

# Comparison of the Performances of Conventional Non-Fractal and Fractal Linear Array Antenna

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**Abstract** — The purpose of this paper is to introduce the concept of fractals and its use in antenna arrays for obtaining multiband property. One type of fractals namely, Cantor set is investigated. Cantor set is used in linear array antenna design. Therefore this array know fractal Cantor linear array antenna. A comparison with conventional non-fractal linear array antenna is made regarding the beamwidth, directivity, and side lobe level. Using a program we developed in Matlab, we plotted the radiation pattern of fractal and conventional non-fractal linear array antenna.

## I. INTRODUCTION

In many applications it is necessary to design antennas with very directive characteristics (very high gains) to meet the demands of long distance communication; this can be accomplished by antenna array [1]. The increasing range of wireless telecommunication services and related applications is driving the attention to the design of multifrequency (multiservice) and small antennas. The telecom operators and equipment manufacturers can produce variety of communications systems, like cellular communications, global positioning, satellite communications, and others, each one of this systems operates at several frequency bands. To give service to the users, each system needs to have an antenna that has to work in the frequency band employed for the specific system. The tendency during last years had been to use one antenna for each system, but this solution is inefficient in terms of space usage, and it is very expensive. The variety of communication systems suggests that there is a need for multiband antennas. The use of fractal geometry is a new solution to the design of multiband antennas and arrays. Fractal geometries have found an intricate place in science as a representation of some of the unique geometrical features occurring in nature. Fractal was first defined by Benoit Mandelbrot [2] in 1975 as a way of classifying structures whose dimensions were not whole numbers. These geometries have been used previously to characterized unique

occurrences in nature that were difficult to define with Euclidean geometries, including the length of coastlines, the density of clouds, and branching of trees [3]. Fractals can be divided into many types, as shown in figure (1).

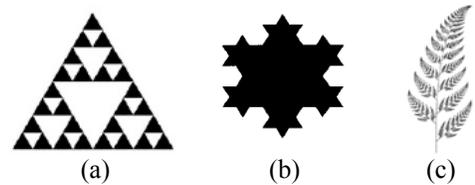


Fig.1. Three fractal examples (a) Sierpinski gasket (b) Koch snowflake (c) Tree.

## II. CONVENTIONAL NON-FRACTAL LINEAR ARRAY ANTENNA

An array is usually comprised of identical elements position in a regular geometrical arrangement. A linear array of isotropic elements  $N$ , uniformly spaced a distance  $d$  apart along the  $z$ -axis, is shown in figure (2) [4].

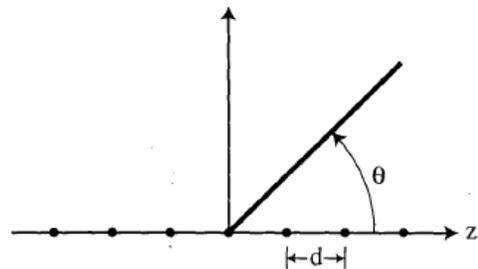


Fig.2. Linear array geometry of uniformly spaced isotropic sources.

The array factor corresponding to this linear array may be expressed in the form [1,5]

$$AF(\psi) = \begin{cases} a_0 + 2 \sum_{n=1}^N a_n \cos(n\psi) & \text{for } (2N+1) \\ & \text{elements} \\ 2 \sum_{n=1}^N a_n \cos\left(\left(\frac{2n-1}{2}\right)\psi\right) & \text{for } (2N) \\ & \text{elements} \end{cases} \quad (1)$$

$$\psi = kd \cos \theta + \alpha \quad (2)$$

$$k = \frac{2\pi}{\lambda} \quad (3)$$

### III. FRACTAL LINEAR ARRAY ANTENNA

Fast recursive algorithms for calculating the radiation patterns of fractal arrays have recently been developed in [6-8]. These algorithms are based on the fact that fractal arrays can be formed recursively through the repetitive application of a generating array. A generating array is a small array at level one ( $P=1$ ) used to recursively construct larger arrays at higher levels (i.e.  $P>1$ ). In many cases the generating subarray has elements that are turned on and off in a certain pattern. A set formula for copying, scaling, and translating of the generating array is then followed in order to produce a family of higher order arrays.

The array factor for a fractal antenna array may be expressed in the general form [6-8]

$$AF_p = \prod_{p=1}^P GA(\delta^{p-1}\psi) \quad (4)$$

where  $GA(\psi)$  represents the array factor associated with the generating array. The parameter  $\delta$  is a scaling or expansion factor that governs how large the array grows with each successive application of the generating array and  $P$  is a level of iteration. This arrays become fractal-like when appropriate elements are turned off or removed, such that

$$a_n = \begin{cases} 1 & \text{if element } n \text{ is turned on} \\ 0 & \text{if element } n \text{ is turned off} \end{cases}$$

One of the simplest schemes for constructing a fractal linear array follows the recipe for the cantor set [9]. Cantor arrays own also multiband properties, so it has multi frequencies ( $F_n$ ):

$$F_n = \frac{F_0}{\delta^n} \quad n = 0, 1, 2, \dots, P-1 \quad (5)$$

where  $F_0$  is the design frequency

The basic Cantor array may be created by starting with a three element generating subarray, and then applying it repeatedly over  $P$  scales of growth. The generating subarray in this case has three uniformly spaced elements, with the center element turned off or removed, i.e., 101. The Cantor array is generated

recursively by replacing 1 by 101 and 0 by 000 at each level of the construction. Table (1) provides the array pattern for the first four levels of the Cantor array.

Table (1) First four levels of the fractal Cantor linear array

P	Elements array Pattern	Active Elements	Total Elements
1	101	2	3
2	101000101	4	9
3	10100010100000000101000101	8	27
4	101000101000000000101000101000 0000000000000000000000000101000 101000000000101000101	16	81

The array factor of the three element generating subarray with the representation 101 is

$$GA(\psi) = 2 \cos(\psi) \quad (6)$$

Which may be derived from eq. (1) by setting  $N=1, a_0=0$ . Substituting eq. (6) into eq. (4) and choosing an expansion factor of three ( $\delta=3$ ), the results in an expression for the Cantor array factor given by

$$AF_p(\psi) = \prod_{p=1}^P GA(3^{p-1}\psi) = 2 \prod_{p=1}^P \cos(3^{p-1}\psi) \quad (7)$$

### IV. COMPUTER SIMULATION RESULTS

In this work, MATLAB programming language version 7.2(R2006a) used to simulate and design the conventional non-fractal and fractal linear array antenna and their radiation pattern. Let, a linear array will be design and simulate at a frequency ( $F_0$ ) equal to 8100MHz, (then the wavelength  $\lambda_0 = 0.037$  m), with quarter-wavelength ( $d = \lambda/4$ ) spacing between array elements and 16 active elements in the array and progressive phase shift between elements ( $\alpha$ ) equal to zero. The level four of Cantor linear array (101) have the number of active elements of 16 and the total elements number of 81. This array will operate at four frequencies depending on the eq. (5). These frequencies are 8100MHz, 2700MHz, 900MHz, and 300MHz. Depending on the frequencies of the fractal Cantor linear array will be design and simulate of conventional non-fractal linear array antenna then compare the radiation field pattern properties for them. The array factor for fractal and non-fractal linear array antenna is plotted with uniformly amplitude distribution which they are feeding to active elements.

The field patterns are illustrating as shown in figure (3). While, the values of the side lobe level, half power beamwidth, and directivity are illustrating in table (2).

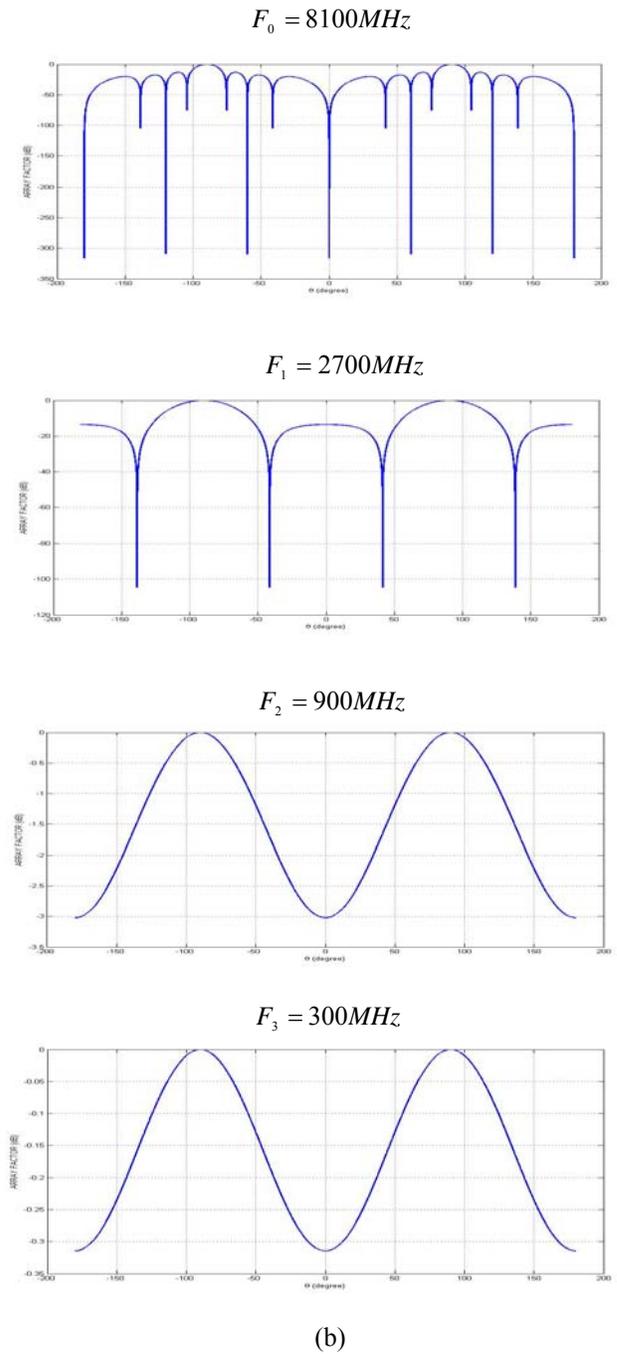
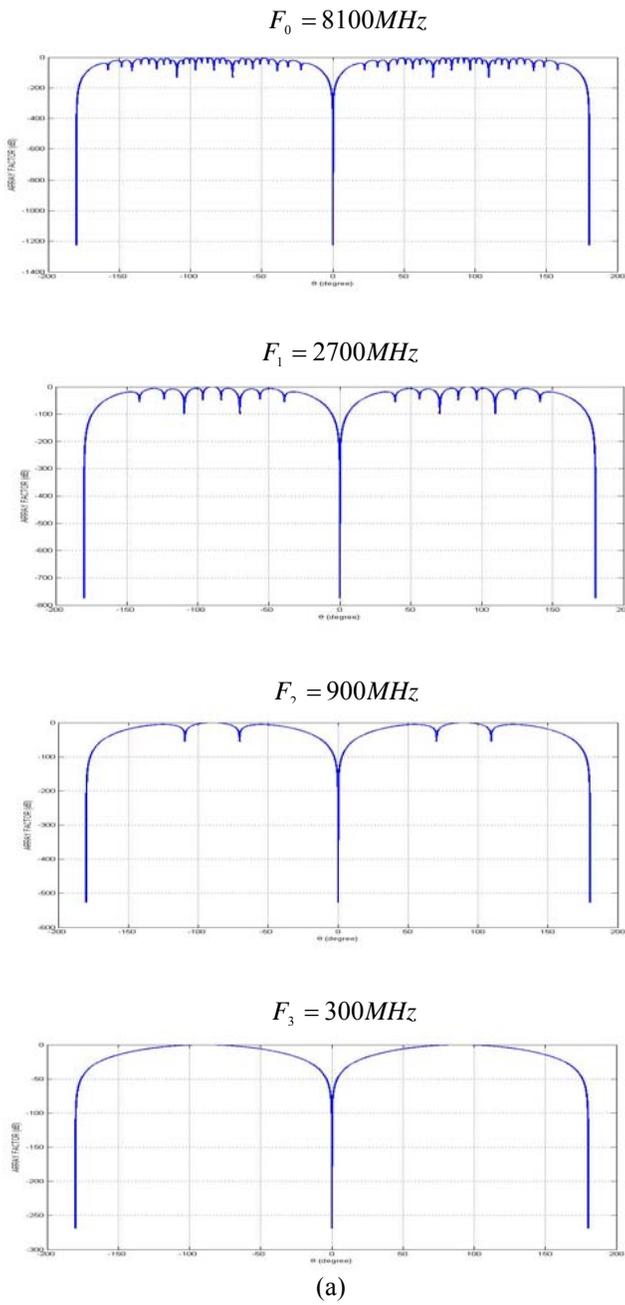


Fig.3. Array factor of a linear array antenna: (a) fractal Cantor linear array (b) conventional non-fractal linear array.

Table (2) SLL, D, and HPBW for uniform amplitude distribution linear array antenna

(a) Fractal linear array

F (MHz)	D (dB)	HPBW (degree)	SLLmax (dB)
8100	12.0436	2.0233	- 5.451
2700	9.1969	6.0721	- 5.446
900	6.204	18.2852	- 5.446
300	3.1848	56.9372	- $\infty$

(b) Non-fractal linear array

F (MHz)	D (dB)	HPBW (degree)	SLLmax (dB)
8100	9.1202	12.7372	- 13.148
2700	4.6369	38.8742	- 13.593
900	0.893	-----	- $\infty$
300	0.106	-----	- $\infty$

V. CONCLUSION

At design frequency  $F_0 = 8100MHz$ , the field pattern for conventional non-fractal linear array antenna has the side lobes and narrow beam width, in other word, the system work as a normal array antenna. But at frequencies very low from the design frequency such as 300 MHz, the array antenna operates as a point source. While fractal linear array antenna at all frequencies not operates as a point source so we conclude that the fractal Cantor linear array have capable to operating in multiband while, the conventional non-fractal linear array have not capable to operate in multiband. Also The field pattern of the fractal linear array antenna have high side lobe level, lower half power beam width and high directivity, while, the conventional non-fractal linear array antenna have lower side lobe level, high half power beam width and lower directivity.

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