

# Kalman Filter Based Equalization for ICI Suppression in High Mobility OFDM Systems

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## Abstract

Orthogonal Frequency Division Multiplexing (OFDM) is known to be resistant to frequency selective fading environments. However, when employed over doubly selective channels with high Doppler, rapid time variations destroy the subcarrier orthogonality and introduce inter-carrier interference (ICI). Thus receiver equalization in such scenarios is a non-trivial task and involves high computational complexity. This paper addresses the problem of ICI suppression by viewing the system as a state space model. Kalman filter, which is known to yield the optimum solution to linear filtering problem, is employed as an equalizer to estimate the unknown state of the system, which comprises the transmitted symbols. Besides this, the use of convolutional coding for such systems is explored. This method is seen to provide a good performance at high Doppler spreads, comparable to other equalizers, while incurring a low computational complexity.

## 1. Introduction

Orthogonal Frequency Division Multiplexing (OFDM) is a modulation scheme widely used in high data rate communication over wireless channels. The main reason for this is its resistance to frequency selective fading environments and its ease of implementation through fast Fourier transform (FFT) algorithms. The equalization task is reduced to the design of a single tap filter. However, the operation of OFDM systems relies on the orthogonality of subcarriers and is perturbed by time selective fading which leads to inter-carrier interference (ICI). This causes the symbol received on each subcarrier to be affected by symbols on the other subcarriers. Several methods have been proposed in the literature so far for equalization of OFDM over doubly selective channels and ICI mitigation, in particular.

Zhao and Haggman in [1] have described a self-cancellation method for OFDM systems, in which transmit data symbols are coded across multiple subcarriers. The signals received on adjacent subcarriers are linearly combined with appropriate coefficients to minimize the residual ICI. It is shown that by using just two or three adjacent symbols, the method provides substantial improvement in the carrier-to-interference power ratio

(CIR). However, the method requires the use of higher order modulation to avoid bandwidth penalty.

Reference [2] has proposed time-domain channel estimation followed by symbol detection based on ZF and MMSE criterion with successive detection. In [3], the authors have analyzed ICI and ISI for large delay spread systems, with insufficient cyclic prefix, and presented an iterative method for joint ICI and ISI mitigation. The method offers substantial gains within 2 to 3 iterations; provided the channel state information is available at the receiver. In [4], Chen and Yao have proposed a technique for ICI mitigation and channel estimation based on pilot tones assuming that the channel taps vary linearly over an OFDM frame.

In [5], Seyedi and Saulnier have proposed a general ICI self-cancellation technique based on time-domain windowing for systems affected by frequency offset as well as Doppler spread. Reference [7] describes the design of a time domain ICI mitigation filter based on the maximization of signal to interference plus noise ratio (SINR). The filter design is proposed for SISO as well as MIMO OFDM systems.

In [10], the ICI generation mechanism is viewed in terms of convolution of the Doppler spectrum with Dirichlet sinc. The receiver uses time domain windowing to achieve ICI shortening and rendering the ICI matrix sparse. This is followed by iterative MMSE symbol estimation. In [11], the authors have designed a time-varying finite impulse response (TV-FIR) equalizer based on basis expansion model (BEM). The equalizer design uses both ZF and MMSE criteria and is extended to the case of multiple receive antennas. In [12], BEM based time varying FIR equalizer (TV FIR TEQ), that works jointly over all the subcarriers, is designed for large delay spread systems affected by both ICI and IBI. This design is transferred to frequency domain to design a per-tone equalizer (PTEQ).

The review of recent work indicates that the use of OFDM over rapidly time varying systems is limited by ICI and equalization of OFDM over doubly selective fading channels is a mammoth task. Besides this, the high performance equalizers are computationally taxing unless some simplifying assumption is made.

This paper addresses the joint problem of channel equalization and ICI suppression in high mobility OFDM systems. Kalman filter is known to provide the optimum linear solution when applied over a time varying Gaussian filtering problem. In this paper, the equalization problem

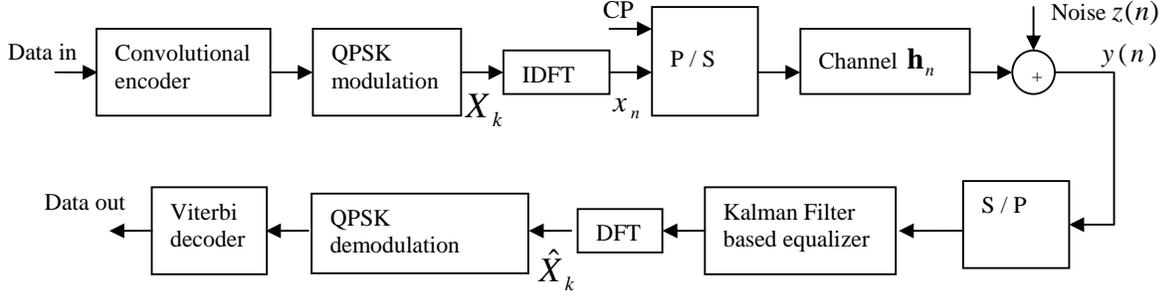


Figure 1: Block schematic of the proposed OFDM

has been defined as a state-space model followed by the use of Kalman filter. Moreover, convolutional coding is employed to improve the system performance. The scheme is found to outperform several contemporary equalizer designs while involving a very low computational complexity. The gain obtained with the use of convolutional codes over an uncoded OFDM system is also demonstrated.

The paper uses the following notation: boldface letters for matrices and vectors, superscripts  $*$  and  $H$  for complex conjugate and Hermitian operations respectively,  $\mathbf{I}_L$  denotes an identity matrix of dimensions  $L \times L$  while  $\mathbf{O}_{L \times L}$  denotes an  $L \times L$  matrix of all zeros.  $E[\cdot]$  denotes the expectation operation. The paper is organized as follows. Section 2 describes the system model followed by a discussion of the proposed technique in section 3. Section 4 describes and compares the computational complexity of the proposed system with other equalization approaches. The technique is validated through Monte Carlo simulations which form section 5. Finally, the paper is concluded in section 6.

## 2. System Model

We consider an OFDM system as in fig. 1 with  $N$  tones or equivalently  $N/2$  parallel sub-channels. Input data are buffered, convolutionally encoded and QPSK modulated into symbols  $X_k$ , where  $X_k$  denotes the  $k^{\text{th}}$  sample and  $0 \leq k \leq N-1$ .

OFDM modulation is accomplished by taking  $N$ -point IDFT of the block  $\mathbf{X} = [X_0 X_1 \dots X_{N-1}]^T$ . A cyclic prefix (CP) of length  $gi$  is appended to form the transmitted block as

$$\mathbf{x} = [x_{-gi} x_{-gi+1} \dots x_{-1} x_0 x_1 \dots x_{N-1}]^T,$$

Where  $x_{-i} = x_{N-i}$  for  $1 \leq i \leq gi$ .

Doubly selective fading channel following wide-sense stationary uncorrelated scattering (WSS-US) is assumed. The classical time autocorrelation function, according to Jakes' model, at a time lag of  $\Delta t$  is

$$\phi_t(\Delta t) = J_0(2\pi f_d \Delta t) \quad (1)$$

where  $J_0(x)$  is the zeroth-order Bessel's function of the first kind and  $f_d$  is the maximum Doppler frequency. Classical Doppler spectrum for each of the  $L$  channel taps is approximated by an independent autoregressive process of order-2 (AR-2). The channel tap vector at each instant of time is denoted by

$\mathbf{h}_n = [h(n,0) h(n,1) \dots h(n,L-1)]$ , where  $h(n,l)$  is the  $l^{\text{th}}$  tap at  $n^{\text{th}}$  time instant.

Considering the AR-2 model, we have

$$h(n,l) = a_1 h(n-1,l) + a_2 h(n-2,l) + v(n,l) \quad (2)$$

where  $a_1$  and  $a_2$  are the AR-2 coefficients chosen so as to provide a good match between the autocorrelation functions of the Jakes' model and the AR-2 model.

For a discrete lag of  $m$  sampling intervals, we have from (1)

$$\phi(m) = E[h(n,l)h^*(n-m,l)] = J_0(2\pi f_d m T_s) \quad (3)$$

which leads to

$$a_1 = -2r_d \cos(0.8\omega_d) \text{ and } a_2 = r_d^2 \quad (4)$$

where  $r_d = \left(1 - \frac{\omega_d}{\pi}\right)$  and  $\omega_d = 2\pi f_d T_s$ .

and  $v(n,l)$  is the modeling noise for  $l^{\text{th}}$  tap at time instant  $n$ .

The time varying frequency response of the channel is defined as

$$H(m,k) = \frac{1}{\sqrt{N}} \sum_{l=0}^{L-1} h(m,l) e^{-j2\pi k l / N} \quad 0 \leq k \leq N-1 \quad (5)$$

The received symbol corrupted by fading channel and AWGN can be written as,

$$y(n) = \sum_{l=0}^{L-1} h(n,l)x_{n-l} + z(n) \quad (6)$$

Demodulation involves taking DFT of the received block after removing the cyclic prefix to get  $[\hat{X}_0 \hat{X}_1 \dots \hat{X}_{N-1}]$ .

The demodulated signal at the  $k^{\text{th}}$  tone is

$$\begin{aligned} \hat{X}_k &= \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} y(n) e^{-j2\pi mk/N} \\ &= \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} \sum_{l=0}^{L-1} h(n,l) x_{n-l} e^{-j2\pi mk/N} \\ &\quad + \sum_{n=0}^{N-1} z(n) e^{-j2\pi mk/N} \end{aligned} \quad (7)$$

This on solving yields

$$\hat{X}_k = \sum_{m=0}^{N-1} ICI(k,m) X_m + Z_k \quad (8)$$

where

$$ICI(k,m) = \frac{1}{N} \sum_{n=0}^{N-1} H(n,m) e^{j2\pi(m-k)n/N} \quad (9)$$

and  $Z_k$  is the noise on  $k^{\text{th}}$  subcarrier.

### 3. Proposed Technique

The system is viewed as a state-space model and the equalizer at the receiver is designed based on Kalman filter [8]. We assume that exact CSI is available at the receiver; the channel tap vector generated using AR-2 model is considered known prior to equalization. At each instant of time, the state vector  $\mathbf{S}_n$  is taken to comprise the  $L$  consecutive transmitted values which affect the received symbol at that instant. Thus (6) provides the basis for forming the measurement equation. The unknown state vector  $\mathbf{S}_n$  needs to be estimated at each instant of time. The state vector transitions from one time instant to the next in accordance with the process equation (11).

$$y(n) = \mathbf{h}_n \mathbf{S}_n + z(n) \quad (10)$$

$$\mathbf{S}_n = \mathbf{A} \mathbf{S}_{n-1} + \mathbf{V}_n \quad (11)$$

Here, the unknown system state at instant  $n$ ,  $\mathbf{S}_n$  is an  $L \times 1$  vector

$$\mathbf{S}_n = [x_n \quad x_{n-1} \quad \dots \quad x_{n-L+1}]^T$$

Measurement matrix is simply the  $1 \times L$  channel tap vector at instant  $n$  (as in Section II)

$$\mathbf{h}_n = [h(n,0) \quad h(n,1) \quad \dots \quad h(n,L-1)]$$

The measurement noise vector  $z(n)$  comprises the AWGN sample at instant  $n$ .

The transition matrix

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & \dots & \dots & 0 \\ 1 & 0 & \dots & \dots & 0 \\ 0 & 1 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \dots & 0 & 1 & 0 \end{bmatrix}_{L \times L}$$

Process noise vector

$\mathbf{V}_n = [x_n \quad 0 \quad \dots \quad 0]^T$  is an  $L \times 1$  vector where  $x_n$  is the transmitted symbol at instant  $n$ .

A Kalman filter is then employed to estimate the unknown state of the system. Received signal  $y(n)$  at each instant is given as input observation to Kalman filter algorithm [13] and the following estimation equations are used:

$$\begin{aligned} \mathbf{S}_{n|n-1} &= \mathbf{A} \mathbf{S}_{n-1} \\ \mathbf{K}_{n|n-1} &= \mathbf{A} \mathbf{K}_{n-1} \mathbf{A}^H + \mathbf{Q}_1 \\ \mathbf{G}_n &= \mathbf{K}_{n|n-1} \mathbf{h}_n^H [\mathbf{h}_n \mathbf{K}_{n|n-1} \mathbf{h}_n^H + \mathbf{Q}_2]^{-1} \\ \mathbf{S}_n &= \mathbf{S}_{n|n-1} + \mathbf{G}_n [y(n) - \mathbf{h}_n \mathbf{S}_{n|n-1}] \\ \mathbf{K}_n &= [\mathbf{I}_L - \mathbf{G}_n \mathbf{h}_n] \mathbf{K}_{n|n-1} \end{aligned} \quad (12) \dots (16)$$

where  $\mathbf{G}_n$  is the  $L \times 1$  Kalman gain vector at instant  $n$ ,  $\mathbf{S}_{n|n-1}$  is the state estimate at instant  $n$ , given the observations upto time  $n-1$ ,  $\mathbf{Q}_1$  and  $\mathbf{Q}_2$  are the  $L \times L$  and  $1 \times 1$  size covariance matrices of  $\mathbf{V}_n$  and  $z(n)$  respectively, and  $\mathbf{K}_n$  is the covariance matrix of estimation error.

At any instant  $n$ , the estimated state vector  $\mathbf{S}_n$  gives  $L$  consecutive transmitted values. However, the best estimate obtainable is that of the symbol transmitted  $L-1$  instants ago, i.e. of the symbol  $x_{n-L+1}$ . As the equalizer is employed for  $N$  consecutive time instants, the estimates of symbols are accumulated into a frame and thereafter demodulated using FFT.

In order to improve the performance of the proposed scheme at high Doppler spreads, we incorporate convolutional coding in our system together with Viterbi decoder at the receiver.

### 4. Computational Complexity

In this section we compare the computational complexity of the proposed equalization scheme with the traditional block equalization scheme and the scheme proposed in

[12]. If the block linear equalizers (based on MMSE or ZF criteria) are employed, a matrix inverse of the order  $N \times N$  needs to be computed per OFDM symbol. This requires  $N^3$  floating-point operations per second (FLOPs) for the equalizer to be designed. Subsequently, estimation of the transmitted block requires further  $N^2$  multiplication and addition operations per OFDM symbol. For the FIR equalizer designed in [11], the design complexity is of the order of  $[(Q + Q')(L + L')]^3$  FLOPs and the implementation requires  $(Q'+1)(L+1)$  multiplications and additions per subcarrier, where the equalizer is assumed to consist of  $L'+1$  taps, each of which is represented in terms of  $Q'+1$  complex exponential basis functions. The equalizer design in [12] also involves similar complexity order although some simplified designs have been proposed. However, in our system, the equalizer requires simply an  $L \times L$  matrix inverse to be computed per subcarrier. Considering that the number of channel taps  $L$  is much smaller than the number of OFDM subcarriers  $N$ , it is clearly evident that our scheme offers considerable computational advantage, while providing a comparable performance to the other equalizers in question.

## 5. Simulation and Results

In the simulation, we have used a (2, 1, 7) convolutional code whose description may be found in [14]. The code rate is  $1/2$  and memory order is 7. The modulation scheme used is 4-QPSK with equi-probable modulated symbols given by  $X_k = \left(\frac{1}{\sqrt{2}}\right)(\pm 1 \pm j)$ . We have assumed an OFDM system with 128 subcarriers. A 6-tap channel where each tap is complex Gaussian with zero mean and unit variance is generated. Each tap is independently governed by an AR-2 process and updated at each time instant in accordance with (2). The system is affected by complex additive white Gaussian noise with variance  $\sigma^2$ . We consider high mobility systems with a velocity of 120 km/hr operating over the GSM band of 900 MHz. This corresponds to a maximum Doppler frequency of  $f_d = 100\text{Hz}$ . The sampling interval is  $T_s = 50\mu\text{s}$ , which for QPSK modulation corresponds to a symbol rate of 20 kilo-symbols/sec.

In this paper, the channel taps are assumed to be known at the receiver. However, in the absence of this information, suitable channel estimation algorithms may be employed based on training and/or semi-blind approaches, like Expectation – Maximization (EM) algorithm, or even blind algorithms which do not require any training. Suitable techniques for channel estimation in high Doppler spread systems are under investigation.

The receiver employs a Viterbi decoder after demodulation to recover the data bits. Fig. 2 shows the typical variations of a channel tap with time. The magnitude of one of the channel taps is monitored over 5 OFDM symbol durations

at  $f_d T_s = 0.005$ . Fig. 3 compares the performance of the Kalman filter based equalization scheme with the other equalizers in [12]. These include the traditional block linear MMSE, the TV FIR TEQ designed with unit-norm constraint (TEQ UNC) and PTEQ designed using basis expansion model without over sampling ( $P=1$ ). It is seen that the BER curves all the three aforesaid equalizers saturate at a BER of well above 0.03 as the SNR is increased. Our equalization scheme provides an improved performance while being computationally simpler as illustrated in Section 4.

Fig. 4 compares the BER performance of the uncoded system with the proposed convolutionally coded system. It is evident that coding provides a tremendous reduction in the SNR required to achieve a certain error performance. For instance, at an SNR of 20 dB, our equalization method without convolutional coding gives a BER of 0.0197 while a coded system with exactly the same parameters gives a BER performance of 0.0031. For comparison, we have shown the curve obtained when an uncoded system is operating at the same Doppler spread without using an equalizer. Such a system is seen to perform very poorly and saturates at a BER of 0.35. Besides this, the theoretical bound obtained for a TIV Rayleigh fading channel [15] is also plotted. Our scheme is seen to perform within 1dB of the theoretical lower bound especially at an SNR upto 20 dB.

## 6. Conclusion

The paper considers the performance of Kalman filter based equalizer for doubly selective OFDM systems affected with high Doppler. Apart from providing a considerable saving in computational complexity, the scheme is seen to perform well in terms of BER, when compared with other equalization techniques. Besides this, the use of convolutional coding for performance improvement is also explored.

## 7. References

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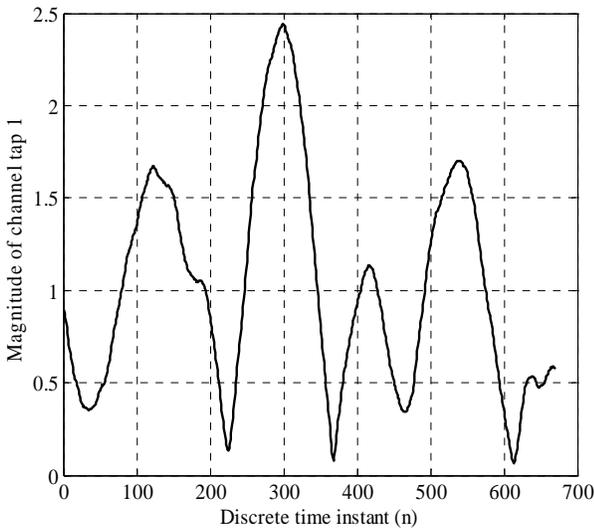


Figure 2: Channel tap variation with time over 5 OFDM symbols,  $f_d T_s = 0.005$

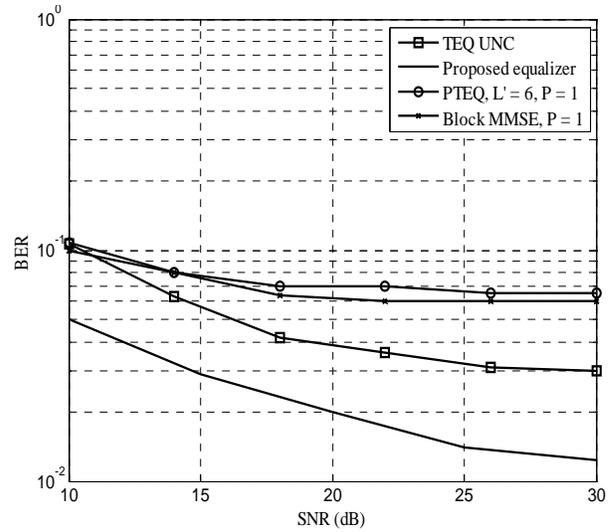


Figure 3: Comparison of BER performance of different equalization schemes

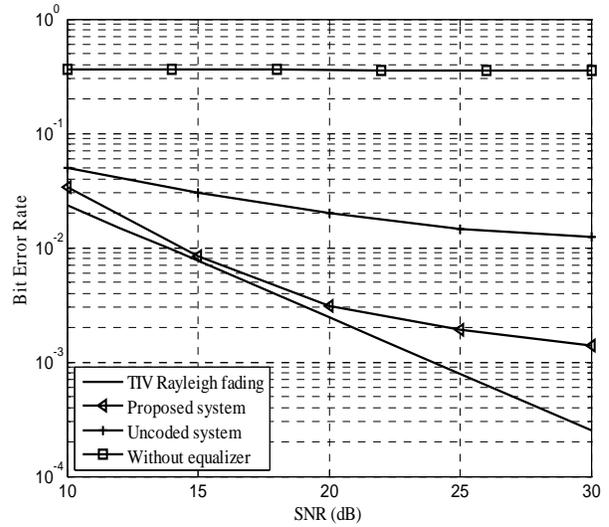


Figure 4: Comparison of proposed scheme with an uncoded system