Performance Degradation of M-QAM-OFDM Systems Jointly Affected by Nonlinear Distortion and IQ Imbalance in AWGN Channels

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Abstract—The OFDM systems are highly sensitive to the nonlinear distortion, introduced by the high power amplifier (HPA) at transmitter and to the inphase-quadrature (IQ) imbalance of the down-converter at the receiver. In this paper, the joint effects of these impairments on the performance of the OFDM systems with M-QAM modulation are investigated. Moreover, an analytical formulation of the bit-error rate (BER) and total degradation (TD) performances of the M-QAM-OFDM systems in AWGN channels as a function of the output back-off (OBO) and IQ imbalance parameters are given. The computer simulation results confirm the accuracy and validity of our proposed analytical approach.

Keywords - OFDM, nonlinear distortion, IQ imbalance, BER performance, theoretical analysis

I. INTRODUCTION

The orthogonal frequency-division multiplexing (OFDM) modulation format has been proposed as the standard for a variety of digital communication applications, such as digital video broadcasting-terrestrial (DVB-T), digital audio broadcasting (DAB) and for wide-band wireless communication systems [1]. However, due to a greatly variable envelope or high peak-to-average power ratio (PAPR), a major impairment for the OFDM systems, especially in broadcasting applications, is the presence of a nonlinear HPA at the transmitter, which introduces nonlinear distortion [1]. The high sensitivity to IQ imbalance, is also another serious impairment [2].

Up to now, in the literature, the effects of the nonlinear distortion introduced by HPA and the IQ imbalance of down converter on the performance of the OFDM system have been investigated separately in many papers [2]–[8]. However, there is no papers dealing with the joint effects of nonlinear distortion and the IQ imbalance on the OFDM system performance.

The aim of this paper is to study analytically performance of OFDM systems that jointly takes into account IQ imbalance and nonlinear distortion in AWGN channels. The performance figure herein considered include the BER and TD, as a function of the OBO values and IQ imbalance parameters.

The paper is organized as follows. Section II, describes the system model. The nonlinear distortion of HPA and the IQ imbalance of the down-converter are also introduced. In section III, the joint effects of the nonlinear distortion and the IQ imbalance on the BER and TD performances of the OFDM systems are analytically evaluated. Section IV is devoted to the numerical results and discussions. Finally, conclusions are given in section V.

II. SYSTEM MODEL

A. Transmitter

In this paper, we focus on the problem of receiver IQ imbalance while the transmitter enjoys perfect IQ balance. A baseband equivalent scheme of the considered OFDM system can be shown as in Fig. 1. The samples of complex baseband OFDM signal $x(t)$ with $N$ subcarriers transmitted during time interval $t \in [0, T_b]$ is generated by taking inverse discrete Fourier transform (IDFT) of data symbols $d_k$,

$$x[m] = x(mT_s) = \frac{1}{N} \sum_{k=0}^{N-1} d_k[k] e^{j(2\pi k/N)T_s}$$  \hspace{1cm} (1)$$

where $d_k[k]$ represents the complex data symbol for the $k$th subcarrier that is generated at rate $1/T_s$ and $T_b = NT_s$ is the OFDM symbol period. The data symbols belong to an alphabet of elements, which depend on the modulation format adopted, and have the same probability.

The HPA is modelled as a memoryless device by means of its AM/AM and AM/PM curves. Due to the central limit theorem, a baseband OFDM signal $x(t)$ is a complex Gaussian process with Rayleigh
envelope distribution [1]. Hence, by using the Bussgang theorem, amplified baseband signal $y(t)$ can be expressed as [7]

$$y(t) = \kappa_0 x(t) + n_d(t)$$  \hspace{1cm} (2)

where the complex coefficient $\kappa_0$ and the autocorrelation function $R_{n_d}(\tau)$ of nonlinear distortion noise $n_d(t)$ depend on the AM/AM and AM/PM curves of the HPA and on the input back-off (IBO) to the HPA [7]. The IBO is defined as the ratio between the input saturating and the mean input power. It can be shown that when $N$ is very large, as in DVB-T, the nonlinear distortion noise treat as an additive Gaussian process [7]. In order to obtain $\kappa_0$ and autocorrelation function $R_{n_d}(\tau)$ and hence the power spectral density (PSD) of the nonlinear noise process, we follow the method presented in [7],[8]. According [7], those can be easily calculated by closed form expressions when the nonlinear distortion is represented by a Bessel series expansion. Consequently, we have a complete characterization of both the effects of linear gain $\kappa_0$ and nonlinear distortion noise $n_d(t)$ produced by the HPA.

B. Receiver

The complex baseband signal received through the AWGN channel represented in Fig. 1 is expressed by

$$z(t) = y(t) + n_c(t)$$  \hspace{1cm} (3)

where $n_c(t)$ is a complex zero-mean AWGN with one-sided PSD $N_0$. We assume a receiver with IQ imbalance due to down-converter. Let $\varepsilon$ and $\delta$ denote a gain imbalance and phase imbalance between the I and Q branches, respectively. The complex baseband equation in the time domain for the IQ imbalance effect on the received complex baseband signal $z(t)$ is given by [2] as

$$w(t) = \mu z(t) + \lambda z^*(t)$$  \hspace{1cm} (4)

where distortion parameters $\mu$ and $\lambda$ express the impact of IQ imbalance on the received signal and are related to the gain and phase imbalances as follows

$$\mu = \cos(\delta/2) + j\varepsilon \sin(\delta/2)$$  
$$\lambda = \varepsilon \cos(\delta/2) - j\sin(\delta/2)$$  \hspace{1cm} (5)

Note that if the I branch is balanced with the Q branch, no distortion will exist in (3) since $\varepsilon = 0$ and $\delta = 0$ which results $\mu = 1$ and $\lambda = 0$. Moreover, the gain imbalance is stated in dB as $20\log(1+\varepsilon/1-\varepsilon)$.

The signal $w(t)$ is successively sampled at the rate $1/T_s$, thus the symbols received on the $k$th subcarrier are expressed, after the discrete Fourier transform (DFT) processing, by

$$d_c[k] = \sum_{m=0}^{N-1} w(mT_s) e^{-j(2\pi m/N)T_s}$$  \hspace{1cm} (6)

where $d_c^m[k] \triangleq d_c^m[-k]$ is the transmitted OFDM symbol, mirrored over the subcarriers and $N_n[k]$ terms is equal to

$$N_n[k] = \mu (N_d[k] + N_c[k]) + \lambda \left( N_d^* [k] + N_c^* [k] \right)$$  \hspace{1cm} (7)

The Gaussian noise term $N_c[k]$ obtained by DFT of the thermal Gaussian noise of the receiver has zero mean and variance $\sigma^2_n[k] = \sigma^2_n = N_0/2$ and the term $N_d[k]$ obtained by DFT of the nonlinear distortion noise has zero mean and variance $\sigma^2_d[k]$. It is easy to show, the effect of the nonlinear distortion noise on each OFDM subcarrier is different [7].

The decision variable for the $k$th subcarrier is then obtained by compensating for all the attenuations and rotations introduced by the HPA (i.e. $\kappa_0$) and by the IQ imbalance (i.e. $\mu$). Hence, the scaled decision variable $r[k]$ can be expressed by

$$r[k] = \left( \frac{1}{\kappa_0 \mu} \right) d_c[k]$$  
$$= d_c[k] + \left( \frac{\lambda}{\mu} \right) d_c^* [k] + \left( \frac{1}{\kappa_0 \mu} \right) N_n[k]$$  \hspace{1cm} (8)

As an example, a received constellation of a 16-QAM-OFDM system under two different cases for the noise-free channel is shown in Fig. 2.

III. ANALYTICAL APPROACH

In this section, we evaluate the BER and TD performances for the OFDM systems in presence of both nonlinear distortion and IQ imbalance. Recently, an analytical expression for the BER performance of OFDM systems in presence of IQ imbalance as functions of the gain and phase imbalance is derived [5]. We try to extend the theoretical approach proposed in [5] to study the joint effects of nonlinear distortion and the IQ imbalance on the OFDM system performance.

In general, the symbol error rate (SER) performance for a M-QAM-OFDM system in AWGN channel is defined as the average of SER performances over all subcarriers, i.e.
where \( N_a \) represents the number of active subcarriers used to transmit information within the total number \( N \) of subcarriers. The theoretical solution to the SER performance for rectangular M-QAM is easily determined from [9]

\[
SER = 1 - P_C
\] (10)

where \( P_C \triangleq (1 - P_I)(1 - P_Q) \) is the probability of correct decision for the M-QAM system. \( P_I \) and \( P_Q \) are the probability of error of PAM with \( \sqrt{M} \) signal points for each I and Q signal components, respectively.

To calculate the SER for the transmitted symbol over the \( k \)th subcarrier, assuming the \( i \)th transmitted symbol and interference term are caused by the \( j \)th symbol, we first obtain the corresponding \( P_I^j (i) \), \( P_Q^j (i) \) and \( P_C^j (i) \) and then, we average the above \( P_C^j (i) \) expression over all of complex data symbols and the interference terms.

Defining, \( s [k] \triangleq d_i [k] \) as transmitted symbol over the \( k \)th subcarrier, \( e [k] \triangleq (\lambda/\mu) d_i^a [k] \) as the scaled version of interference term and \( n [k] \triangleq (1/\kappa_0\mu) N_n [k] \) as the total noise samples, the scaled decision variable \( r [k] \) in (8) can be expressed by

\[
r [k] = s [k] + e [k] + n [k]
\] (11)

For DVB-T standard which employs M-QAM modulation, assuming a signal constellation with distance \( 2d \) between adjacent symbols, \( s [k] \) and \( e [k] \) belong to \( A \) and \( B \) sets, respectively.

\[
A = \left\{ d \left( 2m - 1 - \sqrt{M} \right) + jd \left( 2n - 1 - \sqrt{M} \right) \right\}
\]

\[
B = \left\{ \frac{\lambda}{\mu} d \left( 2m - 1 - \sqrt{M} \right) - j\frac{\lambda}{\mu} d \left( 2n - 1 - \sqrt{M} \right) \right\}
\] (12)

where \( (m, n) = \left\{ 1, 2, ..., \sqrt{M} \right\} \). Since the M-QAM signal mean power is

\[

P_A = \frac{2(M - 1)}{3} d^2
\] (13)

then, for the HPA input signal with the mean power \( \sigma_z^2 = N_a P_A \) in the M-QAM-OFDM system, it is easy to show

\[
d^2 = \frac{\sigma_z^2}{N_a} \frac{3}{2(M - 1)}
\] (14)

Moreover, since nonlinear noise and thermal noise are mutually independent, the probability density function (pdf) of the nonlinear plus thermal noise is also Gaussian and variance of total noise samples \( n [k] \) can be easily evaluated as

\[
\sigma_T^2 [k] = \left( \frac{1}{\kappa_0} \right)^2 + \left( \frac{\lambda}{\kappa_0 \mu} \right)^2 \left( \sigma_n^2 + \sigma_z^2 [k] \right)
\] (15)

From now onwards we will consider only \( k \)th subcarrier. Therefore, for simplicity and convenience, we can drop \( k \) from the above notations so that the received decision variables for the I and Q branches, can be written as

\[
r_I = s_I + e_I + n_I
\]

\[
r_Q = s_Q + e_Q + n_Q
\] (16)

where I and Q subscribes present real and imaginary parts, respectively. To illustrate proposed method that has been used in this paper, the signal constellation for 16-QAM is depicted in Fig. 3a assuming that all the attenuations and rotations will be compensated for before the decision process. The decision thresholds for a prototype symbol affected by interference due to IQ imbalance are also shown in Fig. 3b. With the aid of Fig. 3c., the symbol error probability conditioned to the \( i \)th symbol and interference term caused by the \( j \)th symbol for I component, i.e. \( P_I^j (i) \), can be determined by

\[
P_I^j (i) = \left\{ \begin{array}{ll}
Q \left( \frac{d-e_i}{\sigma_T} \right), & i \in \Lambda_L \\
Q \left( \frac{d+e_i}{\sigma_T} \right), & i \in \Lambda_R \\
Q \left( \frac{d-e_i}{\sigma_T} \right) + Q \left( \frac{d+e_i}{\sigma_T} \right), & else
\end{array} \right.
\] (17)
where,
\[
\Lambda_L = \{1, 2, \ldots, \sqrt{M}\}
\]
\[
\Lambda_R = \{M - \sqrt{M} + 1, M - \sqrt{M} + 2, \ldots, M\} \quad (18)
\]

Using Fig. 3d., similar results are obtained for \(P_Q^i(i)\) with these differences that \(e_1, \Lambda_L\) and \(\Lambda_R\) are substituted by \(e_Q, \Lambda_U\) and \(\Lambda_D\), respectively, where,
\[
\Lambda_U = \{\sqrt{M}, 2\sqrt{M}, \ldots, \sqrt{M}\}
\]
\[
\Lambda_D = \{1, 1 + \sqrt{M}, \ldots, 1 + (\sqrt{M} - 1)\sqrt{M}\} \quad (19)
\]

The probability of correct decision \(P_C\) should be evaluated by averaging the conditional probabilities \(P_C^i(i)\) over all of the data symbols and the interference terms,
\[
P_C = \frac{1}{M^2} \sum_{i=1}^{M} \sum_{j=1}^{M} P_C^i(i)
\]
\[
= \frac{1}{M^2} \sum_{i=1}^{M} \sum_{j=1}^{M} \left[ (1 - P_Q^i(i)) (1 - P_Q^j(i)) \right] \quad (20)
\]

Therefore, according to (10) SER for the \(k\)th subcarrier is can expressed by
\[
(SER)_k = 1 - (P_C)_k \quad (21)
\]

Finally, from (9) and assuming a Gray-coded signal set, BER can be written as
\[
BER = \frac{1}{\log_2 M} \left( \frac{1}{N_a} \sum_{k=1}^{N_a} (SER)_k \right) \quad (22)
\]

Note that the BER performance is generally sketched in respect of the signal to noise ratio (SNR) per bit \(\gamma \equiv E_b/N_0\). Hence, we need find a relation between \(\gamma\) and \(\sigma_n^2\). The SNR at the receiver is generally defined as the power ratio between the received signal and the thermal noise. Consequently, SNR per bit can be expressed by
\[
\gamma = \frac{1}{N_a \log_2 M} \frac{\sigma_y^2}{2\sigma_n^2} \quad (23)
\]

where \(\sigma_y^2\) is the mean power of the HPA output signal. However, the BER performance are not directly imposed by the \(\gamma\), because an increase of variance of nonlinear distortion noise deteriorates the BER performance while increasing the \(\gamma\).

The TD is a well known performance figure used in literature which describe the difference between the maximum power of the HPA and the output power of a linear amplifier required to guarantee a predefined BER. The TD versus the OBO to obtain a target BER can be defined as [3]

\[
TD [\text{dB}] = (\gamma_{NL} [\text{dB}] - \gamma_L [\text{dB}]) + \text{OBO [dB]} \quad (24)
\]

where the OBO is defined as the ratio between the maximum and the mean output power, \(\gamma_{NL}\) is the required SNR per bit in dB to obtain a target BER for a given value of the OBO, and \(\gamma_L\) is the required SNR per bit to obtain the same BER in absence of nonlinear HPA. The TD depends on the working point and there is an optimum working point of the system corresponds to the minimum of function (24). As a consequence, the minimization of the TD can be considered a fair criterion for the selection of the optimum OBO value. Obviously, the TD can also be analytically evaluated by substituting in (24) the quantities obtained for the BER performance.

### IV. Numerical Results

In order to confirm of the proposed analytical approach, the BER and TD performances comparison between analytical results and computer simulation is demonstrated in this section.

The OFDM signal for simulations is similar to the DVB-T standard [10]. 1705 active subcarriers makes use of the 2K-Mode with 16-QAM and 64-QAM modulations for each subcarrier. To obtain the following numerical results, we have considered the HPA model described in [8]. The AM/AM and AM/PM distortion curves are shown in Fig. 4.

Figs. 5-6 compare the analytical and the simulated BER performances for the M-QAM-OFDM system with different the OBO values and selected IQ imbalance values as well as the ideal case (only AWGN) as reference. The comparison between the theoretical curves and the simulated points confirm the correctness of the analytical approach. However, the computer calculation time to obtain the OFDM system BER performance has been dramatically reduced. By comparison of different cases, we observe that the sensitivity of M-QAM-OFDM signals to IQ imbalance and nonlinear distortion increases with the alphabet size M. Moreover, the nonlinear distortion effects could be reduced by backing off the amplifier at the expense of loss in power efficiency of the HPA.

The analytical results concerning TD performance for the OFDM-based system with 16-QAM and 64-QAM modulations in the AWGN channel are reported in Fig. 7. Simulation results are not shown here for the clarity of the presentation. The TD degradation as a function of OBO have been evaluated for 10^{-3} target BER and obtained for different IQ imbalance.
parameters. Moreover, if we compare the results obtained for the M-QAM-OFDM with those obtained in [8] for ideal case (no IQ imbalance), we will find that they are exactly similar.

V. CONCLUSIONS

In this paper, we have studied the BER and TD performances of the M-QAM-OFDM systems subject to both the nonlinear HPA and the IQ imbalance of down-convertor in AWGN channels. We have developed an analytical description of the joint effects of these imperfections on the performance of the OFDM systems and derived closed form BER expression for AWGN channels. Theoretical results show perfect matching with those obtained by simulations for the M-QAM-OFDM systems, which makes the theoretical method able to be used to study the joint effects of the nonlinear distortion and the IQ imbalance without the need to run extensive simulations.

REFERENCES