

Adaptive Multiuser Decision Feedback Detection using Erasure Algorithm based Partial Parallel Interference Cancellation

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Abstract

In this paper, adaptive decision feedback detector (ADFD) based on parallel interference cancellation (PIC) approach is proposed for code division multiple access (CDMA) wireless systems, which not only suppresses the intra-cell interference but also cancels the inter-cell interference. We incorporate the erasure algorithm based soft-slicer (E-slicer) to control the error propagation effect in the proposed decision feedback technique, which supersedes the hard decisions based approach. The computer simulation results are presented to show the substantial improvement in the bit error rate (BER) performance of the presented minimum mean square error (MMSE) ADFD-PIC technique over the conventional techniques, under the frequency-selective smoothly time-varying fading channels.

Index Terms – Decision feedback detection, multiuser detection and interference cancellation (IC).

1. Introduction

Adaptive decision feedback detector has been presented, which combines the multiuser detection of intra-cell users with interference suppression of inter-cell users in CDMA cellular systems. To demodulate K users at base station simultaneously, the multiuser decision feedback strategy for CDMA system was first proposed in [1], [2], and was motivated by the earlier work on multi-input/multi-output decision feedback equalizers (DFEs). In the absence of error propagation, the capacity of CDMA system with the MMSE DFD is close to the capacity of an orthogonal multiple access scheme for the high load values [3], which enables power saving relative to the linear techniques.

A. Duel-Hallen has presented two multiuser decision feedback detectors in [2] *i.e.*, S-DFD (successive –) and P-DFD (parallel –) structures. Previously presented work on DFDs using the successive interference cancellation scheme depicts that the total delay involved in the decision process of S-DFD is more in comparison to the P-DFD, which limits the application of former decision feedback scheme. For the P-DFD, the tentative decisions of all the active users are fed back simultaneously, which are obtained using the linear MMSE receiver without interference cancellation.

The tentative decisions may be unreliable due to the overwhelming nature of multiple access interference (MAI), which leads to error propagation in the subsequent stages of the multistage detector. The cancellation of spurious signals with incorrect decision feedback leads to the interference enhancement. An intuitive approach is to cancel a fraction of the estimated interference, if a symbol estimate is thought

to be unreliable. In [4], D. Divsalar *et al.* have proposed the partial PIC approach, in which the partial cancellation factors (PCFs) are introduced to control the interference cancellation level as a remedy.

During the last decade, different parallel interference cancellation schemes have been proposed in literature for joint interference suppression and multiuser data detection [4]-[9]. There are two types of PICs classified according to the tentative decision device: hard-decision PICs (HPIC), in which the output of slicer is used for IC; and soft-decision PICs (SPIC), in which the output of correlator or linear receiver is used for IC [4]. The HPICs perform better than SPICs because the decision statistic is biased when linear estimates of the symbol are used for IC [5].

We may improve the performance of SPICs by reducing the decision statistic bias using the partial cancellation with PCFs (less than unity). However, this approach may be modified by using the variable PCFs based on the value of correlator output. There are two methods to obtain the optimal PCFs: the optimal values are determined either by theoretical analysis under some simplifying assumptions [6], or by using least mean square (LMS) algorithm in the training mode [7], [8]. However the former method is only valid for small number of users, and the latter adaptive method is not applicable in the time-varying multipath fading environment.

In a more sophisticated approach, Y. Hsieh *et al.* have proposed a two-stage decoupled partial SPIC receiver based on resampling strategy [9]. The optimal PCFs can be calculated efficiently online in the time-varying environment, when the exact channel response, noise variance and spreading sequences are assumed to be known. However, the perfect power control and resampling impose extra computational burden on the receiver.

For asynchronous CDMA system, R. Ratasuk *et al.* have proposed the P-DFD structure based on PIC criterion, which attempts to cancel all the interference simultaneously [10]. The presented results show that the P-DFD outperforms the linear receiver under fading conditions. For the unknown channel, the estimates have been obtained by minimizing the least-squares cost function in the training mode. Under the time-varying environment, it is difficult to estimate the covariance matrix of received signal vector. Therefore, the ill-conditioned covariance matrix introduces numerical problems. Moreover the covariance matrix inversion operation increases the computational complexity, which is undesirable in practice.

Further it has been shown that the P-DFD consists of a

linear MMSE filter followed by an error-estimation filter [11], in which the lower-diagonal constraint on the feedback filters is removed. The simulation results show that the error propagation effect severely degrades the performance of the P-DFD. Moreover, the application of the above MMSE P-DFD is limited to the “very” slowly time-varying channels.

In binary phase shift keying (BPSK) transmission, we transmit chip waveforms at the chip rate in the CDMA wireless systems. At receiver end, the detection takes place at the data rate. Therefore the intersymbol interference (ISI) is not only due to the chip waveforms of past data bits, but also due to the chip waveforms of present data bits of all the active users. However, the ADFD-PIC suppresses ISI using the feedforward and feedback filters.

In all the above discussed PICs, the MAI is considered as the main factor in the system performance degradation. In a recent paper on DFE [12], the E-slicer has been proposed for the asynchronous CDMA receiver. It has been designed to reduce the effects of residual MAI in the decision process. The erasure algorithm may be used to generate the variable PCFs depending on the output of multiuser linear filter. In this paper, we focus on the adaptive MMSE multiuser parallel decision feedback detector, which uses the E-slicer to control the error propagation effect.

The paper is organized as follows. In section 2, we first describe the CDMA system model and give details about the frequency-selective channel model for multipath fading environment. We next present MMSE multiuser P-DFD structure in section 3; and incorporate the erasure algorithm to reduce the error propagation effect in the PIC scheme due to the wrong tentative decisions in the feedback unit. Section 4 includes the adaptive implementation of the proposed erasure algorithm based multiuser ADFD-PIC *i.e.*, E-ADFD-PIC. Simulation results are presented in section 5. Finally, conclusions are given in section 6.

2. System model

In the following, we consider a spread spectrum binary communication system, employing normalized modulation waveforms $s_1(t), s_2(t), \dots, s_K(t)$, such that

$$s_k(t) = \sum_{j=0}^{N-1} c_j^k \psi(t - jT_c) \quad (1)$$

where c_j^k is j th chip ($\pm 1/\sqrt{N}$) in spreading code of k th user, T_c is the chip period, N is the length of spreading code in terms of chip periods, $1/\sqrt{N}$ is the energy normalization factor, and $\psi(t)$ is the real transmitted chip waveform shape, which has unit energy in the time interval $0 \leq t \leq T_c$ *i.e.*, $\psi(t) = 0$ for $t \notin [0, T_c]$. The transmitted bandpass signal for k th user may be written as:

$$x_k(t) = \text{Re} \left[\left\{ A_k \sum_i b_k(i) s_k(t - iT_b) \right\} e^{j\omega_c t} \right] = \text{Re} \left[\hat{x}_k(t) e^{j\omega_c t} \right] \quad (2)$$

where $b_k(i)$ is a real valued transmitted data symbol ± 1 ,

$s_k(t)$ is the spreading signature sequence of user k , A_k is the amplitude level ($A_k = \sqrt{2P_k}$), T_b is the symbol period ($T_b = NT_c$) and ω_c is the carrier frequency. Each user's transmitted signal (with signal power level P_k) is assumed to pass through an independent Rayleigh fading channel, which transforms the bandpass signal for k th user as:

$$r_k(t) = \text{Re} \left[\left\{ \sqrt{2P_k} \sum_i b_k(i) \sum_{l=0}^{L_k-1} \gamma_{lk}(t) s_k(t - iT_b - \tau_{lk}) \right\} e^{j\omega_c t} \right] \\ = \text{Re} \left[\hat{r}_k(t) e^{j\omega_c t} \right] \quad (3)$$

where $\hat{r}_k(t)$ is equivalent lowpass signal, L_k is the number of multipaths for k th user, the complex quantity $\gamma_{lk}(t) = |\gamma_{lk}(t)| e^{-j\omega_c \tau_{lk}}$ represents the complex attenuation factor of the l th path and τ_{lk} is the total propagation delay.

We assume that the fading channel response changes at the symbol rate. The channel order ($L_k - 1$) is kept less than the processing gain N (*i.e.*, the maximum delay spread of channel is smaller relative to the symbol period). For k th user, the discrete-time received signal in i th data symbol interval's n th chip is represented as:

$$\hat{r}_k(iN + n) = \sqrt{2P_k} \sum_{l=0}^{L_k-1} h_{lk}(iN + n - l) b_k(iN + n - l) c_{((n-l)_N)}^k \\ \text{for } n = 0, 1, 2, \dots, N - 1$$

where

$$h_{lk}(iN + n - l) = \int_{iT_c}^{(i+1)T_c} g_k(t' + (iN + n - l)T_c; t') \psi(t' - lT_c) dt',$$

$$g_k(t; \tau) = \sum_{l=0}^{L_k-1} \gamma_{lk}(t) \psi(t - \tau_{lk}), \text{ and the expression } (x)_N$$

denotes “ $x \bmod N$ ”. For further analysis, we consider the system model described in matrix form [13].

Without the loss of generality, we assume that the number of multipaths $L_1 = \dots = L$ and $A_1 = \dots = A_k = 1$ *i.e.*, $\sqrt{2P_k} = 1$. The smoothly time-varying Rayleigh fading channel coefficient $h_{lk}(i)$ is considered to be the autoregressive (AR) process [13]. The received signal vector $\hat{r}_k(i)$ consists of N consecutive stacked samples, where i is the data symbol index. The $N \times 1$ dimensional vector $\hat{r}_k(i)$ can be written as:

$$\hat{r}_k(i) = \left[\hat{r}_k(iN) \dots \hat{r}_k(iN + j) \dots \hat{r}_k(iN + N - 1) \right]^T$$

The composite signal vector $\hat{r}(i)$ can be written as:

$$\hat{r}(i) = \hat{r}_{eq}(i) = \sum_{k=1}^K \hat{r}_k(i) + \hat{z}(i) \quad (4)$$

where, $\hat{z}(i) = \left[z(iN) \dots z(iN + j) \dots z(iN + N - 1) \right]^T$ denotes the noise sample vector. The data bit vector $b(i)$, $L \times 1$ channel coefficient vector $\bar{h}^k(i)$ and $N \times L$ signature-

sequence-matrices C^k and \tilde{C}^k for k th user's i th data symbol can be defined as $b(i) = [b_1(i) \ b_2(i) \ \dots \ b_K(i)]^T$,

$$\bar{h}_k(i) = [h_{0k}(i) \ h_{1k}(i) \ \dots \ h_{(L-1)k}(i)]^T \text{ and}$$

$$C^k = \begin{bmatrix} c_0^k & \dots & 0 \\ c_1^k & \dots & 0 \\ \vdots & \ddots & \vdots \\ c_{N-1}^k & \dots & c_0^k \\ \vdots & \ddots & \vdots \\ c_{N-1}^k & \dots & c_{N-L}^k \end{bmatrix} \quad \tilde{C}^k = \begin{bmatrix} 0 & c_{N-1}^k & c_{N-L+1}^k \\ 0 & 0 & \vdots \\ \vdots & \vdots & c_{N-1}^k \\ \vdots & \vdots & \vdots \\ 0 & 0 & \dots & 0 \end{bmatrix}$$

We define the data symbol detection equation as:

$$\hat{r}_{eq}(i) = C(i)H(i)b(i) + \tilde{C}(i)H(i-1)b(i-1) + \hat{z}(i) \quad (5)$$

where $H(i) = \text{diag}[\bar{h}_1(i), \bar{h}_2(i), \dots, \bar{h}_K(i)]$,

$$C(i) = [C^1 \ C^2 \ \dots \ C^K] \text{ and } \tilde{C}(i) = [\tilde{C}^1 \ \tilde{C}^2 \ \dots \ \tilde{C}^K].$$

Further, we can divide all the active users (in a particular cell) in two sets of the detected ' D ' and the undetected ' $U = K - D$ ' users, which leads to

$$C(i) = [C^D(i) \parallel C^U(i)] \quad (6)$$

where, $C^D(i) = [C^1 \ \dots \ C^D]$ and $C^U(i) = [C^{D+1} \ \dots \ C^K]$.

Similarly, the channel coefficient matrix is subdivided as:

$$H(i) = \text{diag}[H^D(i), H^U(i)] \quad (7)$$

where, $H^D(i) = \text{diag}[\bar{h}_1(i), \bar{h}_2(i), \dots, \bar{h}_D(i)]$ and

$H^U(i) = \text{diag}[\bar{h}_{D+1}(i), \bar{h}_{D+2}(i), \dots, \bar{h}_K(i)]$. The data bit vector is rewritten as:

$$b(i) = [b^{DT}(i) \parallel b^{UT}(i)]^T \quad (8)$$

where, $b^D(i) = [b_1(i) \ b_2(i) \ \dots \ b_D(i)]^T$ and

$b^U(i) = [b_{D+1}(i) \ b_{D+2}(i) \ \dots \ b_K(i)]^T$. The substitution of the

$$\hat{r}_{eq}(i) = C^D(i)H^D(i)b^D(i) + C^U(i)H^U(i)b^U(i) + \tilde{C}(i)H(i-1)b(i-1) + \hat{z}(i) \quad (9)$$

3. Multiuser P-DFD using E-slicer

In this section, we present details about the MMSE multiuser P-DFD receiver design. Let the input vector to the decision device be $\hat{y}(i) = [\hat{y}^1(i) \ \hat{y}^2(i) \ \dots \ \hat{y}^K(i)]^T$ and the corresponding estimated data symbol vector at slicer output is $b_e(i) = [b_{e1}(i) \ b_{e2}(i) \ \dots \ b_{eK}(i)]^T$. In this PIC approach, we use the tentative decisions $b_t(i) = [b_{t1}(i) \ b_{t2}(i) \ \dots \ b_{tK}(i)]^T$ of all the users to cancel interference simultaneously. The output $\hat{x}_{lin}(i) = [\hat{x}_{lin,1}(i) \ \hat{x}_{lin,2}(i) \ \dots \ \hat{x}_{lin,K}(i)]^T$ of the linear MMSE multiuser detector (F_{lin}) is fed to the E-slicer to

generate $W_{pcf} b_t$ (as shown in Fig. 1). The PCF matrix is

$W_{pcf}(i) = \text{diag}[w_{pcf}^1(i), w_{pcf}^2(i), \dots, w_{pcf}^K(i)]$, where w_{pcf}^k

is PCF (real) assigned to the tentative decision of k th detected user. The P-DFD shown in Fig. 1 is modified to give E-DFD-PIC by using $B_w = BW_{pcf}$, where F and B are $N \times K$ feedforward and $K \times K$ feedback matrices respectively. The input to decision device (slicer) is

$$\hat{y}(i) = F^H(i)\hat{r}_{eq}(i) - B^H(i)[W_{pcf}(i)b_t(i)] \\ = F^H(i)\hat{r}_{eq}(i) - B_w^H(i)b_t(i) \text{ with } (W_{pcf} = W_{pcf}^H) \quad (10)$$

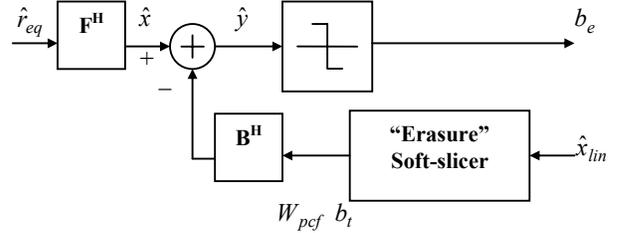


Fig. 1. P-DFD using E-slicer.

The error at the E-DFD-PIC output is $e_{dfd}^F(i) = b_e(i) - \hat{y}(i)$.

The corresponding error covariance matrix is defined as:

$$\Gamma_{dfd}^F(i) = E[e_{dfd}^F(i)e_{dfd}^{FH}(i)] \quad (11)$$

Assuming all the tentative decisions and estimated decisions correct, we define the scalar cost function *i.e.*, $J_{dfd}^F(i)$ as

the trace of $\Gamma_{dfd}^F(i)$. The optimum filters are derived by

$$\frac{\partial J_{dfd}^F(i)}{\partial F} = 0 \text{ and } \frac{\partial J_{dfd}^F(i)}{\partial B} = 0 \text{ (MMSE criterion) [14],}$$

which results in

$$F(i) = [R_{eq}]^{-1} C(i)H(i)[I_K + B(i)W_{pcf}(i)] \quad (12)$$

where, the covariance matrix of input signal vector is defined as $R_{eq} = E[\hat{r}_{eq}(i)\hat{r}_{eq}^H(i)]$.

$$F(i) = F_{lin}(i)[I_K + B_w(i)] \quad (13)$$

Therefore the optimum forward filter is a concatenation of the linear MMSE filter $F_{lin}(i)$, and $[I_K + B_w(i)]$ *i.e.*, the error estimation filter. Where, I_K is the $K \times K$ dimensional identity matrix. The proposed E-DFD-PIC is equivalent to the (scaled) conventional parallel interference canceller, [4], [10], for $L=1$ and $w_{pcf}^k(i) = \delta_w^k$ (less than unity).

4. Adaptive multiuser DFD-PIC

In the previous section, the implementation of the DFD-PIC requires the knowledge of the covariance matrix R_{eq} .

Particularly in the time-varying environment, it is difficult to estimate R_{eq} accurately. In the equation (13), the pseudo-inverse [15] of the matrix $F_{lin}(i)$ is used to derive the

feedback filter as:

$$B_w(i) = B(i)W_{pcf}(i) = [F_{lin}^H(i)F_{lin}(i)]^{-1}F_{lin}^H(i)F(i) - I_K \quad (14)$$

In the above equation, the linear MMSE filter coefficients are obtained recursively as:

$$F_{lin}(i+1) = F_{lin}(i) + \mu \hat{e}_{eq}(i) e_{lin}^H(i) \quad (15)$$

where, μ is the step-size and $e_{lin}(i) = b(i) - \hat{x}_{lin}(i)$ in the training mode. Since each column of the matrix $F(i)$ represents the weight vector corresponding to a particular user, therefore we can use the single user feedforward filter to obtain the optimum feedforward filter as $F(i) = [F^1(i) \ F^2(i) \ \dots \ F^K(i)]$. It is apparent that the linear MMSE multiuser filter $F_{lin}(i)$ partially suppresses MAI and ISI. However, the feedback filter $B_w(i)$ cancels the residual interference under perfect decision feedback condition.

Considering the case of a single undetected user, we may define the input to the decision device as:

$$\hat{y}^k(i) = F^{kH}(i)\hat{r}_{eq}(i) - B^{kH}(i)[W_{pcf}^D(i)b_t^D(i)] \quad (16)$$

The $(K-1) \times 1$ output vector of the E-slicer *i.e.*, $W_{pcf}^D b_t^D$ is pumped into the $(K-1) \times 1$ feedback filter $B^k(i)$, where $D = K - 1$. The feedforward filter $F^k(i)$ is obtained using the adaptive decision feedback strategy [12], which is the k th column of the matrix $F(i)$. Note that the step size μ is kept same to balance the equality (13). The complex computations involved in the calculation of $F^k(i)$ reduces due to the application of LMS algorithm. Moreover, the estimation of R_{eq} is not required in the proposed method.

5. Simulation results

For simulating CDMA system, the Gold-sequences of length $N = 31$ are generated. The problem of asynchronous reception may be resolved by using the over-sampling and fractional-tap F_{lin} and F^k filters [12]. To generate the time-varying environment, the $L \times 1$ dimensional vector $\bar{h}_{k,o}(i)$ is considered to be fast fading mobile communication multipath channel (Rayleigh). The subscript $(\cdot)_o$ denotes the optimum value. The Jakes' model is widely accepted as the realistic fading channel model, which is simulated by using $AR(2)$ process [eq. (1), 16] as:

$$\bar{h}_{k,o}(i) = -K_1 \bar{h}_{k,o}(i-1) - K_2 \bar{h}_{k,o}(i-2) + U(i) \quad (17)$$

where $U(i) = [u_0(i) \ u_1(i) \ \dots \ u_{L-1}(i)]^T$, such that $u_l(i)$ is a complex zero-mean white Gaussian process. The scalar coefficients in the above equation are $K_1 = -2r_d \cos(\sqrt{2}\pi f_D T_b)$ and $K_2 = r_d^2$, which takes account of the maximum Doppler frequency f_D of the underlying fading channel, the sampling time T_b and the

pole radius r_d corresponding to the steepness of the peaks of power spectrum. For accurate modeling [eq. (70), 17], the value of pole radius is given as $r_d = (1 - 2f_D T_b)$.

Two simulation examples are presented, in which we have considered the carrier frequency $f_c = 2GHz$, sample rate $1/T_b = 10kHz$ ($T_c = 3.226\mu sec$) and step size $\mu = 0.01$. In the training mode, we consider $b_t(i) = b_e(i) = b(i)$. Note that the DFD is switched to the decision directed mode after transmission of 500 training bits. For the multipath delay spread $T_m = 9\mu sec$, the number of multipaths is $L = (T_m/T_c) + 1 \approx 4$ for each user's transmission channel. We consider the maximum Doppler spread $f_D = 1Hz$ for all the active users. No power control scheme has been used. The presented results are based on an ensemble average of 150 independent simulation runs. We have shown the average bit error rate for each detector,

which is defined as $BER = \sum_{k=1}^K BER_k / K$.

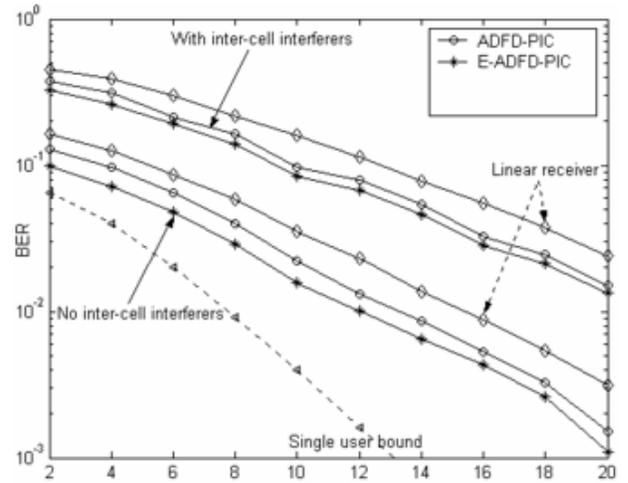


Fig. 2. BER Vs SNR.

Example 5.1: In this simulation, the effects of SNR on the performance of ADFD-PICs are analysed. For four users system, the results shown in Fig. 2 depict that the BER reduces with increasing SNR. The effects of other-cell interference are observed by considering the two inter-cell interferers (with $L = 4$ and $f_D = 1Hz$). The success rate of all the PICs degrades. However, the E-ADFDPIC outperforms the ADFD-PIC with and without inter-cell interferers. It may be inferred from the results presented in [10] that the P-DFD offers approximately $2dB$ gain relative to the linear receiver. However, the proposed E-ADFDPIC provides approximately $2.75dB$ performance gain over the linear receiver in the smoothly time-varying environment.

Example 5.2: For four users system, we analyze the affects of the maximum Doppler spread and number of users on the performance of the proposed multiuser receiver. In this case,

the maximum Doppler spread is varied up to $f_D = 5\text{Hz}$ at $\text{SNR} = 6\text{dB}$. The simulation results presented in Fig. 3 show that the BER increases with increasing f_D i.e., the proposed technique is valid for slow moving mobile users.

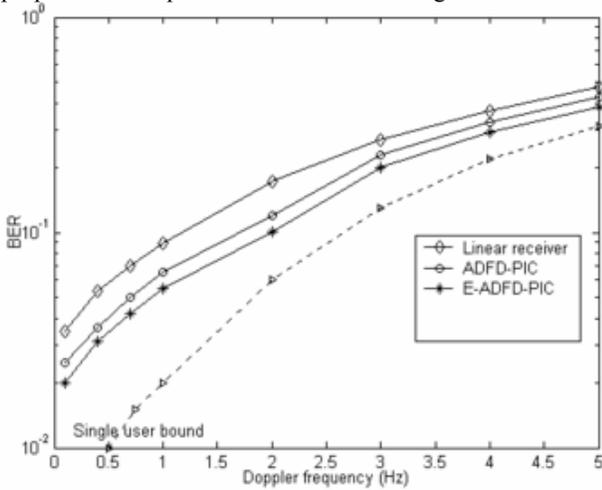


Fig. 3. BER Vs Doppler frequency.

6. Concluding remarks

In this paper, we have proposed an adaptive decision feedback detector based on the parallel interference cancellation scheme for CDMA system, which uses the erasure algorithm based soft-slicer. The ISI due to the chip waveforms of the present data bit and MAI are suppressed first by the linear MMSE feedforward filter, and then the residual MAI, ISI and OCI is cancelled using the feedback filter. The effects of error propagation are controlled by incorporating the erasure algorithm in feedback unit. The E-ADFD-PIC provides approximately 2.75dB performance advantage relative to the linear MMSE multiuser receiver in the smoothly time-varying multipath fading environment. Moreover, the adaptive implementation reduces the computational complexity due to the application of LMS algorithm. The simulation results depict that the proposed receiver may be used efficiently for slow moving users.

Appendix

Let the i th data symbol of the k th user be b_k . The estimated value of the data symbol at the output of F_{lin} is $\hat{x}_{lin,k}$. The corresponding error is $e_{lin}^k = b_k - \hat{x}_{lin,k}$. The erasure algorithm [12] is stated as:

$$\left\{ \begin{array}{l} |\hat{x}_{lin,k}| > |2A_k| \quad \text{or} \quad \left(1 - \frac{|e_{lin}^k|^2}{|A_k|^2} \right) < 0 \longrightarrow w_{pcf}^k = 0 \\ 0 < |\hat{x}_{lin,k}| \leq |2A_k| \longrightarrow w_{pcf}^k = \left(1 - \frac{|e_{lin}^k|^2}{|A_k|^2} \right) \end{array} \right.$$

In the decision directed mode, we use the tentative decision $b_{tk} = \text{sgn}(\hat{x}_{lin,k})$ instead of b_k . The output of the E-slicer is $w_{pcf}^k b_{tk}$. The proposed system is equivalent to the ‘‘Brute Force’’ interference canceller [4], if we let each $w_{pcf}^k(i) = 1$ for $k = 1, 2, \dots, K$.

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