A NEW TECHNIQUE TO REDUCE THE CROSS TERMS IN WIGNER DISTRIBUTION

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ABSTRACT

We present a new method for time-frequency representation (TFR), which combines a Fourier Bessel (FB) series expansion and the Wigner-Ville distribution (WVD). The FB series decomposes a multi-component signal into a number of single component signals before the WVD is applied. and then WVD is applied on the single component signals to analyze its time-frequency distribution (TFD). The simulation results show that the proposed technique based on FB decomposition is a powerful tool to analyze multi-component signals without $cross\ terms$.

1. INTRODUCTION

In many engineering applications such as speech analysis, speech synthesis, radar, sonar and telecommunications, the signals under considerations are known to be non-stationary, for which the signal parameters are time-varying. For spectral analysis of such type of signals, discrete Fourier transform (DFT) can not be employed. Time-frequency analysis [1], among other methods, was proposed to deal with such signals. The short time Fourier transform (STFT) is one of the earliest methods used for time-frequency analysis. A moving window cuts out a slice of the signal, and the Fourier transform of this slice gives the local properties of the signal. The STFT can be interpreted as the output signals from a band pass filter bank. The spectrogram, which is the squared magnitude of the STFT, is used for the analysis of non-stationary signals. The result depends strongly on the choice of the window function and this necessitates a trade off between time localization and frequency resolution [1]. Another commonly used TFD is the Wigner-Ville distribution (WVD) [1, 2]. Theoretically the WVD has an infinite resolution in time due to the absence of averaging over a finite time interval. Also for infinite lag length, it has infinite frequency resolution. The WVD being quadratic in nature introduces cross terms, for a multi-component signal. The cross terms can have significant amplitudes and they can corrupt the transform spaces. A particular application where this effect might have serious implications in speech analysis, since speech can be modeled as a sum of amplitude modulated (AM) and frequency modulated (FM) signals corresponding to formant frequencies [3, 4]. In the last two decades, the research has been aimed at effective suppression of cross terms and improvement of the frequency resolution, preserving the desired properties of the TFD [1]. A denoising approach [5] based on shift invariant wavelet packet decomposition has been proposed for adaptive suppression of cross terms. Recently, a WVD based on signal decomposition approach, realized by a perfect reconstruction filter bank (PRFB), has been proposed [6]. The PRFB decomposes the multi-component signal into its components. The summation of their WVDs, results in a WVD whose cross terms and noise, are significantly reduced [6]. In this paper , a new technique based on FB series expansion has been proposed. This combines a FB series expansion and the WVD. The FB series decomposes the multi-component signal into mono-component signals like filter bank approach in [6]. The technique presented here is conceptually simpler and free from cross terms.

2. THE WIGNER-VILLE DISTRIBUTION

The WVD of a signal $x(t)$ is defined in the time domain as

$$W_x(t,\omega) = \int_{-\infty}^{\infty} x\,(t+\tau/2)\,x^*(t-\tau/2)e^{-j\omega\tau}d\tau \qquad (1)$$

Where $x^*(t)$ is the complex conjugate of $x(t)$. Similarly, in the frequency domain, the WVD is defined as follows:

$$W_x(t,\omega) = \int_{-\infty}^{\infty} X\,(\omega+\xi/2)\,X^*(\omega-\xi/2)e^{-j\xi t}d\xi \qquad (2)$$

where $X(\omega)$ is the Fourier transform of $x(t)$. The various interesting properties of the WVD [2] such as preservation of time and frequency support, instantaneous frequency, group delay, etc., make the WVD useful tool for signal analysis. The main drawback of this method is that it is bilinear in nature, introducing the cross terms in the WVD domain, which make the transform difficult to interpret [7]. the WVD of the sum of n signals $x(t) = \sum_{i=1}^{n} x_i(t)$ is given by

$$W_x(t,\omega) = \underbrace{\sum_{i=1}^{n} W_{x_i}(t,\omega)}_{autocomponents} + \underbrace{\sum_{k=1}^{n-1}\sum_{l=k+1}^{n} 2Re[W_{x_k x_l}(t,\omega)]}_{cross-component} \quad (3)$$

then the WVD of $x(t)$ has n autocomponents and $\binom{n}{2}$ cross-components, i.e., a cross term for every pair of autocomponents. The geometry of these cross terms has been well documented in [8]. if the signal $x(t)$ is a linear chirp, $x(t) = e^{j(\omega_0 t + \frac{1}{2}\beta t^2)}$ Let $x_1(t) = x(t-t_1)e^{j\omega_1 t}$ and $x_2(t) = x(t-t_2)e^{j\omega_2 t}$ be time and frequency shifted versions of a base signal $x(t) = \sum_{i=1}^{2} x_i(t)$ Therefore the WVD of $x(t)$ is

$$\begin{aligned} W_x(t,\omega) &= 2\pi\delta(\omega - (\omega_1+\omega_0) - \beta(t-t_1) \\ &\quad +2\pi\delta(\omega - (\omega_2+\omega_0) - \beta(t-t_2) \\ &\quad +4\pi\delta(\omega - (\omega_m+\omega_0) - \beta(t-t_m) \\ &\quad +2cos[\omega_d(t-t_m) - t_d(\omega-\omega_m) + \omega_d t_m] \end{aligned} \quad (4)$$

Here, $\delta(\omega)$ is the Dirac delta function that is zero everywhere except at the origin, $\omega_d = (\omega_2 - \omega_1), t_d = (t_2 - t_1), \omega_m = (\frac{(\omega_1+\omega_2)}{2}), t_m = (\frac{t_1+t_2}{2})$ In the above equation, the cross terms $a)$ occur mid time, mid frequency, $b)$ oscillate at a frequency proportional to the difference in frequency and time of the signals, $c)$ oscillate in the direction orthogonal to the line that connects the autocomponents, and $d)$ could have an amplitude twice as large as the product of the magnitudes of the WVD of the two signals under consideration.

3. FB SERIES EXPANSION

we wish to expand $x(t)$ over some arbitrary interval $(0, a)$ the zero order Bessel series expansion becomes [9]

$$x(t) = \sum_{m=1}^{P} C_m J_0(\frac{\lambda_m}{a}t),\ 0 < t < a \quad (5)$$

with the coefficients, C_m, calculated from

$$C_m = \frac{2\int_0^a t x(t) J_0(\frac{\lambda_m}{a}t)dt}{a^2 [J_1(\lambda_m)]^2} \quad (6)$$

and λ_m, $m = 1, 2, ..., P$, are the ascending order positive roots of $J_0(t) = 0$. The integral in the numerator of (6) is the finite Hankel transform. We note that the FB series coefficients C_m are unique for a given signal $x(t)$, similar to the fourier coefficients. Unlike the sinusoidal basis functions in the fourier series, the Bessel functions decay within the range a, similar to the rise and fall of speech within a pitch interval. Typically, using 15 to 25 ordered or selected coefficients has been shown to yield a reasonable speech quality in the reconstruction [10]. Since the spectrum of $s(t) = J_0(\frac{\lambda_m}{a}t)$ is given by

$$S(\omega) = \frac{1}{\sqrt{(\frac{\lambda_m}{a})^2 - \omega^2}}, for\ |\omega| < \frac{\lambda_m}{a} \quad (7)$$

Each term $C_m J_0(\frac{\lambda_m}{a}t)$ in the reconstruction in (5) has an approximate bandwidth $\omega_B \cong \frac{\lambda_m}{a}$; hence the reconstruction using the first N terms has a maximum bandwidth of $\omega_{max} \cong \frac{\lambda_N}{a}$. If signal $x(t)$ is the sum of n signals (i.e. multi-component), $x(t) = \sum_{i=1}^{n} x_i(t)$ Then, each component of the signal can be expanded into a FB series

$$x_i(t) = \sum_{m=1}^{P} C_{m_i} J_0(\frac{\lambda_m}{a}t),\ 0 < t < a \quad (8)$$

from (9) we have,

$$x(t) = \sum_{i=1}^{n}\sum_{m=1}^{P} C_{m_i} J_0(\frac{\lambda_m}{a}t),\ 0 < t < a \quad (9)$$

This equation can be written as,

$$x(t) = \sum_{m=1}^{P}(\sum_{i=1}^{n} C_{m_i}) J_0(\frac{\lambda_m}{a}t),\ 0 < t < a \quad (10)$$

Comparing (9) and (5), we have

$$C_m = \sum_{i=1}^{n}(C_{m_i}) \quad (11)$$

Therefore, the resulting sets of the coefficients are the sum of the coefficients of the component signals. The components can be reconstructed from the coefficients only.

4. TECHNIQUE BASED ON FB EXPANSION

The time-frequency analysis of the signal is carried out in the following way. First of all, the multi-component non-stationary signal under analysis is multiplied by a real valued window of appropriate size. In case of multi-component signal one has to choose the size of the window in such a way that we can separate the components. For a quickly changing signal if we take a large size of the window (i.e., more number of sample points) then the fast variations in the signal will be averaged out and we would not get a correct estimate of the frequency present in the local region. Whereas, if the window size is small in case of the slowly varying signal then again the estimate would not be accurate. The windowed signal is expanded in to a FB series. The FB coefficients are calculated from (6). Every component of multi-component signal has non-overlapping coefficients. Since coefficients are real; each component is directly reconstructed from coefficient versus time index plot. We apply WVD for each component to analyze its time-frequency distribution. and finally summation of these distributions gives the WVD of composite signal.

5. SIMULATION RESULTS

We now discuss a few representative examples that were chosen to study the performance of this proposed technique to WVD.

5.1. Gaussian modulated signal

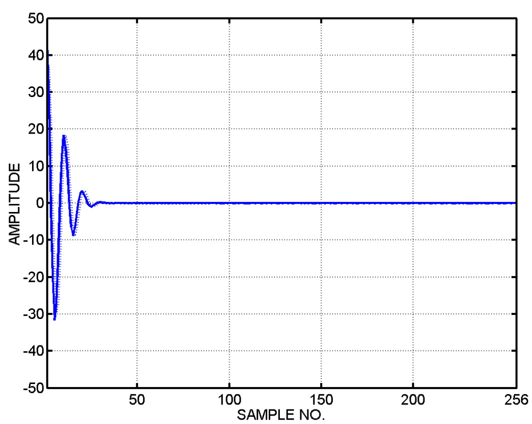

Fig. 1. Original signal (Dotted line), regenerated signal (Solid line), $\tau = 0.05$

For the test signal the Gaussian modulated signal $x[n] = \frac{1}{\tau} exp(-\frac{\pi n^2}{\tau^2}) sin(2\pi f_c n)$ has been considered. where τ is a variable parameter and f_c is the center frequency of the modulated signal. The signal can be regenerated with the fewer FB coefficients compare to the fourier coefficients. A set of 256 samples of the gaussian modulated signal with center frequency 1 kHz is processed. Fig. 1,2,3 and Fig. 4,5,6 shows the original and regenerated signals, power spectral density (PSD) plot of original and regenerated signals

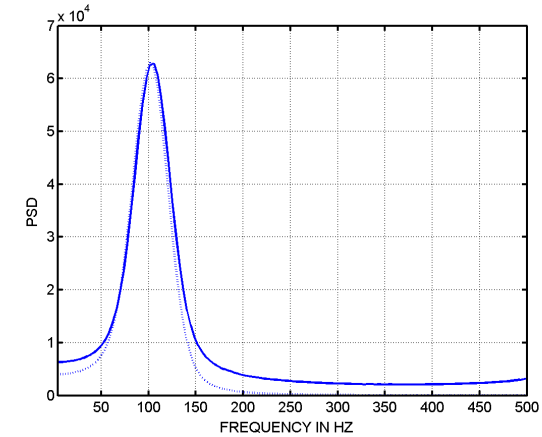

Fig. 2. PSD of: original signal (Dotted line),regenerated (Solid line),$\tau = 0.05$

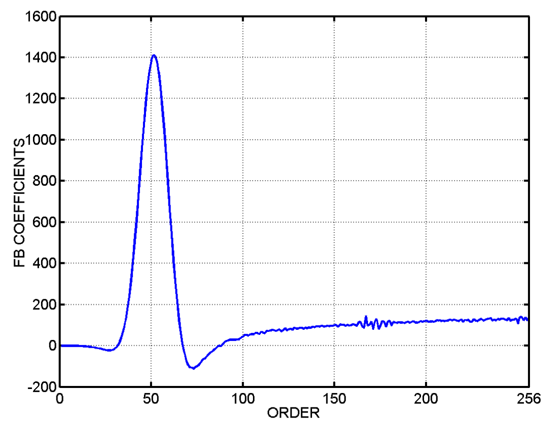

Fig. 3. FB coefficients, $\tau = 0.05$

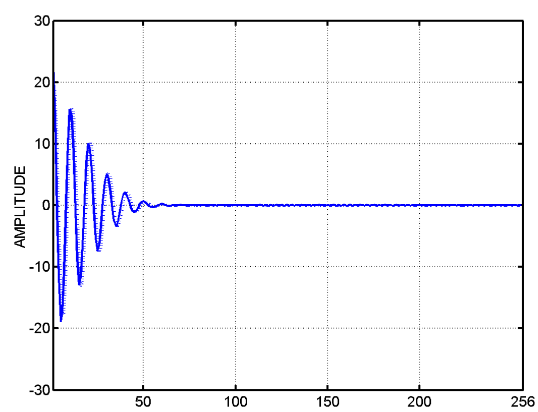

Fig. 4. Original signal (Dotted line), regenerated signal (Solid line), $\tau = 0.2$

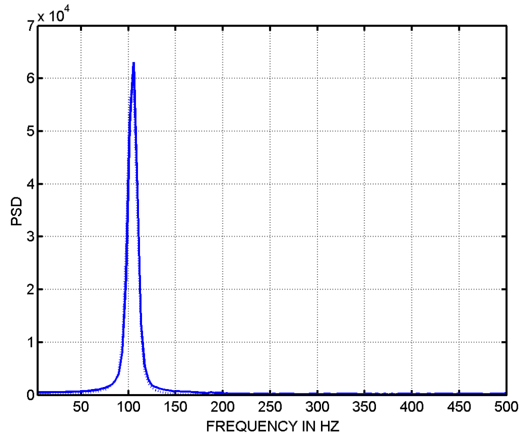

Fig. 5. PSD of: original signal (Dotted line),regenerated (Solid line),$\tau = 0.2$

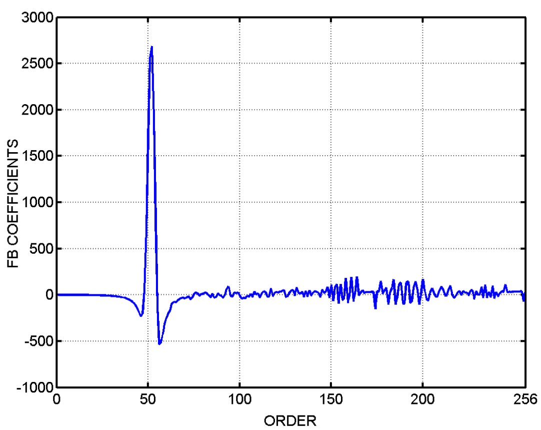

Fig. 6. FB coefficients, $\tau = 0.2$

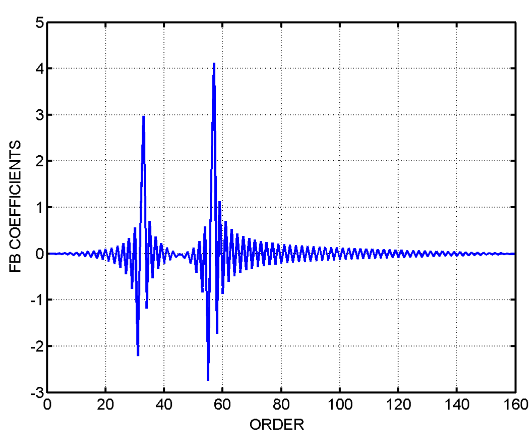

Fig. 7. FB coefficients of two component linear chirp signal

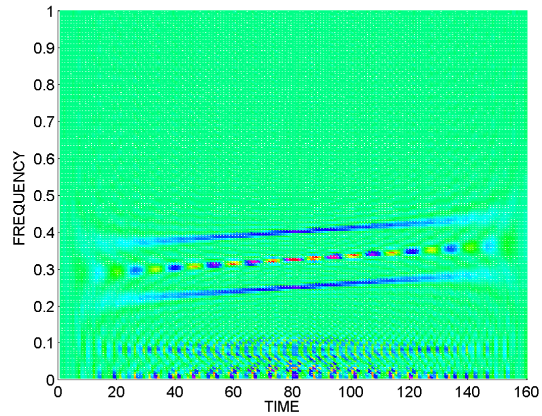

Fig. 8. WVD of two component linear chirp signal

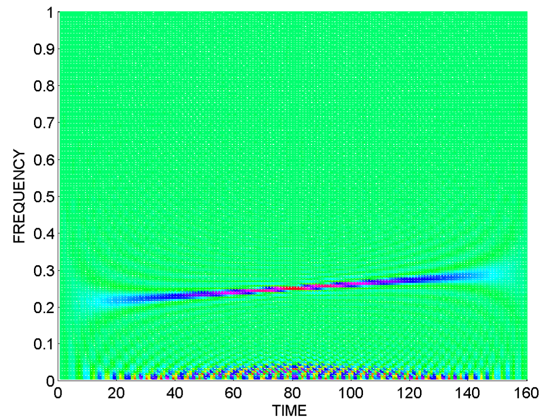

Fig. 9. WVD of first component after separation

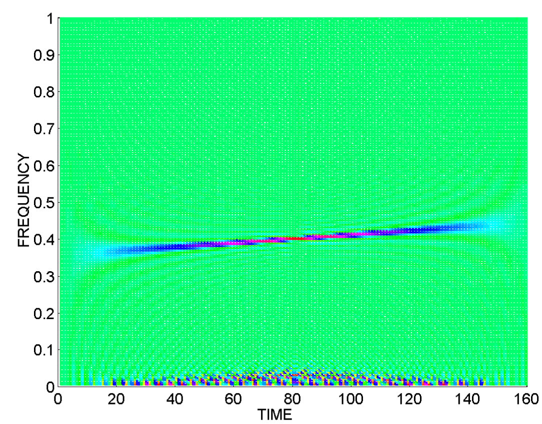

Fig. 10. WVD of second component after separation

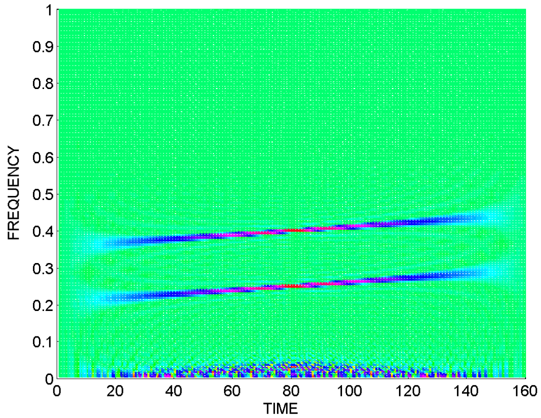

Fig. 11. WVD of two component linear chirp signal after separation

and plot of the FB coefficients at various value of τ. Table 1. shows the number of required FB coefficients in the reconstruction of the signal at two values of τ.

5.2. Two component linear chirp signal model

The discrete sequence $x[n]$ consisting of M single linear chirp signals is represented by, $x[n] = \sum_{i=1}^{M} A_i cos(\omega_{0_i} n + \frac{1}{2}\beta_i (n)^2)$ where A_i is the amplitude of the constituent linear chirp signal, β_i is the chirp rate, and ω_{0_i} is the fundamental angular frequency. For simulation, the signal consisting of $M = 2$ single tone linear chirp signals is sampled at N=160 points. Two set of parameters are chosen as follows: $A_1 = 1$, $\omega_{0_1} = 2\pi \cdot 20.5$, $\beta_1 = 70, A_2 = 1$, $\omega_{0_2} = 2\pi \cdot 35.5$, $\beta_2 = 70$ The WVD of two component linear chirp signal is shown in Fig. 8. It is clear that due to cross terms the amplitude of the corresponding chirp signal does not remain constant. This misleads to 3-component linear chirp signal. By using FB coefficients, both components can be separated out. WVD of two components are shown in Fig. 9 and Fig. 10 respectively. The plot of FB coefficients of two component linear chirp signal is shown in Fig. 7. and summation of these distributions gives the WVD of the composite signal with out cross terms, that is shown in Fig. 11.

S. No.	Value of τ	Required FB Coefficients
1	0.05	46-52 (7)
2	0.2	50-52 (3)

Table 1. Test signal, Gaussian modulated signal

6. CONCLUSION

In this paper, we have shown, a new method based on FB expansion for decomposing a multi-component signal into its mono-components. and cross terms in the Wigner distribution can be removed effectively. We have used this method in a variety of cases with success.

7. REFERENCES

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