ESTIMATION OF DIRECTIONS OF ARRIVAL OF WIDEBAND SOURCES.

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Abstract
Most of the existing methods of direction of arrival (DOA) estimation of wideband sources use spatial as well as temporal processing. These methods are not only computationally complex but also add a large delay, which make them difficult to implement in real life. In this paper, we propose a simple 1-D method of DOA estimation of wideband sources. For uniform linear array (ULA), autoregressive (AR) modeling of the lags of the covariance function is derived, which in turn provides the estimates of directions. Numeric simulations are carried out to study the performance of the suggested method.
KEY WORDS: Wideband sources, Direction of Arrival, Array Processing.

1. INTRODUCTION
The problem of estimating the direction of arrival (DOA) of wideband sources is of considerable interest to the signal processing community. It has applications in various fields such as radar, sonar, radio astronomy etc.
The major recent contributions to DOA estimation of broadband sources include the works of Wax et al. [1], Wang and Kaveh [2], Bienvenu [3], Buckley and Griffiths [4], Grenier [5], Morf and Su [6], Doron et al. [7], and Agrawal and Prasad [8], [9] etc.
These methods attempt to carry out incoherent or coherent aggregation of the parameters using narrowband processing on the one hand [1], [2] and complex spatio-temporal processing on the other [3], [4], [5], [6], [7], [8], [9]. Quite recently, Agrawal and Prasad [9], propose a spatial-only model for the array data, which leads to a practical algorithm for DOA estimation of wideband sources requiring 1-D search.
Like spread sources, the problem of DOA estimation of wideband sources can be seen as a 2-D spectrum estimation. For wideband sources spatial and temporal frequency, whereas, for spread sources, nominal direction and angular spread are the two dimension of interest. Recently Agrawal et.al. [10] have used an AR-model of cross covariance lags to estimate the nominal directions and angular spread of spread sources.
Motivated by these findings, in this paper, we propose a simple yet robust technique that enable high-resolution DOA estimation for wideband sources observed on a uniform linear array (ULA). The cross covariance lag function is modeled as autoregressive (AR) process. The roots of the polynomial corresponding to AR process gives the estimates of DOA. Simulation studies show that new algorithm performs better than [9]. Also unlike [9] the suggested method does not require any multi-dimensional search.

2. PROBLEM FORMULATION
All broadband sources are modeled by having an ideal bandpass power spectrum over a given bandwidth. The signal received by the $m$th sensor
of ULA of $M$ sensors, from the $P$ sources such that the $p$th source is located at $\theta_p$ relative to the broadside of the array, can be written as,

$$y_m(t) = \sum_{p=1}^{P} \int_{f_l}^{f_h} e^{j2\pi f(t+\tau_m(\theta_p))} dS_p(f) + n_m(t),$$

(1)

where $[f_l, f_h]$ is the frequency support and $dS_p(f)$ denotes the measure of the signal spectrum at frequency $f$. Here $\tau_m(\theta_p)$ denotes the propagation delay corresponding to the $m$th sensor for $p$th source with respect to a given reference sensor, for uniform linear array (ULA) $\tau_m(\theta_p) = \frac{m \Delta \sin(\theta_p)}{c}$. Where $\Delta$ is the inter sensor spacing and $c$ is the velocity of the signal wavefront. $n_m(t)$ is the additive white gaussian noise received at $t$th time instant by the $m$th sensor.

Assuming sources to be uncorrelated, the spatial covariance matrix is given by,

$$[R]_{m,n} = \sum_{p=1}^{P} \int_{f_l}^{f_h} \int_{f_l}^{f_h} e^{j2\pi(f (\tau_m(\theta_p) - f' \tau_m(\theta_p)))} E[dS_p(f) dS_p^*(f')].$$

(2)

Making use of the assumption of an ideal bandpass power spectrum for each source, we can write

$$E[dS_p(f) dS_p^*(f')] = \frac{\rho_p}{f_h - f_l} \delta(f - f') \quad \{f, f'\} \in [f_l, f_h].$$

(3)

where $\rho_p$ is the power of the $p$th source and each is uniformly distributed over $[f_l, f_h]$. Using the above formulation the total array covariance matrix is given as,

$$R = \sum_{p=1}^{P} R_p + \sigma_n^2 I$$

(4)

where $\sigma_n^2$ is noise power and $R_p$, the array covariance matrix corresponding to the $p$th source is given by,

$$[R_p]_{m,n} = e^{j\pi(f_h-f_l)\tau_m(\theta_p)} \sin(c\pi(f_h-f_l)\tau_m(\theta_p)).$$

(5)

Unlike MuSiC the array covariance matrix corresponding to broadband sources can not be decomposed into signal and noise subspace. But the information of DOA is still embedded in it. The problem of interest here is to devise a convenient method of finding the source DOA’s from the elements of $R$. In the next section, we propose such an approach.

3. PROPOSED SOLUTION

Using the definition of $\sin(\theta)$ (Euler’s equation), the contribution of each signal to the array covariance matrix $R$ given in (6), can be written as,

$$[R_p]_{m,n} = \frac{\exp(i\pi(f_h-f_l)\tau_m(\theta_p)) - \exp(-i\pi(f_h-f_l)\tau_m(\theta_p))}{2\pi(f_h-f_l)\tau_m(\theta_p)}$$

(6)

$$= \frac{\exp(i2\pi f_h \tau_m(\theta_p)) - \exp(i2\pi f_h \tau_{mn}(\theta_p))}{2\pi(f_h-f_l)\tau_m(\theta_p)}$$

(7)

Define a vector $r$

$$r = [R(2,1), R(3,1), \cdots, R(M,1)]^T$$

(8)

of the cross-correlations between the output of the first sensor and each of the remaining $M-1$ sensors. Therefore,

$$r(m) = \sum_{p=1}^{P} \sigma_p^2 \frac{\exp(i2\pi f_h \tau_{m+1}(\theta_p)) - \exp(i2\pi f_h \tau_{m+1}(\theta_p))}{2\pi(f_h-f_l)\tau_{m+1}(\theta_p)}.$$  

(9)

For ULA the above equation can be simplified as,

$$r(m) = \sum_{p=1}^{P} \sigma_p^2 \frac{\exp(i2\pi m f_h \Delta \sin(\theta_p)/c) - \exp(i2\pi m f_h \Delta \sin(\theta_p)/c)}{2\pi \sin(f_h-f_l) \Delta \sin(\theta_p)/c}.$$

(10)

Let $\tilde{r}$ be a vector of length $(M-1)$ defined as

$$\tilde{r}(m) = m \cdot r(m)$$

(11)

and note that $\tilde{r}$ is a sum of $2P$ modes, viz. $\{\exp(i2\pi m f_h \Delta \sin(\theta_p)/c) \exp(i2\pi m f_h \Delta \sin(\theta_p)/c)\}.$
Hence \( \{r(m)\} \) satisfies an AR equation of order \( 2P \) (see, e.g., [11])

\[
\sum_{n=0}^{2P} \beta_n r(m + n) = 0 \quad m = 1, \ldots, M - 2P - 1
\]

(12)

where \( \{\beta_n\} \) are the coefficients of the annihilating polynomial \( \beta(z) = \beta_0 + \beta_1 z^1 + \cdots + \beta_{2P} z^{2P} \), whose roots are \( \exp(i2\pi mf_h \Delta \sin(\theta_p)/c) \), \( \exp(i2\pi mf_l \Delta \sin(\theta_p)/c) \). The polynomial \( \beta(z) \) can be estimated by the forward-backward predictor method [12]

\[
\hat{\beta} = -(\Gamma^H \Gamma)^{-1} \Gamma^H \gamma.
\]

(13)

\[
\Gamma = \begin{bmatrix}
\hat{r}^*(2P) & \hat{r}^*(2P - 1) & \cdots & \hat{r}^*(1) \\
\hat{r}^*(2P + 1) & \hat{r}^*(2P) & \cdots & \hat{r}^*(2) \\
\vdots & \ddots & \ddots & \ddots \\
\hat{r}^*(M - 2) & \hat{r}^*(M - 3) & \cdots & \hat{r}^*(M - 2P - 1) \\
\hat{r}(2) & \hat{r}(3) & \cdots & \hat{r}(2P + 1) \\
\hat{r}(3) & \hat{r}(4) & \cdots & \hat{r}(2P + 2) \\
\vdots & \ddots & \ddots & \ddots \\
\hat{r}(M - 2P) & \hat{r}(M - 2P + 1) & \cdots & \hat{r}(M - 1)
\end{bmatrix}
\]

and

\[
\gamma = \begin{bmatrix}
\hat{r}^*(2P + 1), & \cdots & \hat{r}^*(M - 1), & \hat{r}(1) \\
\cdots & \cdots & \hat{r}(M - 2P - 1)
\end{bmatrix}^T.
\]

(14)

Here \((\cdot)^*\) denotes the complex conjugate. The above estimate requires that \( M \geq 3P + 1 \). Here we get the estimates of \( \{f_h \sin(\theta_p)\} \) and \( \{f_l \sin(\theta_p)\} \) for all sources and we know the values of \( f_1 \) and \( f_h \). Therefore we need to somehow combine these estimates to obtain the values \( \theta_p \).

Let, for \( P = 1 \), the two estimated modes be \( \hat{\alpha}_a \) and \( \hat{\alpha}_b \).

\[
\hat{\alpha} = \begin{bmatrix}
\hat{\alpha}_a & \hat{\alpha}_b
\end{bmatrix}^T
\]

(15)

\[
\hat{\theta}_1 = \max(\hat{\alpha}/\hat{f}_l)
\]

(16)

\[
\hat{\theta}_1 = \min(\hat{\alpha}/\hat{f}_h)
\]

(17)

Also

\[
\hat{\theta}_1 = \frac{\hat{\alpha}_a + \hat{\alpha}_b}{\hat{f}_h + \hat{f}_l}
\]

(18)

The estimates obtained using (16), (17) and (18) can be averaged to obtain a single estimate of the DOA of the corresponding wideband source. For more sources (i.e. \( P > 1 \)) the above method of combining sources can be generalized directly for well-separated sources. When the sources are close we use covariance matching technique

\[
\{\theta_p\} = \arg\{\min_\theta \|R - R(\theta)\|^2\}
\]

(19)

where the minimization is conducted over all possible combinations of the estimated modes [13].

4. SIMULATION STUDIES

In this section, we give results of the simulation experiments carried out to study and compare the behavior of the proposed DOA estimator for the broadband sources under various conditions of practical interest with the earlier method of Agrawal and Prasad [9]. The simulations reported here are around a uniform linear array of 8 sensors \((M = 8)\) with a sensor spacing of half wavelength corresponding to the center frequency of the band. The spectral support of the signal is taken to be the normalized frequency interval \([.75, 1.25] \).

The root mean square error (RMSE) has been used as the performance measure which is, defined as

\[
RMSE(\hat{\theta}) = \sqrt{\frac{1}{K} \sum_{k=1}^{K} (\theta - \hat{\theta}(k))^2},
\]

(20)

All performance curves given below are obtained by averaging the results over 100 independent trials \((K = 100)\).

Fig. 1 shows the RMSE for the case of a single source placed at \(30^\circ\) with respect to the broadside.
of the array as a function SNR. Where SNR is defined as,

\[
SNR = 10 \log_{10} \sum_{l=1}^{L} \frac{\rho_{pl}^2}{\sigma_n^2}
\]  

(21)

where \( \rho_{pl} \) is the spectral power corresponding to \( l \)th component and \( L \) is the number of spectral components used in the simulation\(^2\).

Fig. 2 shows the RMSE for the case of a single source placed at 30\(^0\) with respect to the broadside of the array as a function number of snapshots used to estimate the array covariance matrix. From both the figures it is clear that the suggested method performs better than [9].

Fig 3 shows the variation in RMSE of the source at 30\(^0\) as a function SNR where array observes two sources present at 30\(^0\) and 60\(^0\). The figure clearly shows that the suggested algorithm behaves quite nicely even in difficult conditions, i.e. the array is comprised of only 8 \((3P + 1 = 7)\) sensors, and each source has a spectral support of \([.75, 1.25]\). As the sources are well separated, therefore the estimates are obtained just by sorting the obtained estimates.

\[\text{Figure 1: RMSE of the estimates of the DOA as a function of SNR using the suggested algorithm (solid line) and using the algorithm of [9](dotted line) when one source is present at 30^0.}\]

\[\text{Figure 2: RMSE of the estimates of the DOA as a function of number of snapshots using the suggested algorithm (solid line) and using the algorithm of [9](dotted line) when one source is present at 30^0.}\]

\[\text{Figure 3: RMSE of the estimates of the DOA as a function SNR where array observes two sources present at 30^0 and 60^0.}\]

5. CONCLUSIONS

In this paper we have suggested an algorithm to obtain the estimates of the DOA of broadband sources. An AR model is obtained from the first column of the array covariance matrix. The roots of the AR polynomial so obtained is shown to give estimates of the direction. Numerical simulations show that the method is quite robust.

REFERENCES


\(^2\)Here \( \rho_{pl} \) is considered as uniformly distributed
Figure 3: RMSE of the estimates of the DOA as a function of SNR using the suggested algorithm when two sources are present at $30^\circ$ and $60^\circ$.


[10] Olivier Besson Monika Agrawal Petre Stoica and Per Ahgren. Estimation of nomi-