cases} \]

We consider the corresponding reduced convolution matrix of the size \((L_{\text{g}} + L_{\text{CSP}} + 1 - N_g) \times (L_{\text{CSP}} + 1)\), where the elements in \( F_{\text{NEW}} \) are the elements of F except the rows corresponding to the \{1, ..., N_g\}, can be represented as,

\[ F_{\text{NEW}} \ast f = h_{\text{NEW}} \]
where \( d = [1, 0, \ldots, 0]^* \) and the error vector, \( \delta = [\delta(1), \delta(2), \ldots, \delta(L_h + L_{CSP} - N_G)] \).

For this situation, we can compute an MMSE equalizer as,
\[
f = (F_{\text{NEW}}^*F_{\text{NEW}})^{-1}F_{\text{NEW}}^*d
\]
(14)
However, in case of the noise, the pre-equalizer might get adversely affected by the noise. So, we consider the effect of noise in our equalization equation,
\[
f = (F_{\text{NEW}}^*F_{\text{NEW}} + \frac{1}{\text{SNR}}I)^{-1}F_{\text{NEW}}^*d
\]
(15)
This expression is used to construct the channel, which takes the CIR and noise into the consideration.

### 2.3 MSIR Method (Maximization of SIR)

This algorithm as proposed in [3], which considers the maximization of SIR, forms the base of the algorithm which is discussed in Section 3.2.4. We consider the situation after the channel being multiplied by the CSP, and concentrate on the terms which contribute to the signal and interference powers respectively, which is represented as follows.

\[
h_{\text{eff}}(n) = \sum_{i=0}^{L_h} h_{\text{eff}}^i(n)\delta(n-i) + \sum_{i=0}^{L_{CSP} - 1} h_{\text{eff}}^i(n)\delta(n-v-i-1)
\]
(16)
Here, \( \{h_{\text{eff}}(n)\} \) represents the Effective CIR, \( \{h_{\text{LAR}}(n)\} \) represents the part of effective CIR, which contributes to the Signal power, and \( \{h_{\text{RES}}(n)\} \) represents the part of effective CIR, which contributes to the interference.

The corresponding matrix formulations are represented as follows,
\[
H_{\text{LAR}} = \begin{bmatrix}
h(d) & h(d-1) & \ldots & h(d - L_{CSP} + 1) \\
. & . & . & . \\
0 & h(d + L_{CSP}) & \ldots & h(d + L_{CSP} - L_{CSP} + 1) \\
\end{bmatrix}
\]
(17)
\[
H_{\text{RES}} = \begin{bmatrix}
h(0) & 0 & \ldots & 0 \\
. & . & . & . \\
h(d-1) & h(d-2) & \ldots & h(d - L_{CSP}) \\
h(d + L_{CSP} + 1) & h(d + L_{CSP}) & \ldots & h(d + L_{CSP} + 2) \\
0 & 0 & \ldots & h(L_h) \\
\end{bmatrix}
\]
(18)

The energy inside the L_{CSP} length is denoted by \( E_{\text{REC}} \).

\[
E_{\text{REC}} = f^H \hat{H}_{\text{LAR}}^H \hat{H}_{\text{LAR}} f
\]
(19)

The energy corresponding to the interference effect is denoted by \( E_{\text{OUT}} \).

\[
E_{\text{OUT}} = f^H \hat{H}_{\text{RES}}^H \hat{H}_{\text{RES}} f
\]
(20)
The SIR is defined as
\[
\text{SIR} = \frac{\sum_{i=0}^{L_{CSP}} h_{\text{LAR}}^i(n)^2}{\sum_{i=0}^{L_{CSP}} h_{\text{RES}}^i(n)^2}
\]
(21)

Hence, we consider the optimization of the above situation by minimizing the \( E_{\text{OUT}} \), while subjecting the \( E_{\text{REC}} \) to 1. Cholesky decomposition of \( E_{\text{OUT}} \) \( (E_{\text{OUT}} = G^H G) \), would yield the matrix \( G \), which is used to compute \( C = G^H E_{\text{REC}} G^H \). The minimum eigenvalue \( \lambda_{\text{MIN}} \) and eigenvector \( u_{\text{MIN}} \) is computed for \( C \), and the CSP is given by,
\[
f = G^H u_{\text{MIN}}
\]
(22)
and the modified SIR is given by,
\[
\text{SIR}_{\text{MAX}} = \log \left( \frac{1}{\lambda_{\text{MIN}}} \right)
\]
(23)
This is similar to the derivation of the optimum matched filter in [9].

### 2.4 MSINR Method (Maximization of SINR)

The algorithm proposed in 2.2 takes care of the energy of the channel outside the desired channel length. However, it doesn’t consider the effect of the noise within the length of CP. There are other methods which aims to optimize based on the SINR in the system as [3] & [10], as discussed in Section 2.2, but they fail to consider the effect of noise. The algorithm proposed in [4] & [5], is an optimization problem over the SINR of the OFDM system. In continuation with the previous notations for channel and CSP, the average energy of the output of the CSP can be written as,
\[
E_{\text{OUT}} = E_{\text{LAR}} + E_{\text{RES}} = \sum_{i=0}^{L_h} h_{\text{LAR}}^i(n)^2 + \sum_{i=0}^{L_{CSP} - 1} h_{\text{RES}}^i(n)^2 + E \left[ \sum_{i=0}^{L_{CSP}} f^2(i) \right] w(n-i)^2
\]
(24)
where \( E() \) is the expectation operator. It must be noted that, the last two components of the above equation correspond to interference and noise terms respectively. We derive the CSP by maximizing the SINR of the output of CSP.

\[
\text{SINR} = \frac{E_{\text{LAR}}}{E_{\text{LAR}} + E_{\text{RES}} + E \left[ \sum_{i=0}^{L_{CSP}} f^2(i) \right] w(n-i)^2}
\]
(25)
We rearrange the terms in \( \{h_{\text{LAR}}(n)\} \) and \( \{h_{\text{RES}}(n)\} \) to form the convolutional matrices, \( H_{\text{LAR}} \) and \( H_{\text{RES}} \), respectively, as shown below (similar to the matrices in section 2.3).

Now, the equation of SINR can be re-written as,
\[
\text{SINR} = \frac{E_{\text{LAR}}}{E_{\text{LAR}} + E_{\text{RES}} + E \left[ \sum_{i=0}^{L_{CSP}} f^2(i) \right] w(n-i)^2}
\]
(26)
The energy within the CP length, denoted by \( E_{\text{REC}} \), is in the numerator of the expression and the energy corresponding to the interference effect is denoted by \( E_{\text{OUT}} \), is in the denominator.

We follow the same method of optimization as done for the algorithm discussed in Section 2.3, ie by minimizing the \( E_{\text{OUT}} \), while subjecting the \( E_{\text{REC}} \) to 1. Cholesky decomposition of \( E_{\text{OUT}} \) \( (E_{\text{OUT}} = G^H G) \), would yield the matrix \( G \), which is used to compute \( C = G^H E_{\text{REC}} G^H \). The minimum eigenvalue \( \lambda_{\text{MIN}} \) and eigenvector \( u_{\text{MIN}} \) is computed for \( C \), and the CSP is given by,
\[
f = G^H u_{\text{MIN}}
\]
(27)
and the modified SIR is given by,
We have different methods of approaching the maximization problem, where the method with eigenvalues and eigenvectors is used to provide an elegant solution. The idea of this method is to maximize the energy which resides inside the CP length, while the power corresponding to noise and interference is minimized. This method without the noise terms is similar to that proposed in [10], and hence can be viewed as an extension to the algorithm proposed in [10].

3. Results:

Since, Min. Energy method (as mentioned in section 2.2) and MSINR method (as mentioned in section 2.4) form extensions of methods 2.1 and 2.3 respectively, we compare the results of Min. energy method with that.

We consider the Min. energy method (as discussed in section 2.2) and MSINR method (as discussed in section 2.4), the SNR has been varied from 1 to 35 dB, with the length of CSP varying from 0 to 100. Table 1 displays performance of the system with Min. energy method, in terms of the ratio of energy within the length of CP to the total energy associated with the received signal. Figure 6 displays the CSP and figure 7 represents the net effective channel at CSP length of 100 and 35 dB SNR, incase of Min. energy method.

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<td>0.0097</td>
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<td>0.0361</td>
<td>0.0845</td>
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Table 1: Performance of the system with Min. energy method, in terms of the ratio of energy within the length of CP to the total energy associated with the received signal

Figure 6: CSP used to shorten the channel

Table 2 displays performance of the system with MSINR method, in terms of the ratio of energy within the length of CP to the total energy associated with the received signal. Figure 8 displays the CSP and figure 9 represents the net effective channel at CSP length of 100 and 35 dB SNR, incase of algorithm 4.

<table>
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<th>1</th>
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<th>40</th>
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<th>100</th>
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<td>0.9541</td>
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</table>

Table 2: Performance of the system with MSINR method, in terms of the ratio of energy within the length of CP to the total energy associated with the received signal

Figure 7: Effective channel after using CSP

Figure 8: CSP used to shorten the channel
3.1 Comparison of the algorithms

Figure 10 represents the comparison between Min. Energy method and MSINR method at 20 dB SNR, while Figure 11 represents the comparison of the same at 35 dB SNR, both at the CSP length of 100.

4. Conclusions:

The spectral efficiency is an increasingly important parameter of consideration in OFDM, since it directly effects the number of users that can be supported for a given bandwidth. Hence, we are limited in the freedom of using a higher length of CP, especially when we are hit by the interference in the high delay spread channels. This work described many methods to derive the Channel shortening pre-filter which when applied, decreases the interference effects on the received signal. We observe that the MSINR method leading to better results, as compared to the Min. Energy method. There are two possible reasons for this occurrence. The noise within the shortening window might be getting amplified in [2] as compared to [4]. We also observe more number of significant taps in the figure corresponding to [4], as opposed to [2], which directly affects the number of computations. Apart from that, the both results reflect the necessity of correctness of the initial estimate, and they are better as the SNR increases, since we get a better estimate of the channel. When the two algorithms are compared for a given SNR, MSINR is observed to have a higher slope of improvement when compared to method discussed in [2] at lower CSP lengths, while the reverse trend is observed for higher orders of CSP. However, the sensitivity of these methods to the Channel estimation errors is to be explored. The two methods involve an additional convolution of the symbols with a pre-filter, and also a channel estimation of the effective channel response. This computational cost is traded with the improvement in terms of SINR. The channel shortening leaves huge prospects in terms of in-depth research analysis. It forms an interesting optimization problem, with important constraints of both additional economic and computation costs.

REFERENCES: