# Angle and Time of Arrival Statistics for Indoor UWB Communication

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***Abstract*—Wireless communication systems require proper understanding of the spatial and temporal characteristics of the propagation channel. This paper presents a geometrical based single bounce model for indoor ultra wideband (UWB) communication in which the multipath reflection is accounted by uniform distribution of scatterers placed in the clusters of elliptical region around the transmitter (Tx) and receiver (Rx). Analytical expressions for the Angle of Arrival (AoA) and Time of Arrival (ToA) probability density functions (pdfs) have been derived for the proposed model. These expressions have been validated through simulation and the results are also compared with the measured data available in literature.**

***Index Terms*—UWB Communication, geometrical based single bounce model, clusters, angle of arrival, time of arrival.**

## I. Introduction

Ultra Wideband (UWB) signals have a -10 dB fractional bandwidth greater than or equal to 20% of the center frequency or have a minimum bandwidth of 500 MHz. As per Federal Communications Commission (FCC) guideline, UWB devices must operate in 3.1 to 10.6 GHz frequency band [1], [2]. Because of the extremely large operational bandwidth, UWB technology is currently being thought as a solution for short range high data rate, wireless home networking and security systems. Accurate knowledge of the wireless propagation channel is essential for the UWB system design as convectional channel models developed for narrowband transmissions are inadequate for UWB transmission. Indoor multipath propagation channel models consider clustering of multipath components, both in time [3], [4], [5] and in angle [6], [7], [8]. The clustering effects give rise to two classes of channel parameters, namely intercluster and intracluster and can be employed for both line-of-sight (LOS) and non-line-of-sight (NLOS) scenarios. The clusters in the indoor environments are formed by the building superstructure and the rays within the clusters are formed by objects in the surrounding area of the transmitter (Tx) and receiver (Rx). The study group IEEE 802.15.SG3a has accepted the modified Saleh-Valenzuela multipath model proposed by Intel corporation as a standard channel model for indoor UWB communication [9]. Modified S-V model gives the knowledge of channel impulse response and delay characteristics from which the temporal parameters such as rms delay spread and mean excess delay can be drawn. However, such model does not convey information about the direction of arrival of signals which is important when dealing with UWB systems containing multiple antennas or directional antennas.

Spatial channel modeling is performed taking into account the location of scatterers/reflectors that describe the Angle of Arrival (AoA) and Time of Arrival (ToA) of the multipath components [10]. In this paper, a spatial channel model employing geometrical based approach is proposed for indoor UWB communication. Geometrically based single bounce model is a widely used radio propagation channel model, where propagation between transmitting and receiving antenna is assumed to take place via single scattering from an intervening obstacle. For microcell or picocell environments, geometrical based Elliptical Scattering Model (ESM) has been proposed in [11] for the uniform distribution of scatterers inside an elliptical region in which Tx and Rx are located at the foci. The ESM model is appropriate for indoor environment where the distance between Tx and Rx is small and the height of the transmitting and receiving antenna is relatively low and same is the case with the indoor UWB communication. In this paper, an approach is given to evaluate the AoA and ToA statistics for indoor UWB communication by extending the geometrical based elliptical scattering model to take into account the clustering effect.

The rest of this paper is organized as follows: Section II describes briefly existing clustering models for the multipath indoor communication. Section III presents the proposed model geometry for the indoor UWB communication and assumptions made for the derivation of analytical results. In Section IV and Section V, we derive the analytical expressions for the AoA pdf and ToA pdf for the model under consideration respectively and verified through simulation and practical results. Section VI concludes the paper.

## II. Clustering Models For The Multipath Indoor Communication

Indoor multipath propagation channel modeling have been reported in literature in both time and angle of arrival of the multipath components [3], [4], [5], [6], [8]. The results presented in [6] are based on the assumption that the channel impulse response as a function of time and azimuth angle is a separable function and can be written as

$$h(t,\theta) = h(t)h(\theta) \tag{1}$$

from which independent descriptions of the multipath time-of-arrival and angle-of-arrival are developed.

### A. Time of Arrival Statistics

The ToA statistics of the modified Saleh-Valenzuela (S-V) model [9] for indoor UWB communication environments is given by the discrete impulse response as

$$h(t) = X \sum_{l=0}^{L-1} \sum_{k=0}^{K_l} \alpha_{k,l} \delta(t - T_l - \tau_{k,l}) \tag{2}$$

where,

$L$=number of clusters;

$K_l$=number of multipath components (MPC) in the $l^{th}$ cluster;

$\alpha_{k,l}$=multipath gain coefficient best fit the log-normal distribution;

$T_l$=arival time of the first ray of the $l^{th}$ cluster;

$\tau_{k,l}$= dealy of the $k^{th}$ rays within the $l^{th}$ cluster relative to the first path arrival time, $T_l$;

$X$=log-normal shadowing.

The clusters arrival time and rays arrival time form a Poisson process with distributions given by

$$p\left(T_l \left| T_{l-1} \right.\right) = \Lambda \exp\left[-\Lambda \left(T_l - T_{l-1}\right)\right], l > 0$$

$$p\left(\tau_{k,l} \left| \tau_{k-1,l} \right.\right) = \lambda \exp\left[-\lambda \left(\tau_{k,l} - \tau_{k-1,l}\right)\right], k > 0$$

where $\Lambda$ is cluster arrival rate and $\lambda$ is ray arrival rate, i.e., the arrival rate of path within each cluster. The channel coefficients are defined as product of small scale and large scale fading coefficients as

$$\alpha_{k,l} = p_{k,l} \xi_l \beta_{k,l}. \tag{3}$$

In the above equation, $\xi_l$ represents the fading associated with the $l^{th}$ cluster, $\beta_{k,l}$ corresponds to the fading associated with the $k^{th}$ ray of the $l^{th}$ cluster and $p_{k,l}$ is equiprobable $(\epsilon\{-1, 1\})$ and accounts for signal inversion due to reflections.

### B. Angle of Arrival Statistics

Several AoA measurement campaigns [6], [7], [8] illustrate that the multipath rays arrive in cluster not only in time but also in angle. Using the separable impulse response of (1), the angular impulse response as proposed in [6] for cluster and ray angle of arrivals can be expressed as

$$h(\theta) = \sum_{l=0}^{L-1} \sum_{k=0}^{K_l} \alpha_{k,l} \delta(\theta - \Theta_l - \omega_{k,l}) \tag{4}$$

where $\Theta_l$ is the mean azimuth angle of arrival of the $l^{th}$ cluster, and $\omega_{k,l}$ is the azimuth angle of arrival of the $k^{th}$ arrival in the $l^{th}$ cluster, relative to $\Theta_l$. In [6] it is proposed that $\Theta_l$ is uniformly distributed in angle and $\omega_{k,l}$ is distributed according to a zero mean Laplacian distribution with standard deviation $\sigma$ as

$$p(\theta) = \frac{1}{\sqrt{2}\sigma} e^{-|\sqrt{2}\theta/\sigma|}. \tag{5}$$

As reported in [7], the relative azimuth arriving angles of the recovered UWB signals in an indoor scenario has a Laplacian distribution.

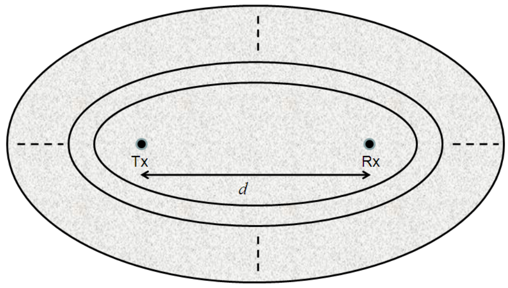


Fig. 1. Geometrical model for indoor UWB communication.

## III. Proposed Model Description And Assumptions

Fig. 1 illustrates the proposed geometrical based single bounce channel model for the indoor UWB communication by extending the ESM model. The clustering phenomenon as accounted in indoor environment of UWB communication has been modeled using multiple elliptical regions having the same foci for all the ellipses and Tx and Rx are placed at the respective foci. Each elliptical region has been assumed to form a single cluster and has uniform distribution of scatterers inside it. For the proposed model, there will be relatively larger number of scatterers in the innermost elliptical region and the density of scatterers decreases away from it which results non uniform distribution of scatterers around the Tx and Rx. Here, we assume $L$ number of elliptical clusters and $d$ is the distance between Tx and Rx as shown in Fig. 1. The semimajor axis and semiminor axis of the ellipse representing $l^{th}$ cluster as shown in Fig. 2 are given by

$$a_l = \frac{c\tau_{ml}}{2}$$

$$b_l = \frac{1}{2}\sqrt{c^2\tau_{ml}^2 - d^2}$$

where $c$ is the speed of light, $\tau_{ml}$ is the maximum delay associated with scatterers within the ellipse of $l^{th}$ cluster. Multipath components with delays greater than maximum delay of each cluster of elliptical regions are ignored as such reflected components will experience greater path loss and hence will have relatively low power compared to those with shorter delays. Before proceeding to formulate, we make the following assumptions [11]:

1) The received signal at the antenna undergoes no more than single scattering by scatterers when traveling from transmitter to receiver i.e. geometrical based single bounce model.
2) All signals received at the antenna are plane waves coming from the horizon, i.e., only azimuthal coordinate is considered and the model essentially represents a 2D scenario.
3) Each scatterer is assumed to be an omnidirectional reradiating element with equal scattering coefficients and uniform random phases.

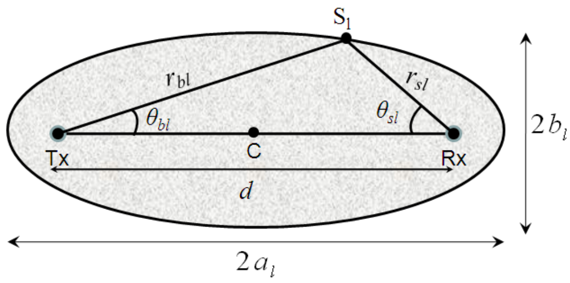


Fig. 2. Geometry for a $l^{th}$ cluster.

4) Effective antenna patterns are omnidirectional for both transmitter and receiver.

The $l^{th}$ ellipse in Fig. 2 may be described in cartesian form as

$$\frac{\left(x_l - \frac{d}{2}\right)^2}{{a_l}^2} + \frac{y_l^2}{{b_l}^2} = 1 \tag{6}$$

or in polar co-ordinate, it can be expressed as [11]

$$r_{bl} = \frac{c^2\tau_{ml}^2 - d^2}{2c\tau_{ml} - 2d\cos(\theta_{bl})} \tag{7}$$

or,

$$r_{sl} = \frac{c^2\tau_{ml}^2 - d^2}{2c\tau_{ml} - 2d\cos(\theta_{sl})} \tag{8}$$

where $r_{bl}$ and $r_{sl}$ are the distance of arbitrary scatterer $S_l$ from the Tx and Rx for the $l^{th}$ cluster as shown in Fig. 2.

## IV. AoA Pdf For The Model

In this section, we derive the pdf of AoA of the multipaths at the transmitter (or receiver) for the UWB indoor scenario. Scatterer density function for uniform distribution of scatters inside the $l^{th}$ elliptical regions are given by

$$f_{x_l,y_l}(x_l, y_l) = \begin{cases} \frac{1}{A_l}, & x_l \text{ and } y_l \in R_{A_l} \\ 0, & \text{else.} \end{cases} \tag{9}$$

Area of the scattering region $(R_{A_l})$ is $A_l = \pi a_l b_l$. In polar co-ordinate system, scatterer density function can also be expressed as

$$f_{r_{bl},\theta_{bl}}(r_{bl}, \theta_{bl}) = r_{bl} f_{x_l,y_l}(r_{bl}\cos(\theta_{bl}), r_{bl}\sin(\theta_{bl})). \tag{10}$$

AoA pdf for $l^{th}$ cluster can be obtained by integrating the polar coordinate system representation of the scatter density function given in (9) with respect to $r_{bl}$ over the range of the boundary of the scatterer region. Due to symmetry of the ellipse, the pdf of AoA will be same at both the antennas i.e. transmitter and receiver [11]. The AoA pdf for a $l^{th}$ elliptical region will be

$$f_{\theta_l}(\theta_l) = \frac{1}{8\pi a_l b_l}\left(\frac{c^2\tau_{ml}^2 - d^2}{c\tau_{ml} - d\cos(\theta_l)}\right)^2 \tag{11}$$

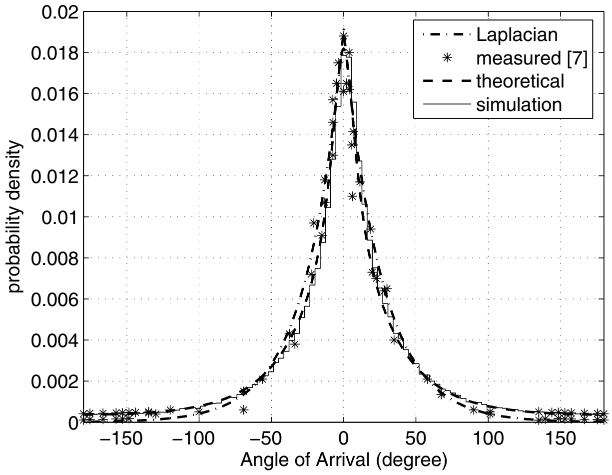


Fig. 3. AoA pdf at Tx or Rx.

where $a_l$ and $b_l$ are the semimajor and semiminor axis and $\tau_{ml}$ is the maximum delay of the $l^{th}$ elliptical region. Here, we consider geometrically based single bounce model and each elliptical region has been assumed to form a single cluster, so the AoA statistics of the multipath components for each elliptical region are disjoint events. Hence, the AoA pdf at Tx or Rx of the multipaths from scattering points within all elliptical regions would be basically the addition of individual AoA pdf at respective side

$$f_\theta(\theta) = \frac{1}{L}\left[f_{\theta_1}(\theta) + f_{\theta_2}(\theta) + f_{\theta_3}(\theta) + \cdots + f_{\theta_L}(\theta)\right] \tag{12}$$

where L is the number of clusters of elliptical region and $f_{\theta_l}(\theta)$ is the pdf of AoA for the $l^{th}$ elliptical cluster and is given by (11)

$$f_{\theta_l}(\theta) = \frac{1}{8\pi a_l b_l}\left(\frac{c^2\tau_{ml}^2 - d^2}{c\tau_{ml} - d\cos(\theta)}\right)^2. \tag{13}$$

Substituting (13) into (12) results the AoA pdf for the assumed geometrical channel model and is expressed as

$$f_\theta(\theta) = \frac{1}{8\pi L}\sum_{l=1}^{L}\frac{1}{a_l b_l}\left(\frac{c^2\tau_{ml}^2 - d^2}{c\tau_{ml} - d\cos(\theta)}\right)^2. \tag{14}$$

The following parameters are taken for the generation of AoA pdf where $\tau_{mi}$ for $i = 1, 2, \cdots, 6$ is the maximum time delay of each cluster. From the measured data as reported in [9], the mean excess delay of the received signal is 5-15 ns, so on the basis of that we have assumed the maximum delay of each cluster as

$$\begin{aligned} &d = 3.2 \text{ meter},\ L = 6, \\ &\tau_{m1} = 11 \text{ ns},\ \tau_{m2} = 13 \text{ ns}, \\ &\tau_{m3} = 15 \text{ ns},\ \tau_{m4} = 17 \text{ ns}, \\ &\tau_{m5} = 19 \text{ ns},\ \tau_{m6} = 21 \text{ ns}. \end{aligned}$$

It has been reported in [7] that the relative azimuth arrival angles of the recovered UWB signals are best fit to a Laplacian density, with a standard deviation, $\sigma$, of $38^o$. Fig. 3 shows the AoA pdf of the proposed model and it is found to match closely with the measured data and the Laplacian distribution.

## V. ToA Pdf For The Model

ToA pdf for a single elliptical region ($l^{th}$ cluster) can be obtained by first deriving ToA cumulative distribution function (CDF) and then differentiating with respect to delay ($\tau_l$). The ToA CDF is calculated as the probability of a scatterer being placed inside the ellipse corresponding to a delay equal to $\tau_l$ as reported in [11]. For the case of uniform scatterer density function as assumed earlier, the ToA pdf for a $l^{th}$ elliptical region is simply

$$f_{\tau_l}(\tau_l) = \frac{1}{A_l}\frac{d}{d(\tau_l)}\left(A_{\tau_l}(\tau_l)\right) \tag{15}$$

where $A_{\tau_l}(\tau_l)$ is the area of elliptical region corresponding to a delay $\tau_l$ and is given by

$$A_{\tau_l}(\tau_l) = \pi a_{\tau_l} b_{\tau_l} = \frac{\pi\tau_l c}{4}\sqrt{\tau_l^2 c^2 - d^2}. \tag{16}$$

Differentiating (16) with respect to $\tau_l$ and simplifying gives

$$\frac{d}{d\tau}\left(A_{\tau_l}(\tau_l)\right) = \frac{\pi c\left(2c^2\tau_l^2 - d^2\right)}{4\sqrt{c^2\tau_l^2 - d^2}}. \tag{17}$$

Substituting (17) into (15) and dividing by $A_l = \pi a_l b_l$ results the ToA pdf as

$$f_{\tau_l}(\tau_l) = \begin{cases} \dfrac{c\left(2c^2\tau_l^2 - d^2\right)}{4ab\sqrt{c^2\tau_l^2 - d^2}}, & \dfrac{d}{c} \leq \tau_l \leq \tau_{ml} \\ 0, & \text{else.} \end{cases} \tag{18}$$

The ToA pdf at Tx or Rx of the multipaths from scattering points within all elliptical regions would be basically the addition of individual ToA pdf at respective side

$$f_\tau(\tau) = \frac{1}{L}\left[f_{\tau_1}(\tau) + f_{\tau_2}(\tau) + f_{\tau_3}(\tau) + \cdots + f_{\tau_L}(\tau)\right] \tag{19}$$

where L is the number of clusters of elliptical region and $f_{\tau_l}(\tau)$ is the pdf of ToA for the $l^{th}$ elliptical cluster and is given by (18) as

$$f_{\tau_l}(\tau) = \begin{cases} \dfrac{c\left(2c^2\tau^2 - d^2\right)}{4a_l b_l\sqrt{c^2\tau^2 - d^2}}, & \dfrac{d}{c} \leq \tau \leq \tau_{ml} \\ 0, & \text{else.} \end{cases} \tag{20}$$

Substituting (20) into (19) results the ToA pdf for the assumed geometrical channel model and is expressed as

$$f_\tau(\tau) = \begin{cases} \displaystyle\sum_{l=1}^{L} \frac{c\left(2c^2\tau^2 - d^2\right)}{4a_l b_l\sqrt{c^2\tau^2 - d^2}}, & \dfrac{d}{c} \leq \tau \leq \tau_{ml} \\ 0, & \text{else.} \end{cases} \tag{21}$$

Fig. 4 show the resulting ToA pdf of the proposed indoor UWB communication channel model for the same parameters as used for the generation of AoA pdf and verified with the simulation.

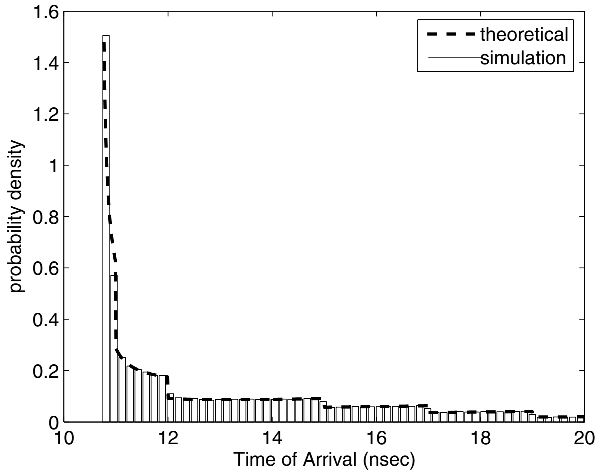


Fig. 4. ToA pdf.

## VI. Conclusion

Empirically it is found that the UWB signals arrive in clusters in both time and angle. In this paper, we proposed a geometrical based clustered elliptical model for the indoor UWB communication. For this model, analytical expressions for the AoA pdf and ToA pdf are derived and the same has been verified through simulation. The results of the AoA pdf has been compared with the measured data available in literature. The proposed model is expected to be useful in evaluating the performance of indoor UWB communication.

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