Optimal Whitening Approach for Improved Channel Estimation in GSM Systems

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Abstract

In GSM applications, the receiver detects the symbols transmitted by different users, after making an estimate of the channels through which the symbols are received. Currently GSM employs the Method of Least Squares (LS) to estimate the unknown channels. We propose to apply an optimal whitening transformation to this LS estimate of the channels to improve the accuracy of the estimate, at moderate to low Signal to Noise Ratio (SNR) prevalent in mobile environments. This estimate also achieves the CRLB. Further, it enables the assignment of non-orthogonal midambles to GSM users accessing the same frequency band in the same time slot in the synchronous case.

Key words: LS Estimation, Best linear Unbiased estimator (BLUE), whitening Transformation, CRLB.

1. Introduction

Cellular networks today try to cater to a large number of users simultaneously. The interference due to other users places a constraint on this goal. In the GSM network, the number of users in a cell is system-dependent with every cell assigned a set of frequency bands, each of which can accommodate upto 8 users in different time-slots.

In GSM networks, the symbols are detected by employing a Trellis search to give a good receiver performance. However, the knowledge of the channel impulse response is inevitable for the detector. The current scheme of estimation in GSM employs the LS estimate to determine the unknown channel coefficients. Amongst all unbiased estimators, the LS estimator is certainly the minimum variance estimator. Also, if the noise in the channel is additive white Gaussian noise (AWGN), it becomes the Best Linear Unbiased Estimate (BLUE). However, when the SNR is low or moderate, the receiving samples 'y', are not sensitive to changes in channel coefficients 'h', in which case the LS estimate of channel coefficients (h_{LS}) becomes a poor estimate of 'h'. The error in estimation of 'h' can have a large dynamic range in its variance and spectral shape.

In this paper, a transformation of the LS estimate of 'h' is chosen such that the covariance of the error in its estimate (and hence of the estimate) is constrained to be white. We apply a whitening transformation W, to h_{LS} , such that the transformed vector is 'as close as possible' in an MMSE sense to h_{LS}. We consider the synchronous case of two users trying to use the same frequency band in the same time-slot. It is also proved that the LS estimator followed by a whitening transformation becomes an estimate that yields the CRLB for biased estimators. For two users using the same frequency at the same time slot (synchronous case), this proposed estimator permits GSM to allocate non-orthogonal midambles to the users as an alternate to the existing system of assigning only uncorrelated midambles to two such users to prevent interference.

Section 2 deals with signal and channel model. Section 3 explores the LS channel estimation currently used in GSM system. Section 4 describes the whitening transformation employed and derives the CRLB. Section 5 concludes the paper. `

2. Signal Model

The structure of the physical content of a time slot, i.e. a burst, specified by the GSM standard, is shown in Figure 1. A midamble of Nm = 26 symbols, defined by the training sequence code (TSC), is placed in the center of each burst, and it is used for estimating radio channel conditions. Two 58-symbol long blocks on each side of the midamble contain data. A guard period (GP) of 8.25 symbols at the end of each burst prevents signals in consecutive time slots from overlapping. Hereinafter, we will ignore both guard period and tail binary symbols (TB), and observe only N_B =142 symbols of data and midamble as the user's burst.

TB	Data symbols	TSC	Data symbols	TB	GP
3	58	26	58	3	8

1time slot =.577ms

Figure1 A Typical GSM Burst

While propagating through the radio channel, the signal is subject to different time varying distortions due to interference, fading and noise. In this paper, we consider a flat fading channel that does not induce inter-symbol interference. In this case, apart from additive noise, the channel introduces only an unknown attenuation that scales the signal amplitude. The total attenuation is the result of the following processes that are assumed mutually independent.

1. *Path loss:* Signal power decreases according to the power law of the distance between the transmitter and the receiver. In the general case, the path loss attenuation can be expressed as

$$\frac{|\omega_p|^2}{|\omega_{p_0}|^2} = \left(\frac{r}{r_0}\right)^{-\alpha}$$

where a is the path loss exponent that depends on the environment, w_p and w_{p0} are amplitude weights at distance r and reference distance r_o, respectively.

2. *Large scale fading* (*shadowing*) is due to large obstacles in the propagation path that block the signal. It is a slowly varying process, modeled stochastically with log-normal distribution:

$$20\log(w_s)\ [\mathrm{dB}] \sim \mathcal{N}(\mu_s, \sigma_s^2)$$

3. *Small scale fading* is the result of multipath propagation. The amplitude of the received faded signal is modeled as a random variable with Rayleigh distribution.

2.1 Users in the Same Cell

Consider the case where two users served by the same base transceiver station (BTS). The cell is modeled as a circular area of radius *R[4]*, with the BTS in the centre and the users uniformly, independently identically distributed (i.i.d.) within. The position of each user is determined by the polar coordinates (r;ø), which are random variables with the following probability density functions (PDFs):

$$p(r) = \frac{2\pi r}{\pi R^2} = \frac{2r}{R^2}, 0 \le r \le R$$

$$p(\varphi) = \frac{1}{2\pi}, 0 \le \varphi \le 2\pi$$

Let w_{s1},w_{p1} be the shadowing and path loss factors between the BTS and MS_1,and w_{s2},w_{p2} the equivalent coef_cients for the BTS - MS_2 link. Since we assume perfect power control performed in the downlink, the base station will adjust the transmit power to be inversely proportional to path loss and shadowing coefficients of each user's signal. Thus, the signal of interest arrives at MS_1 with compensated attenuation factors: $w_{s1}w_{s1}^{-1}=1$ and $w_{p1}w_{p1}^{-1}=1$ while the interfering signal will be received at MS_1 attenuated by relative shadowing and path loss coefficients given respectively by:

$$\omega_s = \frac{\omega_{s1}}{\omega_{s2}}; \omega_p = \frac{\omega_{p1}}{\omega_{p2}} = \left(\frac{r1}{r2}\right)^{-\alpha/2}$$

The baseband signal model can be expressed as:

$$y(i) = x_1(i)h_1 + \omega_s\omega_p x_2(i)h_2 + n(i),$$

where y(i) is a $N_a\times 1$ vector of the received signals at N_a antennas of MS_1 at the i^{th} symbol interval, 1= i = N_B; x_1,x_2 ? { -1; +1} are i^{th} transmitted symbols from BTS to MS_1 and MS_2, n is a complex Gaussian noise vector(zero mean, with variance $\neg^2$) and h_1 and h_2 are channel impulse response vectors containing i.i.d. complex Gaussian coefficients for each receiving antenna. Thus, they can be viewed as unique spatial signatures characterizing each user's signal space.

3. Channel Estimation

Since both the useful and the interfering signal arrive at the mobile receiver from the same BTS, bursts will completely overlap, as shown in Figure 2. Due to attenuation, power levels differ significantly in general. The midambles

$$[x_1(58+1)\dots x_1(58+26)]^T = m_1 \text{ and}$$

$$[x_2(58+1)\dots x_2(58+26)]^T = m_2$$

\will be perfectly aligned. Therefore, in order to enable identification and estimation of different channels for each user, it is necessary that they use different midambles. We assume that MS_1 knows the interferer's midamble m_2.

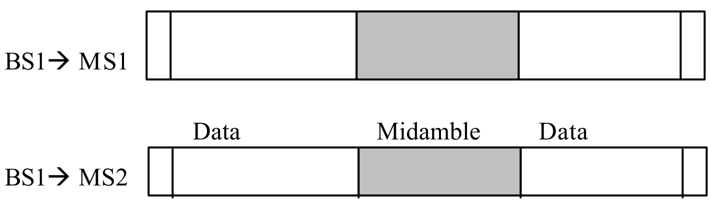


Figure 2 :Overlapping Bursts

Let us consider now a communication system in the presence of co-channel interference that is shown in Figure

3. Two synchronised co-channel signals have independent complex channel impulse response

$h_{L,n} = [\mathbf{h}_{0,n}, h_{1,n} \ldots h_{l,n}]^T$,n=1,2

and where L is the length of the channel memory. The sum of the co-channel signals and noise **n** is sampled in the receiver. The joint demodulation problem is to detect the transmitted bit streams $\mathbf{a}_1$ and $\mathbf{a}_2$ of the two users from the received signal **y**. To assist that joint detection operation the Joint channel estimator provides channel estimates $h_1^{\wedge}$ and $h_2^{\wedge}$

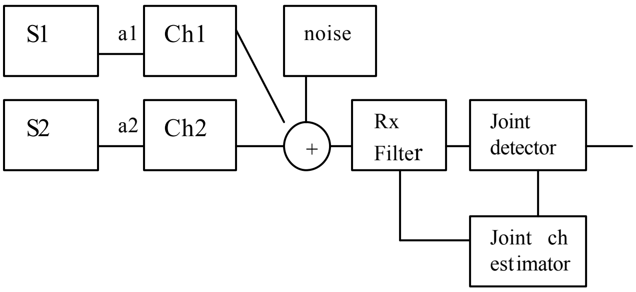


Figure 3: block diagram of GSM system

The complex channel impulse responses of the two synchronous co-channel signals are expressed with a vector $\mathbf{h}^{\sim}$ as follows

$$\tilde{h} = \begin{bmatrix} h_{L,1} \\ h_{L,2} \end{bmatrix}$$

containing the channel taps of the individual signals denoted by

$$h_{L,n} = \begin{bmatrix} h_{0,n} \\ h_{1,n} \\ . \\ . \\ h_{L,n} \end{bmatrix}, n = 1,2$$

Hence, $\mathbf{h}^{\sim}$ has totally 2×(L+1)elements. Both the transmitters send their unique training sequences with a reference length of P and guard period of L bits. The sequences are denoted by

$$m_n = \begin{bmatrix} m_{0,n} \\ m_{1,n} \\ . \\ . \\ m_{p+L-1,n} \end{bmatrix}, n = 1,2$$

The circulant training sequence matrices are denoted by

$$M_n = \begin{bmatrix} m_{L,n} \ldots\ldots\ldots\ldots m_{1,n} m_{0,n} \\ m_{L+1,n} \ldots\ldots\ldots\ldots m_{2,n} m_{1,n} \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ m_{L+p-1,n} \ldots\ldots\ldots m_{P,n} m_{P-1,n} \end{bmatrix}$$

and they are gathered into one large matrix

$$\tilde{M} = [M_1 M_2]$$

With these notations the received signal **y** is again given by

$$\mathbf{y} = \tilde{\mathbf{M}}\tilde{\mathbf{h}} + \mathbf{n} \, .$$

The LS channel estimates can be found simultaneously for the both users by minimizing the squared error quantity, which produces in the presence of AWGN the following solution

$$\hat{\mathbf{h}} = \arg\min_{\mathbf{h}} \left\| \mathbf{y} - \tilde{\mathbf{M}}\tilde{\mathbf{h}} \right\|^2 = (\tilde{\mathbf{M}}^H \tilde{\mathbf{M}})^{-1} \tilde{\mathbf{M}}^H \mathbf{y}$$

3.2 Limitation in the existing GSM System in Channel Estimation

The resultant h is not uncorrelated, as can be proved from its covariance computation:

$$\text{Covariance of } \hat{h} = (M^H M)^{-1} M^H E(y\, y^H)\, M\, (M^H M)^{-1}$$
$$= E\,(nn^H)\,(M^H M)^{-1}$$
$$= \sigma^2\,(M^H M)^{-1} \,,$$

Under conditions of low to moderate SNR, the covariance of the channel estimate can have a large dynamic range,i.e the error in the estimate of 'h' can have a large dynamic range. The covariance matrix is not diagonal,i.e total decorrelation does not exist.In GSM, only four pairs of midambles per slot are orthogonal to each other .This implies that we are forced to allocate only these pairs to users of the same cell, operating in the same time slot.

4. Whitening Approach to Channel Estimation:

The LS estimator seeks the estimate of **h** that results in a data vector **y** that is 'as close as possible' to the original data vector y. However we wish to minimize the error between h and it's estimate. In conditions of low to moderate SNR, the data vector y is not very sensitive to changes in h, so that large error in estimating h may translate into small errors in estimating 'y'. To improve the performance of LS estimator, we make use of a modification of the LS channel estimate, based on the concept of **MMSE whitening**[1],[2].

Theorem 1 (MMSE Whitening Transformation):

Let a e R^m be a random vector with positive definite covariance vector C_a =VDV, where D is a diagonal matrix and V is a unitary matrix .Let w be the optimal whitening transformation that minimizes the MSE between the input a and the output b =Wa with covariance*

$C_b=c^2 I_m$ where c > 0 Then

$$W = aVD^{-1/2}V^* = aCa^{-1/2}$$

Where

1. If c is specified then a = c

2. If c is chosen to minimize the MSE then

$$a = (1/m)\sum_{k=1}^{m}\sqrt{d_k}$$

proof: Let Ca= VDV*, (eigen decomposition).

Define,a =V*a; $\bar{b}$ =V*b;therefore,C_a =D; i.e, the elements of a are uncorrelated. Also, since V* is unitary, $C_b= c^2 I_m$, same as Covariance of $\bar{b}$.Hence MSE of (a ,b)=MSE of (a ,$\bar{b}$).

It is straightforward to show that,

$$\hat{W} = V\hat{\bar{W}}V^*; \tag{1}$$

To determine $\hat{\bar{W}}$, mean square error is given by,

$$\varepsilon_{MSE} = \sum_{k=1}^{m} E((\bar{a}_k - \bar{b}_k)^2) = \sum_{k=1}^{m} d_k + mc^2 - 2\sum_{k=1} E(\bar{a}_k\bar{b}_k) \tag{2}$$

where $d_k = E(\bar{a}_k^2)$ are the eigen values of Ca .

From the Cauchy-Schwartz inequality

$$E(\bar{a}_k\bar{b}_k) \le \left|E(\bar{a}_k\bar{b}_k)\right| \le \left(E(\bar{a}_k^2)E(\bar{b}_k^2)\right)^{1/2} = c^2E(\bar{a}_k^2) \tag{3}$$

With equality if and only if $\bar{b}_k = \left(c/\sqrt{d_k}\right)\bar{a}_k$ with probability 1. $\hat{\bar{W}}$ =$cD^{-1/2}$,From (1)

$$\hat{W} = cVD^{-1/2}V^* = c\,Ca^{-1/2} \tag{4}$$

Minimizing (2) with respect to c,

$$c = (1/m)\sum_{k=1}^{m}\sqrt{d_k}$$

and the optimal whitening transformation is

$$\hat{W} = c\,VD^{-1/2}V^* = cCa^{-1/2}$$

4.1 Whitened Channel Estimate:

Since h_{LS} =h+n, where n = $(M^HM)^{-1}M^Hn$, and (the covariance of the noise component n in h_{LS}) the covariance of h_{LS}, is given by $\neg^2(M^HM)^{-1}$,.

From Theorem 1, it follows that the optimal whitening transformation is proportional to w = $(M^*M)^{1/2}$ so that the WTLS estimator, denoted by H_{WTLS}, has the form

$$H_{WTLS} = ß\,(M^*M)^{1/2} x_{LS} = ß(M^*M)^{1/2}M^*y$$

Minimizing the above equation with respect to ß, the optimal value of ß is given by

$$ß = \frac{Tr\,((M^*M)^{1/2})}{Tr(M^*M)}$$

4.2 Cramer-Rao Lower Bound (CRLB) [for biased estimators]

For biased estimators, whitening estimator attains CRLB[2],[3].

$$\text{Cov(estimate of h)} = [\,I_m + \frac{\partial}{\partial h}B(h)\,]\; J^{-1}\,[\,I_m + \frac{\partial}{\partial h}B(h)]^*$$

$$\text{where } J(h) = -E\,[\frac{\partial^2}{\partial h^2}\log p(h,y)];$$

(Fisher information matrix)

For the whitening estimator, the bias is given by

$$B(h_{WTLS}) = (ß(M^HM)^{1/2} - Im)\,h$$

$$\frac{\partial}{\partial h}(B(h_{WTLS})) = (ß(M^H M)^{1/2} \text{-Im})$$

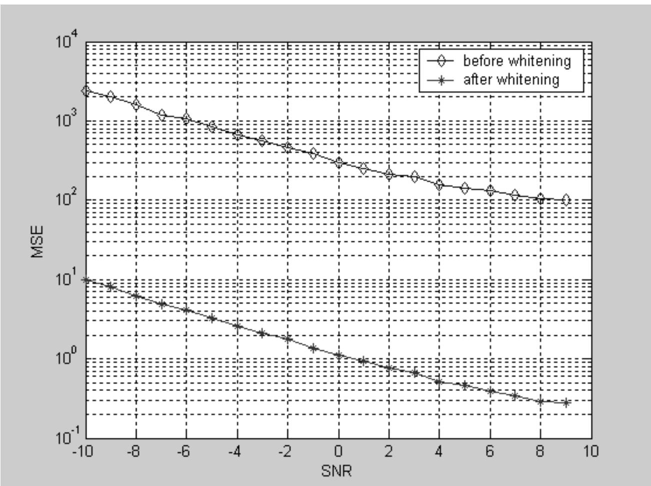
When the noise is Gaussian $J(h) = M^* M$

Therefore the CRLB on the variance of any estimator with bias $B(h_{WTLS})$;

$$Var\ (h_{WTLS}) = ß^2((M^H M)^{1/2}(M^H M)^{-1}(M^H M)^{1/2}$$
$$= ß^2 Im.$$

However, the whitening transformation transforms the least square estimate to an estimate with covariance= $ß^2$Im. Hence the whitening estimator is the optimum estimator of the unknown channel coefficients.

Figure(4) describes the improved performance of the whitening estimator, with respect to the LS estimator by means of a plot of the mean squared error of the estimate as a function of SNR. Here, we have considered low to moderate SNR values ranging from -10db to +10 db.As can be seen,at an SNR of 0db,a mean-squared error of $10^{2..5}$ of the LS estimate has been brought down to 10^0 by using the whitening transformation.



Figure(4): Comparison of MSE between LS and Whitening Approach.

5. Conclusion

In this paper, we propose the application of whitening on two synchronous, power imbalanced users in a GSM-like system. From the results obtained mathematically and through simulations, we can infer that the use of whitening to the LS estimate improves the performance of GSM channel estimation, and hence the overall performance in detection of symbols. Among all the linear estimators for a given bias, this estimator yields the minimum variance, as it theoretically obeys the Cramer-Rao Lower bound for biased estimators. Hence it is bound to outperform all existing methods of channel estimation in GSM environments. We can also employ the whitening approach in the Multi-User-Detection (MUD) of received symbols.

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