Pilot Based Adaptive Channel Estimation for OFDM System
Using GS FAP Algorithm

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Abstract: OFDM is a multi-carrier transmission scheme which effectively combats the problem of ISI in high data rate communication. In this paper, the problem of channel estimation in OFDM system is addressed. This paper discusses an adaptive filtering algorithm called Fast Affine Projection (FAP) algorithm for channel estimation. The FAP algorithm requires matrix inversion which causes numerical instability. FAP combined with Gauss-Seidal (GS) iteration provides good solution to solve this problem. In the proposed method, GS FAP based channel estimation is done in time domain and equalization is performed in frequency domain. The convergence and complexity of GS FAP algorithm is compared with NLMS algorithm. Improvements over existing methods are demonstrated in simulation using UWB channel models.

Keywords: Channel estimation, NLMS, LS, OFDM

1. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) has been applied in wireless communication systems due to its high data rate transmission capability with high bandwidth efficiency and robustness to multipath delay. Ultra-wideband (UWB) communication is an emerging technology for high data rate networks over short range communication. UWB signals with short duration of pulses provide unique advantage in short range applications which include easy penetration through obstacles, high precision ranging and low processing power. UWB OFDM communication was proposed for physical layer in IEEE 802.15.3a standard that covers wide band communication in Wireless Personal Area Networks (WPANs) [1,2]. The channel model for UWB communication is entirely different from narrowband wireless communication and many channel models have been proposed in literatures [3,4]. Dynamic channel estimation is necessary before demodulation of UWB OFDM signal since the radio channel is time varying and frequency selective for wideband systems [5]. This paper mainly focuses on pilot based channel estimation in UWB OFDM system. The most common pilot based channel estimation scheme is the Least Square (LS) based channel estimation [15,16]. In this method, the channel estimates are obtained by simply dividing the received signal with the training symbol [6]. However this method is not optimum when considering the Mean Square Error (MSE). LMS based channel estimation for OFDM communication is proposed in [7]. Kalman filter based per subcarrier frequency domain channel estimation is dealt in [8] and the complexity of this method is comparatively high. Finite Affine Projection (FAP) algorithm is considered to estimate the channel response for OFDM systems [9]. In the proposed method, channel estimation is done in time domain and equalization is performed in frequency domain using LS method. The complexity of FAP algorithm is like LMS algorithm and the convergence is similar to the RLS algorithm. The FAP algorithm requires matrix inversion to find error state covariance matrix which produces numerical instability especially when implementing on real time hardware or fixed point software. Gauss Sedial (GS) iteration provides a good solution to the above problem [10]. In this paper, GS FAP algorithm is proposed for channel estimation in UWB OFDM systems.

The rest of the paper is organized as follows; Section II describes the UWB channel model for IEEE 802.15.3a WPANs with detailed specifications. Section III introduces the system model for UWB OFDM communication. In section IV, a novel channel estimation scheme using training sequences based on GS FAP algorithm is proposed. Section V presents the extensive simulation results under various channel conditions to validate our method.

II. CHANNEL MODEL

UWB channel model is recommended for lognormal distribution rather than Rayleigh distribution which is more suitable for multipath gain magnitude in wideband indoor communication. The famous multipath UWB indoor channel models are tap-delay
line Rayleigh fading model, Saleh-Valenzuela model and $\Delta$-K model \[11\]. Recently Intel proposed a modified S-V model for UWB communication \[4\]. The arrival of multi-path components is modeled by using statistically random process, based on Poisson distribution. The multipath arrival of UWB signals are grouped into two categories, cluster arrival and ray arrival within a cluster. The impulse response of UWB channel can be written as

$$h(t, \tau) = \sum_{l} \sum_{k} \alpha_{k,l} \delta(t - T_l - \tau_k)$$  \(1\)

Where $\alpha_{k,l}$ is the multipath gain coefficient of $k$th ray related to $l$th cluster. $T_l$ is the delay or arrival time of the first path of $l$th cluster. $\tau_k$ is the delay of $k$th path within the $l$th cluster relative to $T_l$. $X$ is the lognormal shadowing term.

The ray arrival and cluster arrival distribution time are given by

$$p(T_l) = \Lambda \exp[-\Lambda(T_l - T_0)] \quad l > 0 \quad (2)$$

$$p(\tau_k) = \lambda \exp[-\lambda(\tau_k - \tau_0)] \quad k > 0 \quad (3)$$

A novel modification method in UWB channel model for multicarrier wideband communication is proposed in \[12\] and the simulated channel parameters are listed in Table.1. More details about UWB channel model can be found in \[3\]. In this paper, the above mentioned channel characteristics are included during simulations.

### III. SYSTEM MODEL

The OFDM transmission model considered in this paper is shown in Fig.1. OFDM transmitter converts input data into N parallel data sequences and they are modulated by the Inverse Fast Fourier Transform (IFFT) in base band and then converted into serial data. Guard Interval (GI) in which zero is inserted between symbols to avoid InterSymbol Interference (ISI) caused by multipath fading \[13\]. The complex base band signal is written as \[1\]

$$r_{re}(t) = \text{Re}\left\{\sum_{k=0}^{N-1} r_k(t - kT_{sym}) \exp(j2\pi f_k t)\right\} \quad (4)$$

Where $\text{Re}(.)$ represents the real part of the complex variable. $r_k(t)$ is the complex base band signal of the $k$th OFDM symbol and it is nonzero over the interval from 0 to $T_{sym}$. $N$ is the number of OFDM symbols. $T_{sym}$ is the symbol interval and $f_k$ is the center frequency for the $k$th band. All of the OFDM symbols $r_k(t)$ can be constructed using an IFFT with certain set of coefficients $x_k(i)$, where the coefficients are defined as either data, pilot or training symbols.

$$s_{sym}(t) = \sum_{i=0}^{N}\sum_{k=0}^{N-1} x_k(i) p_k(t - iT_s) e^{j2\pi f_k(t - iT_p)} \quad (5)$$

Where $\gamma$ and $N_{ST}$ are defined as the subcarrier frequency spacing and the total number of subcarrier. $T_{CP}$ and $T_{GI}$ are the cyclic prefix duration and guard interval duration which are used in OFDM to mitigate the effects of multipath. $p(t)$ is a rectangular symbol pulse waveform defined as

$$p(t) = \begin{cases} 1, & 0 \leq t \leq T_{cp} + T_{sym} \\ 0, & T_{sym} \leq t \leq T_{sym} + T_{cp} + T_{GI} \end{cases} \quad (6)$$

The received signal of one user through UWB channel is written as

$$y(t) = \int_{-\infty}^{\infty} s_{sym}(t - \tau) \otimes h(\tau; t) d\tau + n(t)$$

$$y(t) = \sum_{i=0}^{N_{ST}} \sum_{k=0}^{N-1} z_k(i) p_k(t - iT_s) e^{j2\pi f_k(i - iT_p)} + n(t) \quad (7)$$

Where $z_k(i)$ is the received complex envelop at the $n$th subcarrier.

### IV. CHANNEL ESTIMATION USING FAP ALGORITHM

The Affine Projection Algorithm (APA) is a generalized version of Normalized Least Mean Square (NLMS) algorithm. The each tap weight update of NLMS is viewed as a one dimensional affine projection. In an APA, projections are made in multiple directions. As the projection dimension increases, the convergence rate is increases and so does the computational complexity. A fast version of APA

![Fig.1. System model](image-url)
is called Fast Affine Projection (FAP) algorithm [14]. This section explains the procedure of estimating the channel coefficients using FAP algorithm in an OFDM system which is given in Fig.2. To reduce the mathematical complexity, \( y(t) \) is denoted as \( y_t \). The received scalar complex valued signal \( y_t \) is defined as

\[
y(t) = x^T h + n, \quad \ldots (8)
\]

Where \( X \) is an excitation signal matrix. \( h \) is an unknown channel response and \( w_t \) is an additive white Gaussian noise. The initial conditions are

\[
h_0 = 0, E_0 = 0, R_0 = \delta I, \alpha = 0
\]

Received Symbol \[\rightarrow\]

DFT \[\rightarrow\]

Channel Estimation \[\rightarrow\]

DFT \[\rightarrow\]

Equalization

**Fig.2. Channel estimation block diagram**

At each sample \( t \geq 0 \),

\[
r_t = r_{t-1} + x \alpha_t - x_{t-1} \alpha_{t-1} \quad \ldots (9)
\]

\[
e_t = y_t - \hat{h} x_t - \mu \bar{e} E_{t-1} \quad \ldots (10)
\]

Update \( R_t \) using \( r_t \)

\[
e_t = \begin{bmatrix} e_t \\ (1 - \mu) \bar{e} \end{bmatrix} \quad \ldots (11)
\]

\[
\xi_t = \alpha_t \bar{R}_t \quad \text{or} \quad e_t = \xi_t R_t
\]

\[
E_t = \begin{bmatrix} 0 \\ \hat{E}_{t-1} \end{bmatrix} + \xi_t \quad \ldots (12)
\]

\[
h_t = h_{t-1} + \mu \bar{X}_t - s \bar{E}_{t-1} \quad \ldots (13)
\]

Where \( t \) is the time index, \( h_t = [h_{t-1}, \ldots, h_{t-L+1}]^T \) is an adaptive weight vector of length \( L \). \( \mu \) is the step size and \((.)^T\) denotes the matrix transpose. The excitation signal matrix \( X_t \) of size \( L \times N \) has the structure

\[
X_t = [x_t, x_{t-1}, \ldots, x_{t-L+1}]
\]

where \( \bar{X}_t = [x_t, x_{t-1}, \ldots, x_{t-L+1}] \). \( \bar{e} = R^{-1} \) is the inverse \( N \times N \) regularized autocorrelation matrix of excitation signal. \( R_t = X_t^T X_t + \delta I \). \( \delta \) is the regularization parameter \( l \)

is the \( N \times N \) identity matrix. \( E_t \) is an \( N \times 1 \) vector consisting of a sum of the fast normalized residual echo \( \xi_t \). \( E_{N,t} \) is the last element of \( E_t \). \( \bar{e} \) is a vector consisting of the uppermost \( N \)-1 elements of \( e_t \).

The complexity of FAP algorithm is \( 2L + f(N) \) multiply accumulate operations per sample. The term \( 2L \) is for steps (10) and (14) while the term \( f(N) \) does not depend on \( L \). The direct matrix inversion gives the complexity of \( f(N) = O(N^3) \). Since \( L \gg N \), the matrix \( R_t \) is slowly varying in time and the matrix inversion leads to numerical instability. Assuming that we have already obtained an accurate estimate of the vector \( p_{t-1} \) for sample \( (t-1) \), one GS iteration per sample is enough for nearly optimal performance.

The GS-FAP algorithm is based on one update of \( p_t \) at every sample. This is equivalent to solving the system \( R_t p_t = b \) with one GS iteration

\[
p_t = \frac{1}{(R_t)_s} \left( b_s - \sum_{j=0}^{N-1} (R_t)_{s,j} p_{t,j} + \sum_{j=0}^{N-1} (R_t)_{s,j} p_{t,j} \right)
\]

Where \( b_s \) is the \( i \)-th element of vector \( b \). \( p_t \) is \( i \)-th element of \( p_t \). \((R_t)_{s,j}\) is the \( (i,j) \)-th element of \( R_t \) and \( i=0, \ldots, N-1 \).

Thus the FAP algorithm based on matrix inversion benefits from distribution of the calculation in time. In Eq.14, the channel coefficients are updated recursively for each signal. After detecting one OFDM symbol, the channel coefficients are estimated in time domain and it is transformed into frequency domain to perform equalization.

**V. SIMULATION RESULTS**

In this section, the performance of the proposed pilot based adaptive channel estimation technique is analyzed using UWB channel model. The parameters for the different channel model are given in Table.1 and these parameters are included in simulation. The additive noise used in the simulation is based on Gaussian distribution with a variance \( \sigma^2 \). The parameters of the OFDM are as per IEEE 802.15.3a
standard with a bandwidth of 528 MHz divided into 128 subcarrier and QPSK modulation is considered. To make subcarriers are orthogonal in the presence of multipath, guard interval length of 32 subcarrier is added. The estimation using FAP algorithm is performed in time domain and equalization is done in the frequency domain. The MSE results in Fig. 3 indicate poor performance of NLMS algorithm over FAP due to high convergence and the convergence rate is portrayed in Fig. 4.

![Fig. 3. NMSE Comparison](image)

![Fig. 4. Convergence curve](image)

![Fig. 5. BER analysis](image)

Fig. 5 compares the BER performance of FAP algorithm with NLMS algorithm. The UWB channel model 1 (CM 1) is considered for the simulation and the same results can be achieved for other three channel models. 10,000 channels are randomized for each SNR and the step size considered for BER simulation is 0.1.

**CONCLUSION**

The problem of pilot based adaptive channel estimation in OFDM systems has been considered. A novel GS FAP algorithm based channel estimation for OFDM communication is proposed. The channel parameters are consistently estimated in time domain and equalization is performed in frequency domain. The proposed algorithm provides satisfactory performance than NLMS algorithm in MSE and BER sense over UWB channels. The results given in this paper would be of great importance in the design and analysis of future UWB systems.

**Table 1. Characteristics of UWB Channel model**

<table>
<thead>
<tr>
<th>Channel Model</th>
<th>CM 1</th>
<th>CM 2</th>
<th>CM 3</th>
<th>CM 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Statistics</td>
<td>LOS</td>
<td>NLOS</td>
<td>NLOS</td>
<td>NLOS</td>
</tr>
<tr>
<td>Mean excess delay (nsec)</td>
<td>5.9854</td>
<td>10.4428</td>
<td>16.2088</td>
<td>29.1430</td>
</tr>
<tr>
<td>RMS delay (nsec)</td>
<td>5.6847</td>
<td>8.6050</td>
<td>14.7270</td>
<td>25.9955</td>
</tr>
<tr>
<td>NP (85% energy)</td>
<td>4.25</td>
<td>6.77</td>
<td>9.39</td>
<td>15.77</td>
</tr>
<tr>
<td>NP (10 dB peak)</td>
<td>4.5</td>
<td>6.94</td>
<td>8.59</td>
<td>12.88</td>
</tr>
</tbody>
</table>

**References:**

4. IEEE P802.15-02/279r0-SG3a, UWB channel modeling contribution from Intel http://grouper.ieee.org/groups/802/15/pub/2002/Jul02


