A Computationally Efficient packet wavelet coder using Cellular Neural Network

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Abstract

We present the packet wavelet coder implemented with Cellular Neural Network architecture, and show its superiority over the pyramidal wavelet representation. This paper also demonstrates how the cellular neural universal machine (CNNUM) architecture can be extended to image compression. The packet wavelet coder performs the operation of image compression, aided by CNN architecture. It uses the highly parallel nature of the CNN structure and its speed outperforms traditional digital computers. In packet wavelet coder, an image signal can be analyzed by passing it through an analysis filter banks followed by a decimation process, according to the rules of packet wavelets. The Simulation results indicate that the quality of the reconstructed image is superior by using packet wavelet coding scheme. Our results are compared with that of pyramidal wavelet representation.

1. Introduction

The wavelet coding method has been recognized as an efficient coding technique for lossy compression. The wavelet transform decomposes a typical image data to a few coefficients with large magnitude and many coefficients with small magnitude. Since most of the energy of the image concentrates on these coefficients with large magnitude, lossy compression systems just using coefficients with large magnitude can realize both high compression ratio with good quality reconstructed image at the same time The wavelet transform does a good job of de-correlating image pixels in practice, especially when images have power spectra that decay approximately uniformly and exponentially. For images with non-exponential rates of spectral decay, and for images which have concentrated peaks in those spectra away from DC, one can do considerably better. The optimal sub-band decomposition for an image is one for which the spectrum in each sub-band is approximately flat.

On the other hand Cellular neural networks (CNN) is a analog parallel-computing paradigm defined in space and characterized by locality of connections between processing elements such as cells or neurons .A CNN principal property is the ability to perform massive processing and such property fits perfect in image processing tasks where minimum processing time demanded. In this paper we have presented packet wavelet coder using CNN architecture and compared the results to that of pyramidal wavelets using CNN architecture. Packet wavelets allow the option to zoom in to high frequencies if we like, by analyzing at each level only the band with highest energy. The signal passes through filter banks, and it is sampled following the packet wavelet coder.

In order to implement image coder using CNN, the kernels considered are the templates defined in the CNN mathematical representation. The image is passed through the CNN, which performs filtering operation. Image reconstruction is performed by inverse wavelet transform. The kernels for inverse wavelet transform are also treated as templates. In our work, we have used Quadrature mirror filters (QMF) coefficients for filtering operation.

2. Cellular Neural Networks

Cellular Neural Networks (CNN) is a massive parallel computing paradigm defined in discrete N-dimensional spaces. Following is the Chua-Yang [1] definition:

- A CNN is an N-dimensional regular array of elements (*cells*);
- The cell grid can be for example a planar array with rectangular, triangular or hexagonal geometry, a 2-D or 3-D torus, a 3-D finite array, or a 3-D sequence of 2-D arrays (*layers*);
- Cells are multiple input-single output processors; all described by one or just some few parametric functionals.
- A cell is characterized by an *internal state variable,* sometimes not directly observable from outside the cell itself;
- More than one connection network can be present, with different neighborhood sizes;
- A CNN dynamical system can operate both in continuous (CT-CNN) or discrete time (DT-CNN);
- CNN data and parameters are typically continuous values.
- Cellular Neural Networks operate typically with more than one iteration, i.e. they are *recurrent* networks.

The definition of CNN is, a 2, 3 or N-dimensional array of mainly identical dynamical systems called cells which satisfies two properties namely

1. Most interactions are local within a finite radius R.
2. All state variables are continuous valued signals.

The Cellular neural Network invention was based on the fact that many complex computational problems can be resolved as well defined tasks where signal values, coefficients in the case of images, are placed on a regular geometric 2-D or 3-D grid. The direct interactions between signal values are limited to a finite local neighborhood. Due to their local interconnectivity, cellular neural networks can be realized as VLSI chips, which operate at very high speeds and complexity. Since the range of dynamics is independent of the number of cells, the implementation is highly reliable and efficient. The design of cloning templates is based on the geometric aspects of the problem. The matrix also defines the interaction between each cell and all its neighboring cells in terms of their input state and output variables. In [2], it has been shown that the CNN is also a spatial approximation of a diffusion type partial differential equation. However, CNN is inherently local in nature. So, it cannot be expected to perform global operations of a coding scheme e.g., entropy coding scheme. However, most steps of the scheme can be implemented using CNN and its highly parallel nature makes its speed outperform traditional digital solutions.

Defining Equations of CNN [1]:

State equation:

$$\overset{o}{x}_{ij} = -x_{ij} + \sum_{C(k,l)\in S(i,j)} A(i,j;k,l)\, y_{kl} + \sum_{C(k,l)\in S(i,j)} B(i,j;k,l)\, u_{kl} + z_{ij} \quad ....(1)$$

The Output Equation:

$$y_{ij} = f(x_{ij}) = \frac{1}{2}\left|x_{ij}+1\right| - \frac{1}{2}\left|x_{ij}-1\right| \quad ....(2)$$

Where $x_{ij} \in R,\ y_{kl} \in R,\ u_{kl} \in R \ and\ z_{ij} \in R$ are called state output, input and threshold of cell C(i,j) respectively. A(i,j;k,l) and B(i,j;k,l) are called the feedback and input synaptic operators to be defined below. Figure (1) shows standard non-linearity $g(x^e)$. Hence Cellular neural networks are also called as cellular non-linear networks

Moreover it has also been demonstrated [1] that CNN paradigm is universal, being equivalent to the Turing Machine. A mathematical formal description of the discrete time case is contained in the following equations.

$$x(t+1) = g(x(t)) + I(t) + \sum(A(yk(t), PA(j))) + \sum(B(uk(t), PB(j))) \quad .... (3)$$

$$y(t) = f\ (x(t)) \quad .... (4)$$

where *x* is the internal state of a cell, *y* its output, *u* its external input and *I* a local value called *bias. A* and *B* are two generic parametric functionals, *PA(j)* and *PB(j)* are the parameters arrays (typically the inter-cell connection weights). The neighbor *yk* and *uk* values are collected from the cells present in the two neighborhood *Nr,* for the *feedback* functional *A,* and *Ns,* for the *control* functional *B.* The two neighborhoods are potentially different. The functionals *A* and *B* are also called *templates.* The *instantaneous local feedback function g* expresses the possibility of an immediate feedback effect. This function is typically not used. *f* is the function that gives cell output from the internal state. Generally is used the *chessboard* distance convention, expressed by the equation,

$$d(i,j) = \max(|x(i) - x(j)|, |y(i) - y(j)|) \quad .... (5)$$

In most of cases the system is non–Monrovian, i.e. the future internal state depends also from the past history of the system. In the special case of *time-variant* CNN all the above functions, neighborhoods and parameters can be also function of time. A block-schematic of a generic CNN is shown in Fig 1. Here the feedback template is taken as zero.

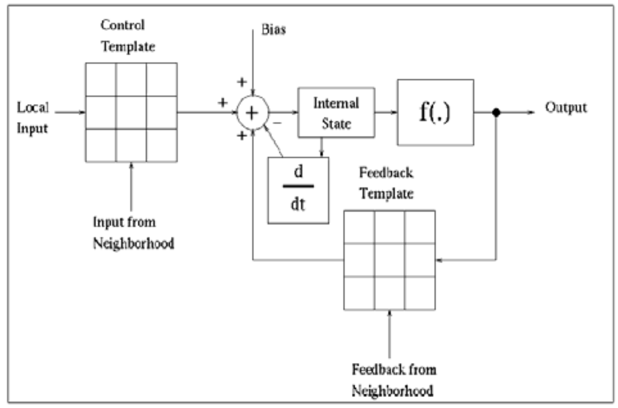

Fig 1.Generic Cellular Neural Network

3. Packet Wavelet Representation

Images are 2-D signals. An image can be analyzed by passed it through four different combinations of low and high pass filter. At each level of resolutions, instead of splitting the signal in to two components, it can be split in to four. A schematic representation of such analysis is shown in Fig.2. This is known as Packet wavelet representation. The tree we construct this way is called structure tree [2]. Zooming into a frequency band with a packet wavelet transform may be achieved by preferentially expanding a chosen band at each level of resolution. It is a usual practice that one chooses the band, which contains the maximum energy. The energy of a band is computed by adding the squares of the values of the individual pixels. In the method used, the band with maximum energy at each level of decomposition is chosen and it is further decomposed into its constituent sub-bands. Here the original image is decomposed using four filters (filter banks) HH, HG, GH and GG in to four bands. We obtain four bands D1, E1, F1 and G1 Then it is sub-sampled by the factor 2,the energy is calculated and the one with maximum energy is once again decomposed using the four filters HH, HG, GH and GG in to four bands and sub-sampled by 2. Now we obtain D2, E2, F2 and G2. The general wavelet decomposition is shown below.

For the reconstruction of original image, we convolve the received D2, E2, F2 and G2 with the filters HHr, HGr, GHr and GGr, and we add all the results to

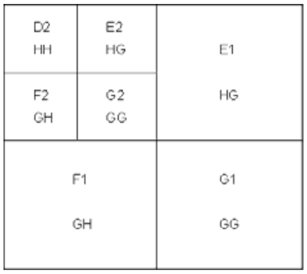

Fig 2.Wavelet Decomposition

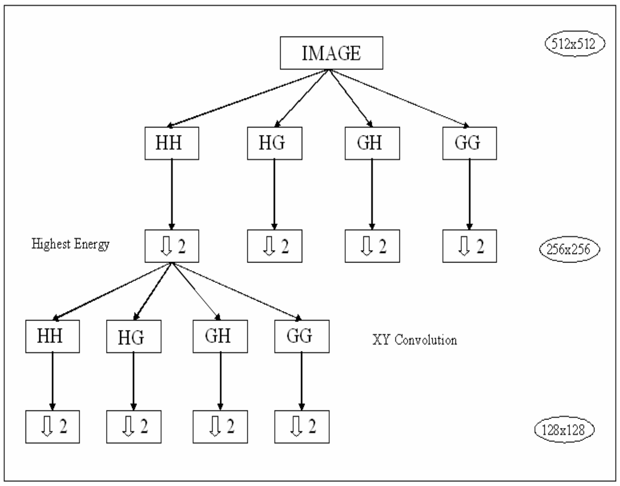

Fig 3.Packet Wavelet Decomposition

obtain the reconstructed image. A schematic representation of the reconstruction Process is shown in Fig.4.

4. Packet Wavelet Coder in CNN

The Packet Wavelet scheme represented in the Fig.2 can be implemented in the CNN computer. The 2D Haar Wavelet has been used implemented in the CNN universal machine, wherein each convolution block is modeled according to [1],

$$\tau \frac{dx^c}{dt} = -g(x^c) + \sum_{d=1}^{9} A_d x^d + \sum_{d=1}^{9} B_d u^d + D_A + D_B$$

where

$$g(x^c) = \begin{cases} -\infty, x^c < -1 \\ 0 \ , |x^c| < 1 \\ \infty \ , x^c > 1 \end{cases} \quad \text{……… (6)}$$

Boundary conditions are stated according to a periodic representation, where r, the neighborhood distance is set to unity and D_A, D_B, the offsets are set to zero. The possibility of electrical saturation in to the CNN chip is controlled by the non –linear function g (x^e). In order to perform the Wavelet transform with the CNN chip, the relationship templates are:

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad B_{HH} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0.5 & 0.5 \\ 0 & 0.5 & 0.5 \end{bmatrix} \quad B_{HG} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -0.5 & 0.5 \\ 0 & -0.5 & 0.5 \end{bmatrix}$$

$$B_{GH} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -0.5 & -0.5 \\ 0 & 0.5 & 0.5 \end{bmatrix} \quad B_{GG} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0.5 & -0.5 \\ 0 & -0.5 & 0.5 \end{bmatrix}$$

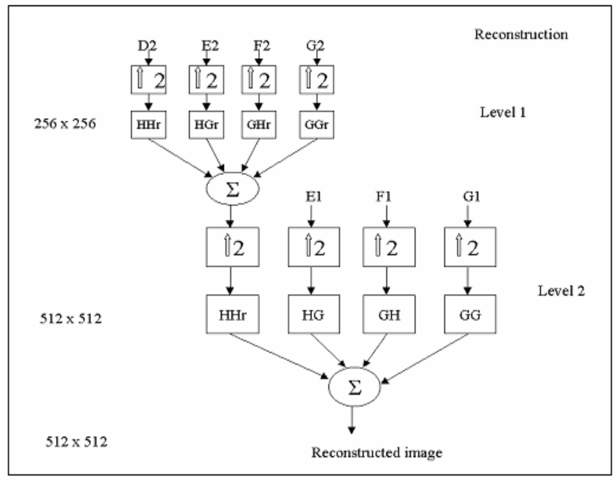

Fig.4.Schematic representation of the reconstruction process

When reconstruction is performed, conditions stated for direct transform are preserved. We only change the B templates [5] which will be;

$$B_{HHr} = \begin{bmatrix} 0.5 & 0.5 & 0 \\ 0.5 & 0.5 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad B_{HGr} = \begin{bmatrix} 0.5 & -0.5 & 0 \\ 0.5 & -0.5 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$B_{GHr} = \begin{bmatrix} 0.5 & 0.5 & 0 \\ -0.5 & -0.5 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad B_{GGr} = \begin{bmatrix} 0.5 & -0.5 & 0 \\ -0.5 & 0.5 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

5. Simulation Results

Extensive simulations with a number of images for the packet wavelet transform implementation in CNN were carried out. To evaluate the performance, the proposed system is applied to six standard test images. The results obtained for a 512*512 image is shown in Table.1. The values of the Peak Signal to Noise Ratio(PSNR) and the energy level of each sub-band during each level of decomposition are recorded for all the images. The compression ratio achieved for the entire test image is equal to 4:1. The concept of packet wavelet transform is implemented in such a way that, at each iteration the energy of the four sub-bands are calculated. The sub-band with maximum energy is now taken out and decomposition is performed on that particular sub-band again. For example the Barbara test image contains a narrow band component at high frequencies. Fingerprint images also contain similar narrow band high frequency components. Table- 2 shows the energy calculations for the different sub bands in the standard test images considered.

The reconstruction for images that contain narrow band frequency components using packet wavelet transform shows improved quality than that of the pyramidal structured wavelets. This is very clearly seen by our simulation results. Convolution is performed at each compression level and the tempering matrices used are remaining unchanged. Similarly, the matrices used for de-convolution are also shown. All simulations are carried out for the case of 2D-Haar wavelet transform. Fig.5. Shows the results obtained by decomposing the fingerprint image and energy of each sub band is indicated.

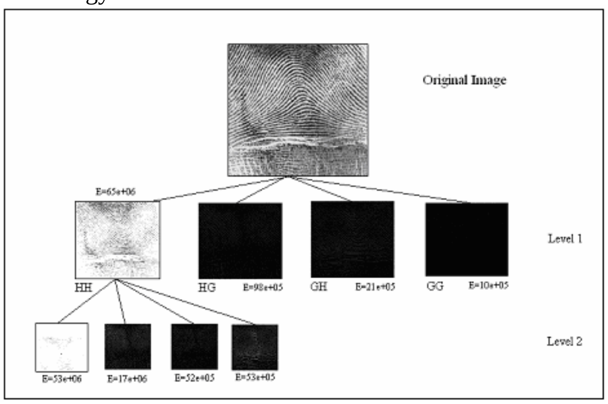

Fig 5.Decompostion of Finger Print Image Based on Packet Wavelets

As it can be seen from the Table.1, that the packet wavelet representation perform well for Barbara and Fingerprint images compared to pyramidal wavelets. This is due to the fact that Barbara and Fingerprint images contain narrow band high frequency components. Fig.6 and Fig. 8 show the two level decomposition of a fingerprint image by using packet wavelet and pyramidal wavelet representation respectively. Fig.7 shows the reconstructed images at level-1and at level-2 using packet wavelet representation. Fig.9.shows the reconstruction based on pyramidal wavelet representation.

Table 1: PSNR (dB) of Different Test Images

Test images	Packet wavelet representation	Pyramidal wavelet representation
Barbara	29.4	24.8
Lena	25.32	24.32
Desert	29.76	28.54
Boston city	29.4	28.45
Boat	25.95	25.32
Finger prints	32.26	28.43

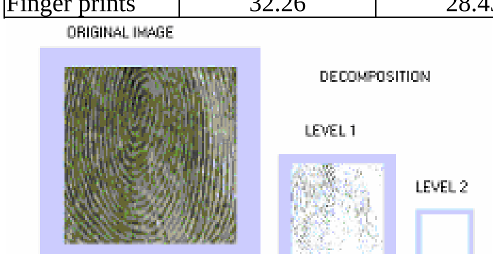

Fig 6. Packet wavelet de-composition

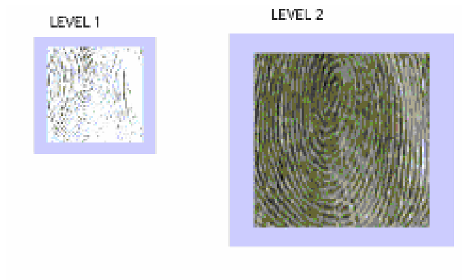

Fig 7.Packet wavelet reconstruction

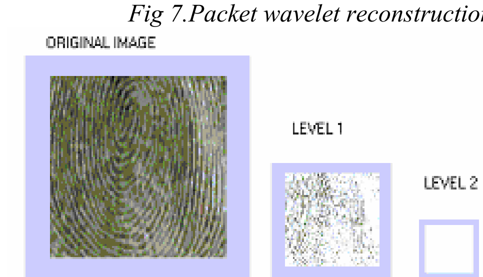

Fig 8.Pyramidal wavelet decomposition

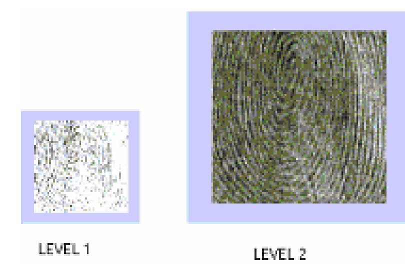

Fig 9.Pyramidal wavelet reconstruction

Table 2: Energy at Four Bands for Different Test Images

Test images	Energy EHH	Energy EHG	Energy EGH	Energy EGG
Barbara	2.9e+09	3.4864e+07	5.8e+06	5.88e+06
Lena	4.62e+09	7384038	345094	1051472
Desert	7.6e+09	1.2e+06	4.04e+06	3.5e+05
Boston city	619963602	3974763	4484129	108336
Boat	4.6e+09	1049904	4734126	779104
Finger prints	65e+06	98e+05	21e+05	10e+05

6. Conclusion

We have presented the packet wavelet based image-coding scheme implemented with the Cellular Neural Network architecture. The idea is to exploit CNN characteristics along with the frequency adaptive nature of the packet wavelet transform. From the simulations results and image reconstruction on various test images, we can say that high compression rates and high visual quality are possible for images that contain narrow band frequency components. Also the visual quality is better than pyramidal wavelets using CNN.

7. References

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