

Semi-blind Channel Estimation and Iterative Detection for MIMO-OFDM Systems

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Abstract—The combination of MIMO and OFDM techniques has been recently explored for providing high speed data communication over fading dispersive channels. Bit interleaved coded modulation (BICM) architecture with iterative detection and decoding techniques (IDD) at the receiver has been found to attain good performance (close to Turbo MIMO) with moderate complexity. The scheme however relies on the availability of perfect channel knowledge at the receiver. This paper proposes a semi-blind channel estimation scheme for MIMO-OFDM systems based on modified EM algorithm which makes use of soft / hard decisions available from VBLAST. The technique is seen to converge within a few iterations and provides a performance comparable to the ideal CSI case for MIMO-OFDM.

I. INTRODUCTION

MIMO systems are effective means of achieving good spectral efficiencies over wireless channels, provided the equipment supports multiple RF chains and additional processing requirements. When coupled with OFDM, the systems have added advantages of robustness to inter-symbol interference, inter-block interference and ease of equalization. Recently, the bit interleaved coded modulation (BICM) architecture has been investigated wherein the receiver exploits the benefits of Turbo decoding principle as efficiently as possible [1][2]. However, the above assumes the availability of perfect channel knowledge at the receiver for applying VBLAST and interference cancellation in the iterative detection decoding (IDD) block. Techniques on channel estimation for MIMO-OFDM are reported in [3], [4] (and ref. therein) - the traditional approach being pilot aided estimation. However, the method soon becomes wasteful of bandwidth as the number of parameters to be tracked increases. This is where the semi-blind techniques gain their importance - an initial estimate is improved upon by the receiver solely with the use of received data. Expectation-Maximization (EM) algorithm [5][6], a semi-blind scheme, has gained wide attention for MIMO channel estimation. Researchers have lately proposed modifications to unbiased and simplify the conventional EM algorithm [7][8]. In this paper, we have addressed the problem of semi-blind channel

estimation for BICM MIMO-OFDM systems, which use the VBLAST-IDD architecture at the receiver. Our work couples EM algorithm with VBLAST in an innovative way such that the E-step of EM algorithm may be altogether avoided. The result is a computationally simpler algorithm, which converges quickly and gives good estimation performance. Both hard and soft output versions of the technique are explored.

The paper is organized as follows: section II describes the system and the channel model, section III details the channel estimation technique, section IV evaluates the performance of the said schemes and section V concludes the paper.

II. SYSTEM MODEL

We consider a convolutionally coded layered MIMO OFDM system with N_t transmit antennas, N_r receive antennas and N useful subcarriers. The block diagram of the transmitter module is shown in Fig. 1a). The input bits are convolutionally encoded, interleaved and demultiplexed into N_t parallel streams. Each of these is mapped onto symbols from a suitable constellation, OFDM modulated through FFT and transmitted via N_t antennas after concatenation of cyclic prefix (CP).

We do not assume independent paths between antennas, since size constraints on user-end equipment may not permit proper antenna separation. We use the Stanford University Interim (SUI) model [9] for MIMO channels with proper input and output mixing matrices to cater for antenna correlation at each end. Multipath fading is modeled as a tapped delay line (TDL) with L taps, $\mathbf{h}^{q,p}$ denoting the $L \times 1$ channel tap vector from p^{th} transmit to q^{th} receive antenna. The TDL weights are taken as independent complex Gaussian random variables with gains characterized by Rayleigh fading. Correlation parameters ρ_t and ρ_r are adjusted to vary the degree of fading signal correlation. The frequency response of the channel between the p^{th} transmit and the q^{th} receive antenna may be expressed as

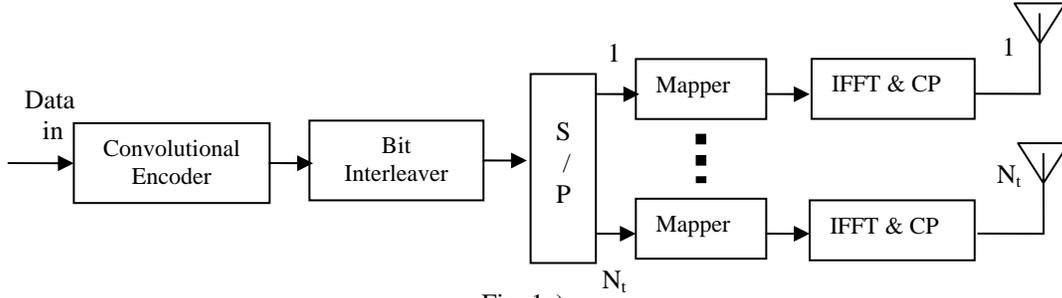


Fig. 1a)

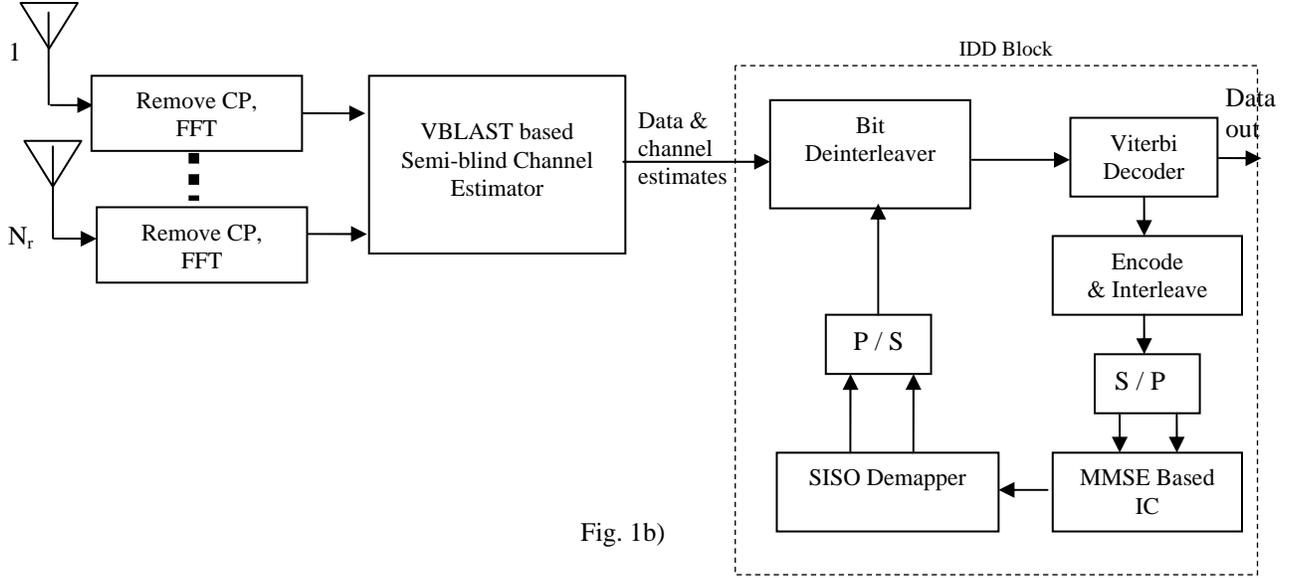


Fig. 1b)

Fig. 1. System block diagram. a) Transmitter side b) Receiver side.

$$H^{q,p}(k) = \frac{1}{\sqrt{N}} \sum_{l=0}^{L-1} h^{q,p}(l) e^{-j2\pi kl/N}, 0 \leq k \leq N-1 \quad (1)$$

Taking DFT at the q^{th} receive antenna, the received signal at subcarrier k becomes

$$Y^q(k) = \sum_{p=1}^{N_t} H^{q,p}(k) X^p(k) + W^q(k), 1 \leq q \leq N_r \quad (2)$$

Where $X^p(k)$ is the symbol transmitted from antenna p at subcarrier k .

Assuming P pilots in an OFDM symbol and gathering the received pilots at q^{th} antenna, we have

$$\tilde{\mathbf{Y}}^q = \tilde{\mathbf{A}} \mathbf{h}^q + \tilde{\mathbf{W}}^q \quad (3)$$

where \mathbf{h}^q is the $LN_t \times 1$ vector of channel taps from all transmit antennas to the q^{th} receive antenna, $\tilde{\mathbf{W}}^q$ consists of P AWGN noise samples with variance σ_w^2 each and

$$\tilde{\mathbf{A}} = [\text{diag}(X^1(k_1), \dots, X^1(k_p)) \tilde{\mathbf{F}}_L^1; \dots; \text{diag}(X^{N_t}(k_1), \dots, X^{N_t}(k_p)) \tilde{\mathbf{F}}_L^{N_t}]_{P \times LN_t} \quad (4)$$

and $\tilde{\mathbf{F}}_L$ is a $P \times L$ matrix obtained from the standard $N \times N$ DFT matrix [10]. The least squares (LS) solution is simply obtained as

$$\hat{\mathbf{h}}^q = \tilde{\mathbf{A}}^{-1} \tilde{\mathbf{Y}}^q \quad (5)$$

This pilot aided channel estimation method is hereafter referred to as PACE. These channel estimates are further refined through semi-blind processing as is described in the section III.

An alternative representation of the transmitted-received symbols is as follows:

$$\mathbf{y}(k) = \mathbf{H}(k) \mathbf{x}(k) + \mathbf{w}(k) \quad (6)$$

where $\mathbf{y}(k)$ is the $N_r \times 1$ received signal vector on subcarrier k , $\mathbf{x}(k)$ is the $N_t \times 1$ transmit vector and $\mathbf{H}(k)$ is the $N_r \times N_t$ matrix whose $(p, q)^{\text{th}}$ element is the frequency response of the channel from transmit antenna q to receive antenna p at subcarrier k , given by $H^{p,q}(k)$ and may be obtained from (1) by reversal of variables.

The conventional VBLAST detection algorithm [11] for MIMO-OFDM systems is a simple approach capable of attaining high spectral efficiency. It basically involves

removing the effect of already detected symbols (assuming those decisions to be correct) and linearly combining the received symbols in such a way that reduces the interference from yet-to-be-detected symbols.

However, in the enhanced VBLAST algorithm for MIMO systems, the matrix for nulling the effects of already detected symbols is determined by MMSE criterion and takes into account the decision errors too. The matrix is found to be ([1][12])

$$\mathbf{G} = \mathbf{H}^{i:N_i}(k)^H [\mathbf{H}^{i:N_i}(k) \mathbf{H}^{i:N_i}(k) + \frac{1}{\sigma_s^2} \mathbf{H}^{1:i-1}(k) \mathbf{Q}_{\hat{e}_k^{i-1}} \mathbf{H}^{1:i-1}(k)^H + \alpha \mathbf{I}_{N_r}]^{-1} \quad (7)$$

where $i \in \{1, 2, \dots, N_r\}$ denotes the current symbol being detected according to optimal order, $\mathbf{H}^{i:N_i}(k)$ is the matrix having $N_r - i + 1$ columns from $\mathbf{H}(k)$ corresponding to undetected symbols, $\mathbf{Q}_{\hat{e}_k^{i-1}}$ is the $(i-1) \times (i-1)$ error covariance matrix for decisions already made and $\mathbf{H}^{1:i-1}(k)$ forms the $i-1$ columns of $\mathbf{H}(k)$, pertaining to already detected symbols. Applying the rows of this equalizer matrix on the received vector in optimal order yields a biased estimate for each transmitted symbol. From these, we may compute the log-likelihood ratios (LLR) for the bits transmitted from each antenna over each subcarrier as

$$L(d^{i,m}(k)) = \log \frac{\text{P}[d^{i,m}(k) = 0 \mid \text{equalizer output}]}{\text{P}[d^{i,m}(k) = 1 \mid \text{equalizer output}]} \quad (8)$$

where $d^{i,m}(k)$ is the m^{th} bit corresponding to the symbol transmitted from antenna i at subcarrier k , M is the constellation size, and $1 \leq m \leq \log_2 M$ [1][12].

In our work, the VBLAST algorithm at receiver has been coupled with the channel estimation mechanism (fig. 1b)); the details of this are explained in section III. The channel estimates together with hard decisions of transmitted data are available to the IDD block. The details of the IDD block may be found in [1], which involves deinterleaving, Viterbi decoding, re-interleaving of bits, MMSE based interference cancellation, and SISO demapping. With successive iterations of IDD, the estimates keep on getting refined. Subsequently the data bits may be retrieved from the output of Viterbi decoder.

III. PROPOSED TECHNIQUE

In this section we describe the semi-blind channel estimation algorithm based on conventional EM technique. Considering (2) and (3), if we collect the received sequence not just for pilot tones but for all the tones, we get

$$\mathbf{Y}^q = \mathbf{A} \mathbf{h}^q + \mathbf{W}^q \quad (9)$$

where \mathbf{Y}^q is the $N \times 1$ vector received on antenna q and

$$\mathbf{A} = [\text{diag}(\mathbf{X}^1) \mathbf{F}_L \dots \text{diag}(\mathbf{X}^{N_r}) \mathbf{F}_L]_{N \times LN_r} \quad (10)$$

\mathbf{X}^p is the $N \times 1$ transmit vector from antenna p and \mathbf{F}_L has first L columns of DFT matrix.

Applying conventional EM to this system, we may define the log-likelihood function of complete information as

$$\mathcal{L} = \log f(\mathbf{Y}^q, \mathbf{A} \mid \mathbf{h}^q) \quad (11)$$

Since \mathbf{A} is unknown, an expectation needs to be taken and (11) becomes

$$\begin{aligned} \mathcal{L} &= \text{E} \left[\log f(\mathbf{Y}^q, \mathbf{A} \mid \mathbf{h}^q) \mid \mathbf{Y}^q, \mathbf{h}_i^q \right] \\ &= \text{E} \left[\log(f(\mathbf{Y}^q \mid \mathbf{A}, \mathbf{h}^q) f(\mathbf{A})) \mid \mathbf{Y}^q, \mathbf{h}_i^q \right] \end{aligned} \quad (12)$$

Where \mathbf{A} is assumed to be independent of the channel, \mathbf{h}_i^q is the estimated \mathbf{h}^q at step i and $f(\mathbf{Y}^q \mid \mathbf{A}, \mathbf{h}^q)$ is the probability density function (pdf) of a multivariate Gaussian random vector of size $N \times 1$ given by

$$f(\mathbf{Y}^q \mid \mathbf{A}, \mathbf{h}^q) = \frac{1}{(2\pi)^{N/2} \sqrt{\det \mathbf{C}}} \exp \left[-\frac{1}{2} (\mathbf{Y}^q - \boldsymbol{\mu})^H \mathbf{C}^{-1} (\mathbf{Y}^q - \boldsymbol{\mu}) \right]$$

with mean vector $\boldsymbol{\mu}$ and covariance matrix \mathbf{C} .

The pdf thus becomes

$$f(\mathbf{Y}^q \mid \mathbf{A}, \mathbf{h}^q) = \frac{1}{(2\pi)^{N/2} \sigma_w^N} \exp \left[-\left(\frac{1}{2\sigma_w^2} \right) (\mathbf{Y}^q - \mathbf{A} \mathbf{h}^q)^H (\mathbf{Y}^q - \mathbf{A} \mathbf{h}^q) \right]$$

whereas the pdf of \mathbf{A} is determined from *a-priori* probabilities of transmitted symbols.

The maximization step involves differentiating \mathcal{L} in (12) with respect to \mathbf{h}^q and finding the channel parameter which maximizes the log-likelihood. Thus we have,

$$\frac{\partial}{\partial \mathbf{h}^q} \left[\sum_{\mathbf{x} \in \mathbf{X}} \log \left(f(\mathbf{Y}^q \mid \mathbf{A}_x, \mathbf{h}^q) f(\mathbf{A}_x) \right) \right] = 0$$

i.e.

$$\hat{\mathbf{h}}^{qH} = \left[\sum_{\mathbf{x} \in \mathbf{X}} \mathbf{Y}^{qH} \mathbf{A}_x f(\mathbf{Y}^q \mid \mathbf{A}_x, \mathbf{h}_i^q) \right] \left[\sum_{\mathbf{x} \in \mathbf{X}} \mathbf{A}_x^H \mathbf{A}_x f(\mathbf{Y}^q \mid \mathbf{A}_x, \mathbf{h}_i^q) \right]^{-1} \quad (13)$$

However, it is pointed out in [13] (for the case of SISO-OFDM) that instead of taking expectation over all possible values of transmitted sequence, if we use an MMSE estimate of the transmitted sequence, a much simpler algorithm with identical performance results. Inspired by this, we propose two modifications to the conventional EM algorithm to make it suitable for our case of layered MIMO-OFDM.

In the first instance, we use the soft VBLAST algorithm described in section II and find the LLRs of all the transmitted bits as in (8). We use these LLRs to find the probability of

each bit being 0 or 1 as

$$P[d^{i,m}(k) = b] = \begin{cases} \frac{\exp(L(d^{i,m}(k)))}{1 + \exp(L(d^{i,m}(k)))}, & b = 0 \\ \frac{1}{1 + \exp(L(d^{i,m}(k)))}, & b = 1 \end{cases} \quad (14)$$

With these, we find the estimate of the transmitted symbol

$$\sum_{s \in S} s P(X^i(k) = s) \quad (15)$$

where S is the constellation being used.

Let $\hat{\mathbf{X}}$ be the so-estimated $N_r N \times 1$ transmit vector and $\mathbf{A}_{\hat{\mathbf{X}}}$ be the corresponding \mathbf{A} matrix defined in (10). If we replace the expectation in (12) by $\hat{\mathbf{X}}$ and $\mathbf{A}_{\hat{\mathbf{X}}}$, we are left with the log-likelihood function

$$\mathcal{L} = \log(f(\mathbf{Y}^q | \mathbf{A}_{\hat{\mathbf{X}}}, \mathbf{h}^q) f(\mathbf{A}_{\hat{\mathbf{X}}})) \quad (16)$$

Maximization of this function leads to

$$\frac{\partial}{\partial \mathbf{h}^q} (\mathcal{L}) = \frac{\partial}{\partial \mathbf{h}^q} \left[\log \left(\frac{1}{(2\pi)^{N/2} \sigma_w^N} f(\mathbf{A}_{\hat{\mathbf{X}}}) \right) - \frac{1}{2\sigma_w^2} (\mathbf{Y}^q - \mathbf{A}_{\hat{\mathbf{X}}} \mathbf{h}^q)^H (\mathbf{Y}^q - \mathbf{A}_{\hat{\mathbf{X}}} \mathbf{h}^q) \right]$$

For equi-probable transmitted symbols, the density of $\mathbf{A}_{\hat{\mathbf{X}}}$ contributes only a constant term, hence we are left with

$$\frac{\partial}{\partial \mathbf{h}^q} \left[\mathbf{h}^{qH} \mathbf{A}_{\hat{\mathbf{X}}}^H \mathbf{A}_{\hat{\mathbf{X}}} \mathbf{h}^q - \mathbf{Y}^{qH} \mathbf{A}_{\hat{\mathbf{X}}} \mathbf{h}^q \right] = 0$$

and hence

$$\hat{\mathbf{h}}^q = \left[(\mathbf{Y}^{qH} \mathbf{A}_{\hat{\mathbf{X}}}) (\mathbf{A}_{\hat{\mathbf{X}}}^H \mathbf{A}_{\hat{\mathbf{X}}})^{-1} \right]^H \quad (17)$$

This is evidently simpler to compute than (13). The aforesaid channel estimation scheme, which is a combination of soft VBLAST and a variant of EM, is hereafter referred to as SVBEM.

In the second instance, the approach is further simplified and plain VBLAST is applied to get an estimate of transmitted symbols $\hat{\mathbf{X}}$ and hence $\mathbf{A}_{\hat{\mathbf{X}}}$. These readily help find the improved channel estimates from (17). This scheme, which combines hard output VBLAST with a variant of EM is hereafter referred to as HVBEM.

The algorithm may thus be summarized as follows:

- 1) Using P pilot tones in the first OFDM symbol of the frame, the receiver obtains rough channel estimates using PACE as in (5).
- 2) The estimates obtained from PACE are used to run soft VBLAST / hard VBLAST algorithm and LLRs / hard estimates of data $\hat{\mathbf{X}}$ are found.
- 3) Corresponding matrix $\mathbf{A}_{\hat{\mathbf{X}}}$ is computed and used in (17) to improve channel estimates.
- 4) Steps 2) and 3) are iterated until convergence.

IV. RESULTS & DISCUSSION

We consider the following simulation environment: a 2×2 MIMO-OFDM system with 128 subcarriers and useful symbol duration of 0.5ms. Fading dispersive channel is modeled with $L = 2$ taps and the correlation coefficients for SUI-MIMO channel model are kept at $\rho_r = 0.2$ and $\rho_r = 0.4$. The convolutional code used is a rate $1/2$ code with a constraint length of 7 and generator polynomials specified by {712} and {476} in octal notation. The encoded bits are mapped onto 4-QPSK constellation with equi-probable unit energy symbols drawn from $(\pm 1 \pm j) / \sqrt{2}$. PACE is carried out by sending 16 pilot tones in the first OFDM symbol of each frame, which consists of 30 OFDM symbols.

First we compare the performances of the two channel estimators in question – SVBEM and HVBEM. The mean square estimation error (MSEE) of the two methods is obtained through (18) and plotted in fig. 2.

$$MSEE = \frac{1}{RN_r N_t L} \sum_{r=1}^R \sum_{q=1}^{N_r} \left\| \mathbf{h}^q - \hat{\mathbf{h}}^q \right\|^2 \quad (18)$$

Here R is the number of independent channel realizations used for averaging (200 in our case). For comparison, we have also shown the MSEE obtained from PACE as in (5) when no subsequent VBLAST-EM processing is used at the receiver. As one may expect, it is evident that the SVBEM scheme performs the best whereas the PACE scheme remains the worst with HVBEM performing between the two extremes. For SNR values beyond 8dB, semi-blind processing through SVBEM gives a gain of 4-6dB over PACE and this gain increases as SNR rises. At an SNR of 20dB, the PACE curve gives an MSE of 0.01 while the VBEM techniques provide an MSEE lower than 0.005. It may be noted that at low SNRs, the HVBEM curve almost coincides with PACE. This is because the LS based PACE is prone to noise and often yields a poor estimate to start with. The HVBEM algorithm thus fails to improve the estimates. In fact at SNR < 5dB, the HVBEM is found to diverge more often than not, thereby leading to an MSEE value slightly higher than PACE. However, the exchange of soft information allows the SVBEM to improve channel estimation even in the low SNR regime. It is also apparent that as the noise levels recede, the HVBEM and SVBEM methods perform almost identically.

Fig. 2 shows the average number of iterations required for the estimation techniques to converge. These are again obtained at different SNR values by averaging over 200 independent trials. It is found that for SNR values beyond 8dB, the proposed methods converge within 4 iterations, and this requirement falls to below 3 iterations as the SNR crosses 12 dB.

Next we demonstrate the BER performance of the system when coupled with the IDD structure (fig. 1b)). We use three iterations of the IDD structure and plot BER curves against received SNR per bit by averaging over multiple channel realizations. It is seen that the performance with channel

estimators is pretty close to that when ideal CSI available. The SVBEM is seen to be roughly 1 dB away from the ideal CSI curve. At low SNRs, the HVBEM performs almost 1dB worse than SVBEM, but this gap goes on increasing with SNR. The SVBEM technique is found to reach a BER of less than 0.002 at an SNR of 15 dB which is acceptable for the performance of most systems.

V. CONCLUSION

The paper details soft and hard versions of a semi-blind channel estimation scheme customized for coded, layered MIMO OFDM systems. Both the schemes converge quickly and are seen to perform well, while being computationally simpler than conventional EM. SVBEM gives a better performance than HVBEM but is slightly more complex.

REFERENCES

- [1] I. Lee, A. Chan and C. -E. W. Sundberg, "Space-time bit interleaved coded modulation for OFDM systems," *IEEE Trans. Sig. Process.*, vol. 52, no. 3, pp. 820-825, Mar. 2004.
- [2] H. Lee, B. Lee and I. Lee, "Iterative detection and decoding with an improved V-BLAST for MIMO-OFDM systems," *IEEE Jnl. Sel. Areas in Commun.*, vol. 24, no. 3 pp. 504-513, Mar. 2006.
- [3] I. Barhumi, G. Leus, M. Moonen, "Optimal training design for MIMO OFDM in mobile wireless channels," *IEEE Trans. Sig. Process.*, vol. 5, pp. 1615-1624, June 2003.
- [4] B. Hassibi and B.M. Hochwald, "How much training is needed in multiple-antennas wireless links?" *IEEE Trans. Info. Theory*, vol. 49, no. 4, pp. 951-963, Apr. 2003.
- [5] T. K. Moon, "The expectation-maximization algorithm," *IEEE Sig. Process. Mag.*, pp. 47-60, Nov. 1996.
- [6] X. Ma, H. Kobayashi, S. C. Schwartz, "An EM-based estimation of OFDM signals," presented at the IEEE Wireless Communication and Networking Conf., WCNC 2002, vol. 1, pp. 228-232, 17-21 March 2002.
- [7] M. A. Khalighi and J. J. Boutros, "Semi-blind channel estimation using EM algorithm in MIMO APP detectors," *IEEE Trans. Wireless Commun.*, vol. 5, no. 11, pp. 3165-3173, Nov. 2006.
- [8] X. Wautelet, C. Herzet, A. Dejonghe, J. Louveaux, L. Vandendorpe, "Comparison of EM-based algorithms for MIMO channel estimation," *IEEE Trans. Commun.*, vol. 55, no. 1, pp. 216-226, Jan. 2007.
- [9] A. Paulraj, R. Nabar, D. Gore, *Introduction to Space-Time Wireless Communications*. Cambridge, UK: Cambridge Univ. Press, ch. 3.
- [10] D. Hu, L. Yang, Y. Shi and L. He, "Optimal pilot sequence design for channel estimation in MIMO-OFDM systems," *IEEE Commun. Letters*, vol. 10, no. 1, pp. 1-3, Jan. 2006.
- [11] G. D. Golden, C. J. Foschini, R. A. Valenzuela, P. W. Wolniansky, "Detection Algorithm and initial laboratory results using V-BLAST space-time communication architecture," *Electronics Letters*, vol. 35, no. 1, pp. 14-16, 7th Jan. 1999.
- [12] H. Lee and I. Lee, "New approach for coded layered space-time architecture for MIMO-OFDM systems," in *Proc. ICC*, May 2005, pp. 608-612.
- [13] S. Jain, P. Gupta, D. K. Mehra, "EM-MMSE based channel estimation for OFDM systems," presented at the IEEE International Conf. on Industrial Technology, ICIT 2006, pp. 2598-2602, 15-17 Dec. 2006.

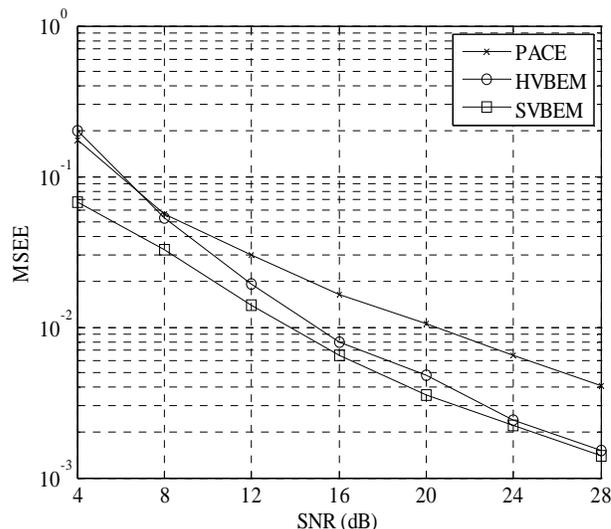


Fig. 2. Mean square estimation error performance of channel estimators compared against pilot estimator

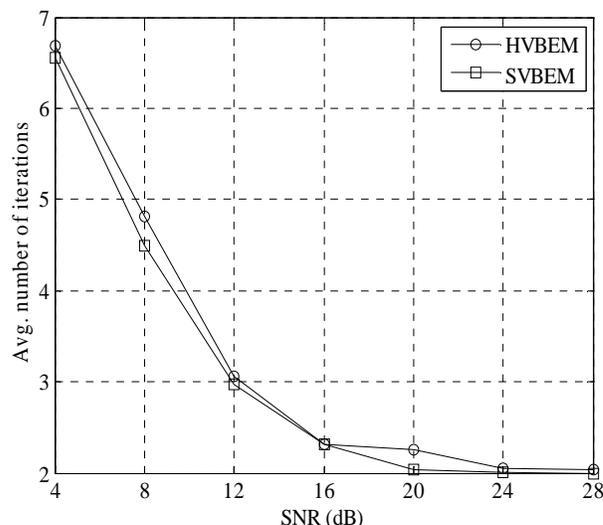


Fig. 3. Average number of iterations required for channel estimators to converge

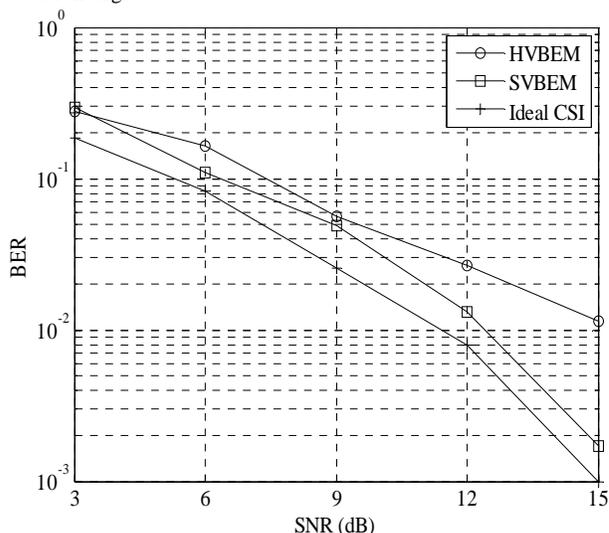


Fig. 4. BER performance of VB-EM compared with ideal CSI