Performance of Symmetric and Asymmetric Turbo Code for Distributed Source Coding in a Flat Fading Channel

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***Abstract*- In this paper, the problem of distributed source coding of image data using symmetric and asymmetric turbo codes has been addressed. Both symmetric and asymmetric turbo codes are promising for distributed source coding (DSC) because of their simple encoding implementation and impressive decoding performance. We evaluate bit error rate (BER) and peak signal to noise ratio (PSNR) performance of a reconstructed image data using the side information transmitted through a Rayleigh flat fading channel unlike earlier proposals which assume perfect side information at the decoder. It is observed that Log-MAP and SOVA decoders perform very well with DSC. This observation is true in symmetric turbo code as well as in asymmetric turbo code configuration. Further the Log-MAP decoder performs marginally better than SOVA decoder in distributed source coding environment. All simulation results are presented for image data in a Rayleigh flat fading channel.**

I. INTRODUCTION

Turbo codes are promising for distributed source coding (DSC) because of their simple encoding implementation and impressive decoding performance [1]. DSC addresses the problem of compression of correlated sources that are not co-located normally. The information-theoretic foundations of DSC are based on Slepian-Wolf (SW) [2] and Wyner-Ziv (WZ) [3] theorems. It has gained high interest recently as potential solution for compressing information in applications requiring simple encoders such as visual surveillance, sensor networks and mobile camera phones. In these applications, the DSC achieves efficient compression in original data using channel code by transmitting only parity bits (fully or partially) without sending systematic bits. The Turbo codes [4] and low-density parity check (LDPC) codes are efficient channel codes that approach the SW bounds more closely. Decoder using these channel codes concatenates the parity bits with the side information and performs soft decoding or maximum a-posteriori (MAP) estimation of original data iteratively. One of the correlated sources is treated as side information at the decoder. The quality of side information at the decoder plays a critical role. The statistical dependence between image data transmitted through different channels is exploited in the channel decoder. It is observed that better channel conditions correspond to better side-information for highly correlated data. It leads better BER as well as PSNR performance with simple computational complexity at the encoder.

We consider the problem of distributed coding of two correlated images (X and Y) such that one correlated image data Y helps the decoder to reconstruct the other image data X. Although in our case both X and Y is fully correlated image data or same image data transmitted through two different paths in a rayleigh flat fading channel. Here both the parity and the systematic bits are sent through a noisy channel unlike conventional distributed video coding schemes. Hence the performance comparison in our work is closer to a realistic scenario. The noisy version of Y, acts as side information in the decoding of image data X. Here we have proposed the asymmetric turbo code in distributed source coding concept. The parallel concatenated turbo codes assumes identical component codes, hence known as symmetric turbo codes, which have either a good “waterfall” bit error rate (BER) performance or a good “error floor” BER performance, but not both. Asymmetric turbo code uses non-identical component codes. Several new classes of asymmetric turbo codes are introduced which improve performance compared to the original turbo codes over the entire range of signal-to-noise ratios. A practical set-up with symmetric and asymmetric turbo codes is described and the performance results are discussed. The rest of the paper is organized as follows. Section II deals with the bounds and principles of the DSC [1] of correlated sources. Section III deals the symmetric and asymmetric Turbo codes for DSC. Section IV shows BER as well as PSNR performance analysis of Log-MAP and SOVA decoders with and without DSC using symmetric and asymmetric Turbo codes. Section V concludes the paper

II. PRINCIPLE OF DSC FOR TWO CORRELATED IMAGES

In this paper, we have considered DSC for two correlated images. The fundamental theories of distributed source coding are based on SW and WZ theorems. SW deals with lossless coding of the correlated sources where as WZ deals with lossy coding of the correlated sources.

A. Slepian-Wolf Theorem (lossless coding)

It states that lossless compression [5] rate bound (joint entropy $H(X,Y)$) can be approached with a vanishing error probability for long sequences, even if the two sources are coded separately, provided that they are decoded jointly and that their correlation is known to both the encoder and the decoder. The achievable rate region (as shown in Fig.1) in this case can be defined as follow:

$$R_X \geq H(X|Y), \quad \textit{conditional entropy} \quad (2)$$

$$R_Y \geq H(Y|X), \quad \textit{conditional entropy} \quad (3)$$

$$R_X + R_Y \geq H(X, Y), \quad \textit{joint entropy} \quad (4)$$

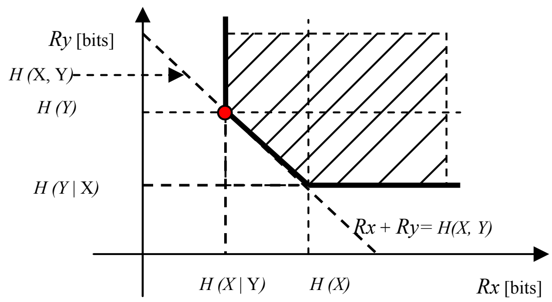

Fig.1. *Achievable rate region for distributed compression of two statistically dependent sources X and Y* [6]

B. *Wyner– Ziv Theorem (lossy coding)*

The lossless coding of sources with side-information is a subset of Slepian-Wolf coding. It corresponds to one of the corner points of the SW rate region (Fig.1). Although, the minimum total rate for the two sources is maintained at *H(X, Y)* (= *H(Y)* + *H (X|Y)*). But Wyner-Ziv [7] formulated the rate distortion function for coding of sources with side-information. So the WZ coding is a lossy version of Slepian-Wolf coding that deals with the source coding of X sequence considering the *Y* sequence, is available at the decoder. Let us assume that *X* and *Y* are two correlated continuous-valued sources to be compressed.

$$D \geq E\left[d\left(X, \hat{X}\right)\right] \quad (5)$$

where *d* is distortion and *D* is the average distortion between *X* and $\hat{X}$. $R_{X|Y}(D)$ is the rate required to encode *X* (*Y* is available at both encoder and decoder), $R_X(D)$ is the minimal rate of encoding *X* without side information and $R^*_{X|Y}(D)$ is the minimum rate needed to encode *X* (the side information *Y* is available at the decoder only). For the source *X,* the average distortion D= $d\left(X, \hat{X}\right)$ satisfy the following inequality:

$$R_{X|Y}(D) \leq R^*_{X|Y}(D) \leq R_X(D) \quad (6)$$

For correlated Gaussian sources, WZ coding suffers no rate loss with respect to joint coding and joint decoding of the two sources. i.e., $R^*_{X/Y}(D) - R_{X/Y}(D) = 0$.

III. Symmetric and Asymmetric Turbo Convolutional Codes in Distributed Coding

Turbo Convolutional Code proposed in 1993 by Berrou *et al*, is known for excellent coding gain. Here we have proposed a distributed source coding scheme based on asymmetric turbo code [5, 8]. The parallel concatenated turbo codes assume identical component codes, hence known as symmetric turbo codes which don't have both the good "waterfall" BER performance and the good "error floor" BER performance simultaneously. Asymmetric turbo code uses non-identical component codes. Several new classes of asymmetric turbo codes are introduced which improve performance compared to the original turbo codes over the entire range of signal-to-noise ratios. The information sequence is encoded twice using an interleaver between two encoders to make the two encoded data sequences approximately statistically independent of each other. Here we have used half-rate Recursive Systematic Convolutional (RSC) encoder shown in Fig.2, with each RSC encoder producing a systematic as well as a stream of parity information. In the symmetric turbo code both RSC encoder 1 and 2 is the same. We can get a different RSC encoder by using different generator polynomial. Also different constant lengths can be used for the same purpose.

At the decoder as shown in Fig.2, two RSC decoders are used. Generally SOVA and Log-MAP algorithm [9] are used as the decoding algorithm, which accept soft inputs and produce soft outputs for the decoded sequence. The component decoders exploit both the inputs from the channel and a-priori information. These soft inputs and outputs provide not only an indication of whether a particular bit is a 0 or a 1, but also a likelihood ratio which gives the probability that the bit has been correctly decoded. In turbo decoding, the magnitude of the Log Likelihood Ratio (LLR) gives sign of the bit, typically represents the soft outputs and the amplitude gives the probability of a correct decision. The LLR of a data bit u_k is denoted as:

$$L(u_k) \triangleq \ln\left(\frac{P(u_k = +1)}{P(u_k = -1)}\right) \quad (7)$$

$L(u_k) \approx 0 \Rightarrow P(u_k = +1) \approx P(u_k = -1) \approx 0.5$, and the value of u_k is uncertain and when $L(u_k) \gg 0 \Rightarrow P(u_k = +1) \gg P(u_k = -1)$, so we can be almost certain that u_k = +1. It is noted, in our case of DSC, both the parity bit-steam as well as the systematic bit-stream are transmitted over a noisy channel. These two components inputs to the turbo decoder are considered to have Gaussian distributions of independent distribution parameters due to different code-rates in separate transmission channel for parity and systematic steams. If (x_{k1}, y_{k1}) and (x_{k2}, y_{k2}) represent systematic and parity components then the conditional LLR can be derived with help of analysis shown in [10]. In this condition, the channel reliabilities can be evaluated using conditional LLR as shown below:

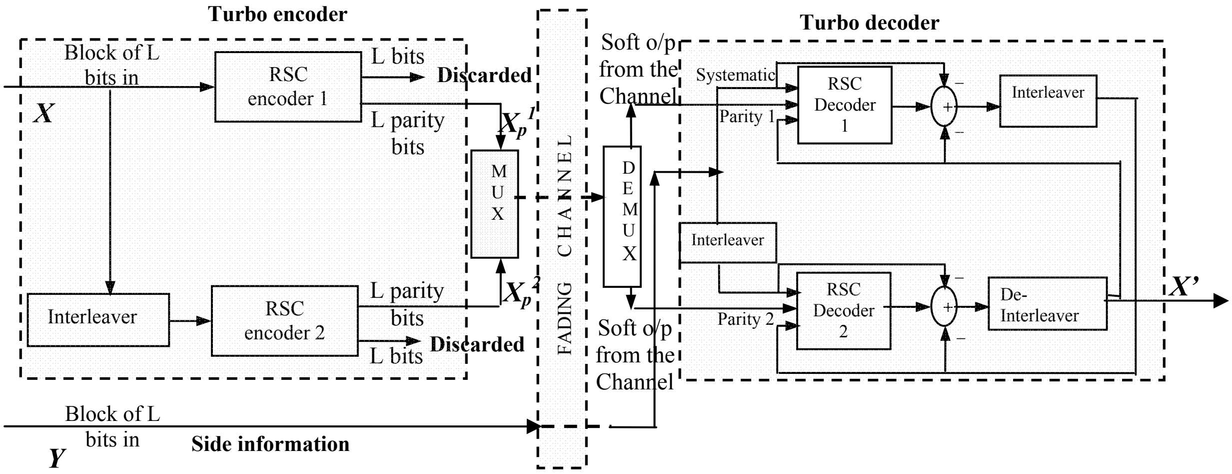

Fig.2. Block diagram of *Distributed Source Coding using turbo code*

$$
\begin{aligned}
L(y_k \mid x_k) &\triangleq \ln\left\{\left(\frac{P(y_{k1} \mid x_{k1}=+1)}{P(y_{k1} \mid x_{k1}=-1)}\right)\left(\frac{P(y_{k2} \mid x_{k2}=+1)}{P(y_{k2} \mid x_{k2}=-1)}\right)\right\} \\
&= \frac{(E_{s1})}{2\sigma_1^2}4a.y_{k1} + \frac{(E_{s2})}{2\sigma_2^2}4a.y_{k2} \\
&= \frac{(E_b * R_1)}{2\sigma_1^2}4a.y_{k1} + \frac{(E_b * R_2)}{2\sigma_2^2}4a.y_{k2} \\
&= L_{c1}.y_{k1} + L_{c2}.y_{k2} \qquad (8)
\end{aligned}
$$

where $L_{c1} = 4a\frac{E_b * R_1}{2\sigma_1^2}$, $L_{c2} = 4a\frac{E_b * R_2}{2\sigma_2^2}$, R_1 and R_2 are code rates for parity and systematic bit transmission, E_b is the transmitted energy per bit, $\sigma_1^2 = \sigma_2^2 = \sigma^2$ is the noise variance and a is the fading amplitude (a=1 for non-fading AWGN channel and for fading channel it is rayleigh coefficients) . L_{c1} and L_{c2} are defined as the channel reliability values, and depends only on the SNR and fading amplitude of the channel. Therefore, the conditional LLR $L(y_k \mid x_k)$ is the matched filter output y_k multiplied by the channel reliability value over a Gaussian or fading channel.

IV. Results and Discussions

Our proposed DSC scheme based on asymmetric turbo code is compared with DSC scheme based on symmetric turbo code. The BER as well as PSNR performance curve has been shown. The peak signal to noise ratio (PSNR) is the most commonly used parameter to measure the quality of a image with the help of mean square error (MSE). For two $m\times n$ monochrome images *I* and *K*, *PSNR* (in dB) is defined as: $PSNR = 10.\log_{10}\left(\frac{255^2}{MSE}\right)$ where $MSE = \frac{1}{m.n}\sum_{i=0}^{m-1}\sum_{j=0}^{n-1}(I(i,j)-K(i,j))^2$ and *I* (or *K)* is the original (or reconstructed) image. The DSC case has been compared with that of "without DSC" case using Log-MAP and SOVA decoders. Performance curves of symmetric turbo code are also shown for comparing the scheme with asymmetric turbo code case. In this case also both Log-MAP and SOVA decoders are used for comparison. The different parameters used in simulation of symmetric and asymmetric turbo code are shown in Table 1. In both the environments of symmetric as well as asymmetric turbo code the baseband transmission through Rayleigh flat fading channel is considered. Although all simulations are carried on fixed foreman image frame but data bits are randomized before transmission through channel such that performance curves are independent of input fixed image data.

TABLE I

Different parameters of symmetric and asymmetric turbo codes in DSC

Type of codes	*Symmetric Turbo Code*	*Asymmetric Turbo Code*
Decoding Algorithm	Log-MAP or SOVA	Log-MAP or SOVA
Generator polynomials	[7,5] in octal	[13 ,11] & [13, 9] in decimal
Frame size	1024 + m (memory)	1024 + m
Coderate (parity bits)	1/2	1/2
Coderate (systematic)	1	1
Transmission	Baseband	Baseband
channel	Rayleigh flat fading	Rayleigh flat fading

A. Simulation environment

The block diagram of basic distributed source coding scheme is shown in Fig. 2. Only luminance component of a Foreman frame (image) is taken for simulation study. First of all we have converted input foreman frame into bits then it is transmitted through channel after randomizing. Here the DSC scheme is for an image. So, in our simulation, ***X*** and **Y** (shown in Fig. 2) is same bit stream corresponding to the input image. These bits are transmitted in a block of L bits. Here, L is interleaver size. The first set of bit stream is encoded and then transmitted after discarding the systematic bits using turbo code. X_p^1 and X_p^2 are transmitted parity bits as shown in Fig.2. The second set of bit stream derived from different correlated frame (here it is same i.e., $Y=X$) is transmitted as a side information for the turbo decoder. The side information bit stream treated as compensation of systematic bit for corresponding turbo decoder. Since the systematic bits are discarded in the first channel after encoding, the parity bits are sent with rate half code instead of earlier rate one-third code. The systematic bits sent through second channel, but with rate one code. In both the channel, noise conditions are kept same to evaluate the performance in more realistic scenario unlike conventional DSC techniques. In symmetric turbo encoding case both recursive convolutional codes are same and it is different in asymmetric case. Also Log-MAP case is compared with the SOVA decoding case for both the symmetric and asymmetric turbo coding environments.

B. Simulation results and discussion for symmetric turbo code

The BER plots shown in Fig. 3a are obtained for symmetric turbo coding "with DSC" and "without DSC" using Log-MAP and SOVA decoders. Plots are shown for fully correlated image data (i.e., correlation=1.00). These curves corresponding to **a**, **b** as shown in Fig. 3a are consistent with the results shown in [9] for "Without DSC" case. Without DSC, is equivalent to the classical turbo encoding and decoding without side information. In this case decoder gets systematic bit like conventional turbo decoder not as side information. Approximately more than 2.5 dB improvements we are getting in distributed source coding for Log-MAP decoder (for curve **d**) and that of 2.0 dB gains for SOVA decoder case (for curve **c**) at lower values of E_b/N_0. Highest PSNR shown in Fig.3b are corresponding to zero mean square error. Negligible error is assumed for zero mean square error. This gives maximum PSNR value of ~148 dB instead of infinity.

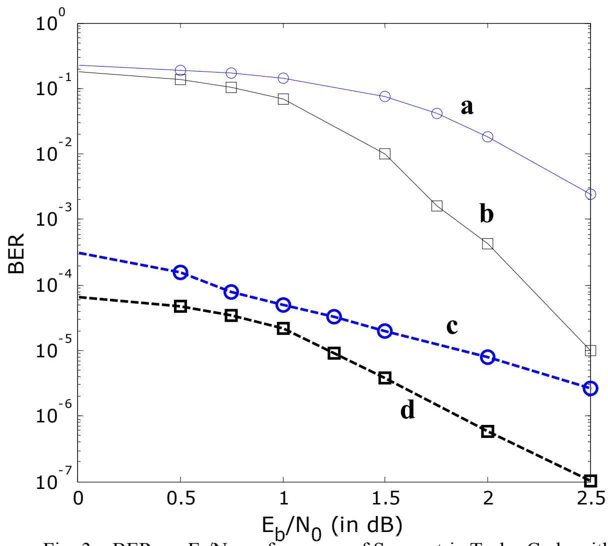

Fig. 3a: BER vs. E_b/N_0 performance of Symmetric Turbo Code with DSC and without DSC: **a**.) SOVA decoder without DSC for code rate 1/3 **b**.) Log-MAP decoder without DSC for code rate 1/3 **c**.) SOVA decoder with DSC **d**.) Log-MAP decoder with DSC

Log-MAP decoders' are showing better performance for both with-DSC and without-DSC than that of SOVA. Most importantly, the coding scheme with DSC shows much better performance than that of without DSC for both the decoders at very low values of E_b/N_0. DSC performance (i.e., curve **d** & **c**) saturates at E_b/N_0 value of 2.5 dB onwards, but without DSC case (i.e., curve **b** & **a**) saturate at 4 dB and 6 dB onwards for Log-MAP and SOVA decoders respectively.

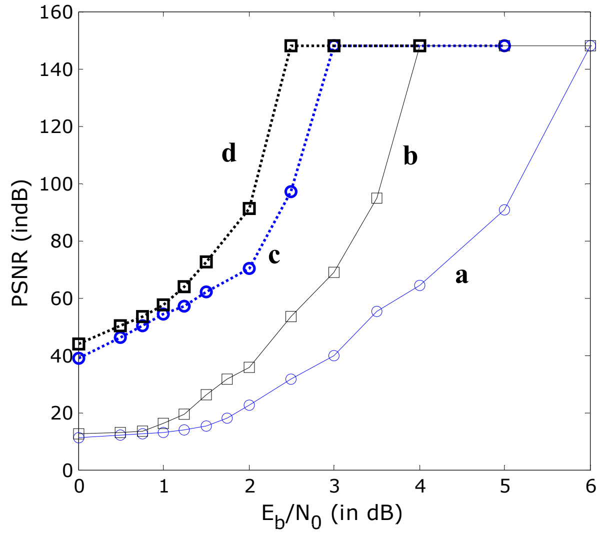

Fig. 3b: PSNR vs. E_b/N_0 performance Symmetric Turbo Code with DSC and without DSC: **a**.) SOVA decoder without DSC for code rate 1/3 **b**.) Log-MAP decoder without DSC for code rate 1/3 **c**.) SOVA decoder with DSC **d**.) Log-MAP decoder with DSC

C. Simulation results and discussion for asymmetric turbo code

The performance curves in asymmetric turbo code environment have been shown in Fig. 4a and Fig. 4b.

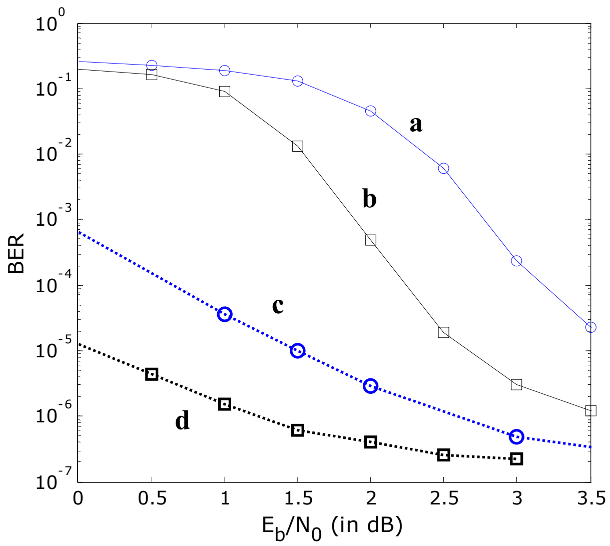

Fig. 4a: BER vs. E_b/N_0 performance of Asymmetric Turbo Code with DSC and without DSC: **a**.) SOVA decoder without DSC for code rate 1/3 **b**.) Log-MAP decoder without DSC for code rate 1/3 **c**.) SOVA decoder with DSC **d**.) Log-MAP decoder with DSC

In this case also plots are shown for fully correlated image data with correlation value 1. One set of plots (i.e., curve **c** & **d**) are shown for distributed source coding (DSC) environment and other set of plots (i.e., curve **b** & **a**) for without DSC. The BER plots in asymmetric case are nearly matching with symmetric one for "Without DSC" case. From Fig. 4a, we find approximately 3.0 dB improvements in distributed source coding for Log-MAP decoder (curve **d**) and that of 2.5 dB gains for SOVA decoder case (curve **c**) at lower values of E_b/N_0. Without DSC, is equivalent to the classical turbo encoding and decoding without side information. It is observed from Fig. 4b that Log-MAP decoders' are showing better performance for both with-DSC and without-DSC than that of SOVA decoders. Most importantly, the coding scheme with DSC shows much better performance than that of without DSC for both the decoders at very low values of E_b/N_0. DSC performance saturates at E_b/N_0 value of 2.0 dB onwards, but without DSC case saturate at 4 dB and 6 dB onwards for Log-MAP and SOVA decoders respectively. For given specification (as defined in Table.1) of turbo code, symmetric coding case shows slightly better performance than asymmetric case without DSC. But asymmetric case show better result for Log-MAP decoder with DSC as shown in Fig.4a as well as Fig. 4b.

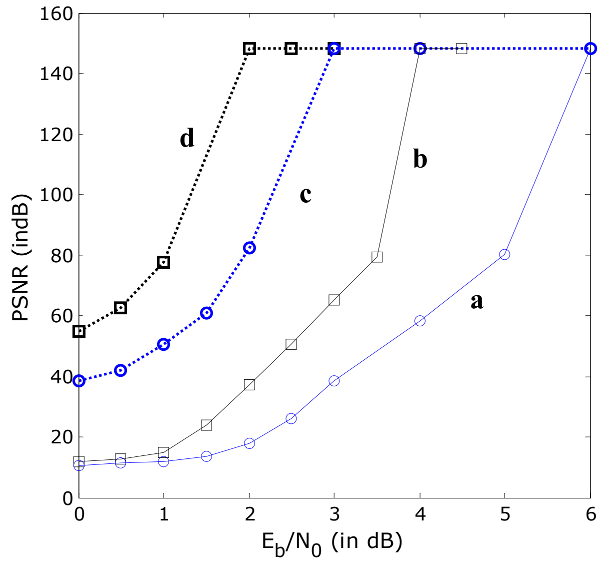

Fig. 4b: PSNR vs. E_b/N_0 performance Asymmetric Turbo Code with DSC and without DSC: **a**.) SOVA decoder without DSC for code rate 1/3 **b**.) Log-MAP decoder without DSC for code rate 1/3 **c**.) SOVA decoder with DSC **d**.) Log-MAP decoder with DSC

V. CONCLUSION

The distributed source coding performs better than joint source encoding and decoding case in terms of the BER and the PSNR performance. This is possible because distribution of the source data makes use of different code rate for coded data transmission through more than one channel, although decoder does joint decoding. This makes the DSC scheme much better than that without DSC for both Log-MAP and SOVA decoders at very low values of E_b/N_0. Log-MAP decoder shows better PSNR performance as well as BER performance than that of SOVA decoder with approximately double the computation time in DSC. Asymmetric turbo code shows almost the same performance as symmetric turbo code using Log-MAP and SOVA decoder without DSC. Also, asymmetric turbo code using SOVA decoder with DSC performs same as that of the symmetric case. However, the Log-MAP decoder with DSC performs better in asymmetric turbo code than symmetric turbo code. In distributed source coding scheme, the side-information is a compensation of systematic data at the turbo decoder. So, with same noisy side information asymmetric turbo code can perform better in terms of PSNR as well as BER performance even in rayleigh fading channels by exploiting the correlation at the decoder.

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