

# FREQUENCY DOMAIN VARIABLE STEP-SIZE LMS ALGORITHM FOR FAST FADING OFDM CHANNELS

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## ABSTRACT

Orthogonal FDM (OFDM) systems are very sensitive to fast fading channels. In this paper, we propose, various tracking techniques applicable for the Frequency Domain Equalisation (FDE) of a fast fading (mobile) OFDM system. We propose a Frequency Domain Variable Step-size (FDVS) technique with an independent Least Mean Squares (LMS) algorithms for each tone. This FDVS-LMS decision-directed tracker can significantly outperform the conventional FDE, for large burst lengths or for high fade rates. Further improvements in error rate performances are possible by incorporating a data-reuse mechanism and also by using a Least-Squares technique to remove noise in the frequency response estimates. Simulated bit error rate performance indicates that this family of FDVS-LMS algorithms are robust to even fade rates of 200 Hz when used for a signal model similar to IEEE 802.11a OFDM standards.

## I. INTRODUCTION

Orthogonal FDM (OFDM) is used in standards such as IEEE802.11a and 802.16a. While the wireless LAN 802.11a standard does not explicitly require channel tracking, the 802.16a wireless LAN standard would definitely have to combat fading, especially when narrow band OFDM is used. Frequent retraining for channel estimation is not desirable, and hence, decision directed tracking is needed.

For OFDM, we propose channel tracking in the frequency domain. A single tap LMS filter is run on each subcarrier. For a fixed step size LMS filter to perform well for each and every fade rate, an optimal step size has to be found by trial and error. This paper presents a modified LMS algorithm which works over a wide

range of fade rates to effectively track the time varying channel. The channel is initially estimated using the preamble data, and the channel estimates are tracked in a decision directed manner. Aside from these initial estimates, the algorithm does not need any knowledge about channel characteristics. This parallel bank of LMS trackers (one per tone) independently and adaptively change the gain constants  $\mu[n, k]$  based on error/lag measurements.

## II. THE SYSTEM MODEL

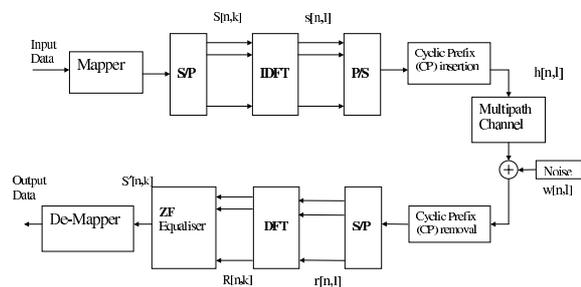


Fig. 1. Block diagram for OFDM system

The block diagram of a OFDM transceiver is shown in Figure 1. We consider an OFDM system with  $N$  subcarriers. The data symbols are denoted by  $S[n, k]$  where  $n \in \mathbb{Z}$  is the OFDM symbol index, and  $k$  corresponds to subcarrier,  $k \in 0, 1, \dots, N - 1$ .  $S[n, k]$  is drawn from a 16 QAM constellation. In 802.11a, four BPSK modulated pilots are inserted at specific subcarrier locations. These pilots are not used in our model. The  $n^{\text{th}}$  OFDM symbol  $s[n, m]$  is obtained by applying a IDFT to  $S[n, k]$  and then prepending a cyclic prefix of length  $L_{cp}$ , i.e.,

$$s[n, m] = \sum_{k=0}^{N-1} S[n, k] \exp \frac{j2\pi km}{N} \quad (1)$$

Each OFDM symbol has length  $N + L_{cp}$ . The time varying wireless channel has a equivalent base-band impulse response  $h[m, l]$  where  $l = 0, 1 \dots L - 1$  and the maximum delay spread of the channel is less than  $L_{cp}$ . The scattering function of the simulated channel has a Jakes Doppler profile. Each channel tap fades independently of the other taps. The received signal is given by

$$r[m] = \sum_{l=0}^{L-1} h[m, l]s[m - l] + w[m] \quad (2)$$

here  $w[m]$  is the additive white noise component at time  $m$ . At the receiver, the cyclic prefix is discarded and  $r[m]$  is demodulated using the DFT operation to yield

$$R[n, k] = \sum_{m=0}^{N-1} r[nP + m] \exp \frac{j2\pi km}{N} \quad (3)$$

where  $P = N + L_{cp}$ . If the variation of  $h[m, l]$  is small within one OFDM symbol period,  $R[n, k]$  is given by

$$R[n, k] = H[n, k]S[n, k] + W[n, k]; \quad (4)$$

where

$$H[n, k] = \sum_{l=0}^{L-1} h[n, l] \exp \frac{j2\pi kl}{N} \quad (5)$$

and

$$W[n, k] = \sum_{m=0}^{M-1} w[nP + m] \exp \frac{j2\pi km}{N} \quad (6)$$

Even though the channel actually varies within a OFDM symbol period, we have considered the case where the intercarrier interference(ICI) introduced has negligible effect on the bit error rate(BER). This allows us to use eqn(4)for decoding data. For higher fade rates around 1000 Hz ICI effects have to be incorporated into the measurement model.

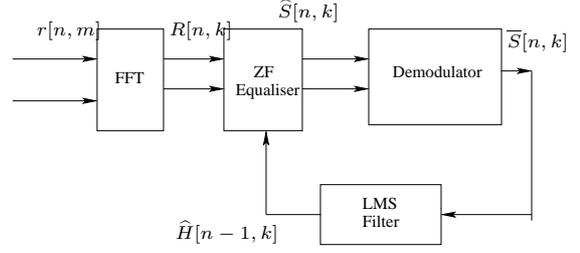


Fig. 2. Block diagram for DDT for OFDM

### III. DECISION DIRECTED TRACKING(DDT) FOR OFDM

The basic principle of DDT is shown in Figure 2.  $R[n, k]$  is equalised using a zero forcing(ZF) technique based on previous estimate of the channel namely,  $\hat{H}[n - 1, k]$ . The initial estimate  $\hat{H}[0, k]$  is obtained using the preamble data.

$$\hat{S}[n, k] = \frac{R[n, k]}{\hat{H}[n - 1, k]} \quad (7)$$

The equalised symbol  $\hat{S}[n, k]$  is then passed through a demodulator (to map the symbol to nearest constellation point), and this detected symbol, denoted by  $\bar{S}[n, k]$  is used as the desired signal for the LMS update equation.

For each subcarrier  $k$ , a single tap frequency domain LMS filter is used. The estimation error  $E[n, k]$  is given by

$$E[n, k] = (\bar{S}[n, k] - \hat{H}_{inv}[n - 1, k]R[n, k]) \quad (8)$$

where  $\hat{H}_{inv}[n - 1, k] = \frac{1}{\hat{H}[n-1, k]}$

The update equation is given by

$$\hat{H}_{inv}[n, k] = \hat{H}_{inv}[n-1, k] + \mu[n - 1, k]E[n, k]R^*[n, k] \quad (9)$$

The fixed gain LMS algorithm performs poorly in a rapidly varying channel when used over a wide range of fade rates. All subcarriers do not show the same variation in their amplitudes. Due to the time varying nature of the frequency response of the channel [4], the optimum step size for each subcarrier can differ.

The optimal step size  $\mu$  for time domain LMS filter was derived in [1]. It assumes a wide sense stationary uncorrelated scattering channel (WSSUS) model and develops a frequency domain Mean Square Error (MSE) analysis for channel tracking problems. It fur-

ther assumes statistical knowledge of the channel and determines  $\mu_{opt}$  as follows

$$\mu_{opt} = \left( \frac{2\omega_D^2}{LE_s^2\sigma_n^2} \right)^{\frac{1}{3}} \quad (10)$$

where  $L$  is actual channel length,  $E_s$  is energy of data symbol  $S[n, k]$ ,  $\sigma_n^2$  is the noise variance and  $\omega_D$  is the normalised maximum Doppler frequency defined by

$$\omega_D = \frac{2\pi v f_c}{f_s c} \quad (11)$$

where  $f_s$  is the symbol rate,  $v$  is the vehicle speed,  $c$  is the speed of light, and  $f_c$  is the carrier frequency.

It can be shown that the optimal step-size for the Frequency Domain(FD) LMS is similar to that given by (10) when a MSE analysis is done for the single tap FD LMS filter. This analysis [7] gives the same optimum step size for all the subcarriers. However here the optimality criterion is MSE, better performance is possible using a FDVS algorithm based on the instantaneous error  $E[n, k]$ . When the statistical knowledge of the channel is assumed to be available many types of optimal algorithms can be designed to track a fading channel [1]. What we propose is a FDVS-LMS algorithm which does not require any statistical knowledge of the channel.

At the start of the frame, the first OFDM data symbol is decoded using the channel estimate obtained using preamble data. These channel estimates,  $\hat{H}[n-1, k]$  are further improved by exploiting the fact that the channel length  $L$  cannot be greater than the cyclic prefix length  $L_{cp}$  [6]. Hence, these channel estimates are very close to the actual channel at the start of the first OFDM data symbol. As the first OFDM data symbol is decoded using these channel estimates, the error  $E[1, k]$  given by eqn(8) is quite small.  $E[1, k]$  is used to set a threshold with which we compare the instantaneous error  $E[n, k]$ . Whenever  $|E[n, k]| \geq \alpha |E[1, k]|$

$$\mu[n, k] = \mu[n, k] + \beta \quad (12)$$

where  $\alpha$  and  $\beta$  are constants selected by the user.

Whenever  $E[n, k]$  is large, it implies that the tracking algorithm is unable to track the time variations closely, and it slowly starts to lag behind the actual channel. As this lag increases, the corresponding error  $E[n, k]$  also increases. What is now needed is a

faster converging algorithm. Thus the the step size of the LMS filters is increased. Since MSE due to lag  $\epsilon_{lag}$  is inversely proportional to the step size [3], the MSE is reduced as the step size is increased. When we use LMS for tracking we have to consider two components of the excess MSE [1], [3], namely

$$\epsilon = \epsilon_{lag} + \epsilon_{self} \quad (13)$$

where  $\epsilon_{self}$  is the excess MSE due to self noise. while  $\epsilon_{lag} \propto \frac{1}{\mu}$ , we know that  $\epsilon_{self} \propto \mu$

This implies that though we are trying to improve our performance by increasing the step size  $\mu$ , if  $\mu$  becomes large,  $\epsilon_{self}$  increases. Therefore, to prevent  $\mu$  from becoming too large we apply the following "sign change" logic. Whenever  $E[n, k]$  changes sign, it indicates that the tracking algorithm's estimate,  $\hat{H}[n, k]$  is very near  $H[n, k]$ . In other words this implies that from lagging behind the channel, the estimate has over-shot the channel or vice versa. As the channel does not abruptly vary from symbol to symbol, a small value of  $\mu$  would suffice when the tracking algorithm is close enough to the channel.

Hence, whenever  $E[n, k]$  changes its sign and continues to change for at least two consecutive symbols (to take care of change in sign due to random noise)  $\mu[n, k]$  is reduced to a low value  $\gamma$ .

The Variable Step-size LMS algorithm (FDVS-LMS) shows a much better performance than the conventional LMS algorithm. All  $\mu[n, k]$ s are given an initial value of 0.001 and are upper bounded by the value 0.75.

To further improve the performance a variant of the data reusing (DR) LMS algorithm [5] can be used. The FDVS logic is used on the data reusing DR-LMS. The main feature of the DR-LMS family is the reuse of each received sample by the algorithm several times before, the new data is received. This is particularly appropriate in OFDM systems where we get new data only once in every OFDM symbol period, i.e once in every  $NT_s$  seconds. The basic update of DR-LMS algorithm can be written as

$$\hat{H}_{inv}^{i+1}[n-1, k] = \hat{H}_{inv}^i[n-1, k] + \mu[n, k] E^i[n, k] R^*[n, k] \quad (14)$$

where

$$E^i[n, k] = (\bar{S}[n, k] - \hat{H}_{inv}^i[n-1, k] R[n, k]) \quad (15)$$

$i = 1, 2, \dots, R$

and  $E^i[n,k]$  represents the output error on the  $i^{th}$  use of the same data, out of a total of  $R$  passes. Here data is reused  $R$  times. The time update recursion is given by

$$\hat{H}_{inv}^1[n-1, k] = \hat{H}_{inv}[n-1, k] \quad (16)$$

$$\hat{H}_{inv}[n, k] = \hat{H}_{inv}^{R+1}[n-1, k] \quad (17)$$

When  $R=1$ , DR-LMS reduces to the standard LMS update.

Using FDVS-LMS with data-reuse yields a significant improvement in performance over the FDVS scheme which does not reuse data. Further improvement in performance can be achieved by exploiting the fact that we are dealing with a channel whose length cannot be greater than the cyclic prefix length[6]. Using a Least Squares Noise Reduction Algorithm (NRA) [2], [6] periodically on the channel estimates obtained using FDVS LMS, further improvement in performance can be achieved. For brevity, we do not discuss this technique further.

#### IV. SIMULATION RESULTS

To analyse the performance of the described concepts simulations have been performed. In the simulations we consider a system similar to IEEE 802.11a operating with bandwidth of 20MHz, divided into 64 tones ( $N=64$ ). The cyclic prefix length  $L_{cp}$  is 16. Here, 11 of the 64 tones are used as the guard band. Sampling is performed with a 20 MHz rate. A OFDM symbol thus consists of 80 samples. 1000 channels are randomised per average SNR. Each channel consists of 16 taps, of which 15 have uniformly distributed delays over the interval 0 to  $0.8\mu s$ , while one tap is always assumed to have zero delay corresponding to a perfect time synchronisation of the sampling instants. The multipath intensity profile is assumed to be  $\phi(\tau) \approx e^{-\tau/\tau_{rms}} \tau_{rms}$  is  $\frac{1}{4}^{th}$  of the cyclic prefix. The modulation scheme selected is 16-QAM with a convolutional code rate of  $\frac{1}{2}$ . The scattering function of the simulated channel has a Jakes Doppler profile and the channel is WSSUS.

Normalised LMS algorithm has been used in the simulations to facilitate easier choice for upper bound for  $\mu$ . Figure 3 shows the BER curves for the different

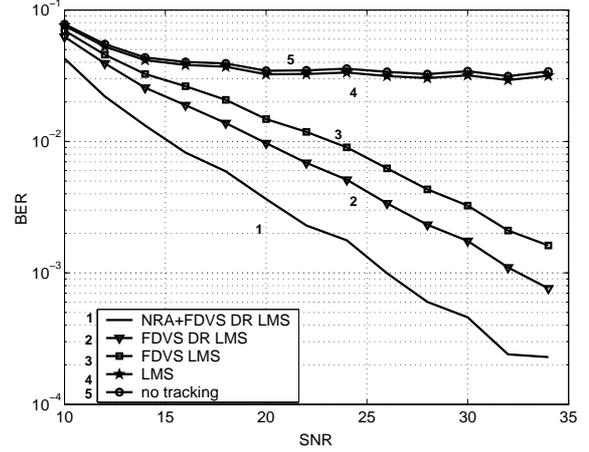


Fig. 3. BER for 16QAM with fade rate 200Hz 50 OFDM symbols

tracking algorithms with 200Hz fading. The FDVS-DR-LMS with a periodic NRA is seen to perform better than the other algorithms. The value of  $\alpha=4$  and  $\beta=.001$  are chosen for (12). Whenever the sign change algorithm detects a state where the tracking algorithm closely follows the actual channel,  $\mu[n, k]$  is reduced to a small value  $\gamma=0.005$ . The reuse factor  $R$  used for data reuse FDVS-LMS is 5.

The FDVS-LMS with data reuse of  $R=5$  and a periodic Least Squares based noise removal technique (with periodicity of once every 5 OFDM symbols) seems to give the best error rate performance. However it is computationally more expensive.

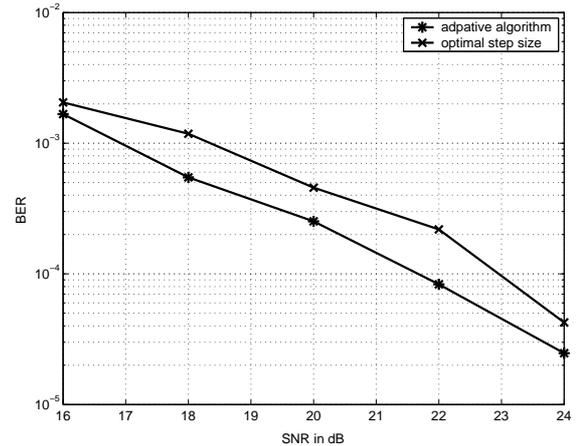


Fig. 4. Comparison of optimal step size algorithm with FDVS algorithm fade rate 400 Hz, QPSK modulation 100 OFDM symbols

Figure 4 shows a comparison between a LMS algorithm with optimal step size and a DR-FDVS LMS algorithm using NRA. Since the FDVS LMS algorithm uses the instantaneous error/lag measurements to vary the step size it outperforms the optimal step size algorithm.

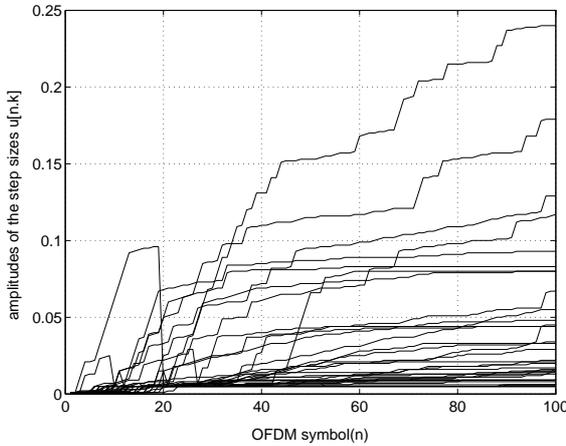


Fig. 5. variation of all the  $\mu[n, k]$ 's with time (SNR = 26 dB, fade rate 200Hz)

Figure 5 shows how the step size corresponding to each subcarrier  $\mu[n, k]$  varies with OFDM symbol number. Each curve in Fig. 4 corresponds to the time variation of the step size of a different subcarrier index  $k$ . It can be seen that the variation in  $\mu[n, k]$  is different for each  $k$ , i.e. each subcarrier is affected differently by the fading channel, and hence would require different step sizes. At some time instants, there is a steep increase in  $\mu[n, k]$ , because the variation in the time varying frequency response is high at that time instant. Some  $\mu[n, k]$ 's do not show much variation because the corresponding subcarrier gains have not varied much over time.

## V. CONCLUSION

A family of Frequency Domain Variable Step-Size (FDVS) LMS based decision-directed channel tracking techniques have been introduced in this paper. The relative performance of the different techniques were analysed using computer simulations. Unlike the time-domain LMS equaliser, the FD LMS algorithm independently adapts the gains  $\mu[n, k]$  at a per tone level. As we are making use of error thresholds to update the step size, it need not be updated every symbol. Instead, the step size is increased only when the channel

estimates show a significant lag compared to the actual channel. The step size is again decreased only when we are sure the channel estimates are quite close to the actual channel. This family of FDVS-LMS algorithms can also be extended to OFDM systems with transmit diversity. For high Normalised Doppler i.e .01 to .1 the FDVS-LMS algorithm can be used with space or frequency diversity schemes to achieve low BER .

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