

# Multiuser Adaptive MMSE Receiver using Multiple Antennas for Asynchronous W-CDMA Systems in Multipath Fading Channels

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## ABSTRACT

The performance of multiuser adaptive minimum mean square error (MMSE) receiver combined with multiple antennas is investigated for wideband code division multiple access systems (W-CDMA) in multipath fading channels. In the adaptive implementation of the receiver, performance of various algorithms like least mean square (LMS), normalized least mean square (NLMS), recursive least square (RLS) and the newly proposed affine projection adaptive (APA) filtering algorithm is considered. With the use of spatial diversity (by having multiple antennas at the receiver) significant improvement can be achieved in the output signal-to-interference-and-noise ratio (SINR). The effect of fading correlation between antenna elements (which is a function of antenna spacing, angle of arrival, angle spread, and carrier frequency) on the performance is evaluated using simulation.

Index terms: - CDMA systems, multiple access interference, multiuser detection, multipath fading, diversity.

## I. INTRODUCTION

The capacity of wireless CDMA systems is limited by the interference generated by the transmission from other mobiles, known as multiple access interference (MAI) and by multipath fading, which also causes intersymbol interference (ISI) [1]. Multiuser detection has been shown to be a very promising method to increase the capacity of the CDMA systems. As an alternative to optimal but exponentially complex maximum-likelihood sequence detector, sub-optimum linear multiuser detectors (like MMSE receivers) have been explored. These time-domain signal processing techniques can be improved upon by using multiple receive antennas and antenna array processing [1], [2], which enhances the output SINR and offers diversity to mitigate the impairments caused by fading [3]. The application of adaptive space-time multiuser detection is considered in [1], [2] among others. In [1], no attention is paid to the

effect of correlated fading and in [2], only synchronous CDMA system has been considered.

In this paper, we consider adaptive implementation of space-time multiuser detector based on MMSE criteria using APA filtering algorithm [4], whose convergence performance is found to be superior to NLMS algorithm, with much less computational complexity as compared to the RLS algorithm. The APA filtering algorithm has not been used elsewhere to the best of authors' knowledge for the adaptive implementation of MMSE receivers in multipath fading channels. We also show that by adding antennas at the receiver, SINR gain can be increased. We highlight the additional diversity gain obtained if either angular spread is increased or independent fading is assumed across antenna elements.

In section II, we describe signal model. Description of architecture of space-time multiuser receiver is given in section III including APA filtering algorithm. Simulation results are presented in section IV. Conclusions are drawn in section V.

## II. SIGNAL MODEL

A standard model for an asynchronous binary phase shift keyed (BPSK) direct-sequence CDMA (DS/CDMA) system is used in this paper [5]. In  $K$  user system, the  $k^{\text{th}}$  active users' baseband transmission can be written as

$$x_k(t) = \sum_i A_k b_k[i] s_k(t - iT - \nu_k) \quad (1)$$

where  $b_k[i]$  is the symbol transmitted during  $i^{\text{th}}$  interval  $(i-1)T \leq t \leq iT$ .  $T$  is the symbol duration and  $b_k[i] \in \{-1, +1\}$ .  $A_k$  and  $\nu_k$  represent the  $k^{\text{th}}$  user's amplitude and delay respectively.  $\nu_k = (d_k + \delta_k)T_c$ , where  $d_k$  is an integer between 0 and  $N-1$  and  $\delta_k$  lies between  $[0, 1)$ .  $s_k(t)$  is the signature sequence associated

with the  $k^{\text{th}}$  user. We consider short code CDMA where same signature sequence is employed for each symbol interval.

$$s_k(t) = \sum_{j=0}^{N-1} a_k(j) \psi(t - jT_c) \quad (2)$$

where  $T_c$  is the chip interval and the processing gain is  $N$ , where  $N = T/T_c$ .  $\psi(t)$  is the chip waveform and  $a_k(j) \in \left\{ \pm \frac{1}{\sqrt{N}} \right\}$  is normalized so that  $s_k(t)$  is having unit energy and duration  $T$ . Received signal corresponding to the  $k^{\text{th}}$  user at  $m^{\text{th}}$  antenna is given by

$$r_{km}(t) = \sum_{l=1}^{L_p} h_{k,l,m}(t) x_k(t - \tau_{k,l}) \quad (3)$$

where  $h_{k,l,m}(t)$  is the channel coefficient corresponding to the  $k^{\text{th}}$  user's  $l^{\text{th}}$  multipath for  $m^{\text{th}}$  antenna and  $\tau_{k,l}$  is the path delay associated with the  $k^{\text{th}}$  user's  $l^{\text{th}}$  multipath.  $L_p$  is total number of multipaths.

We have modified wide sense stationary uncorrelated scattering channel model [6], to include directional dependence to simulate the fading multipath channel. A highly frequency selective, tapped delay line channel model is considered, where multipaths arrive at  $T_c$  interval length following the arrival of the first path i.e.  $\tau_{k,l} = \tau_{k,1} + (l - 1) T_c$ , where  $l = 2, 3, \dots, L_p$ . This situation arises where the delay profile is more or less continuous. Correlated fading channel coefficients between receiving antenna elements are given as

$$h_{k,l,m} = h_{k,1,m} \exp\left(-j2\pi \frac{d}{\lambda} (m-1) \sin \theta_{k,l}\right) \quad (4)$$

where  $m = 1, 2, \dots, M$ .  $h_{k,1}$  is the channel coefficient at the first antenna.  $\theta_{k,l}$  is azimuthal angle of arrival of  $k^{\text{th}}$  user's  $l^{\text{th}}$  path and  $d$  is antenna spacing. For independent fading, all channel coefficients corresponding to different antennas are generated independently, assuming that antenna elements are kept sufficiently apart. The receiver is assumed to be synchronized with the main path (which is taken to be the first path) of the  $k^{\text{th}}$  desired user i.e.  $\tau_{k,1} + v_k = 0$  and the received signal is passed through a chip matched filter and sampled at the chip rate. We next define matrices  $\mathbf{P}_k^+$  and  $\mathbf{P}_k^-$

$$\mathbf{P}_k^\pm = \left[ \mathbf{p}_{k,1}^\pm, \mathbf{p}_{k,2}^\pm, \dots, \mathbf{p}_{k,L_p}^\pm \right] \quad (5)$$

where  $\mathbf{p}_{k,l}^+$  and  $\mathbf{p}_{k,l}^-$ ,  $1 \leq k \leq K$  and  $1 \leq l \leq L_p$ , are the vectors of chip matched filter sampled outputs during  $i^{\text{th}}$  symbol duration corresponding to the inputs  $s_k(t - iT - v_k - \tau_{k,l})$  and  $s_k(t - (i-1)T - v_k - \tau_{k,l})$  respectively [5]. If we run one chip matched filter per user to track the first ray of each user then received signal vector at the  $m^{\text{th}}$  antenna corresponding to  $j^{\text{th}}$  users' (chip matched filter) would be (during  $i^{\text{th}}$  symbol duration),

$$\mathbf{r}_{m,j}[i] = b_j[i] A_j \mathbf{P}_j^+ \mathbf{h}_{j,m}[i] + \sum_{\substack{k=1 \\ k \neq j}}^K [ b_k[i] A_k \mathbf{P}_k^+ \mathbf{h}_{k,m}[i] + b_k[i-1] A_k \mathbf{P}_k^+ \mathbf{h}_{k,m}[i-1] ] + \mathbf{n}_m[i] \quad (6)$$

$$\text{where } \mathbf{h}_{k,m}[i] = [h_{k,1,m}, h_{k,2,m}, \dots, h_{k,L_p,m}]^T \quad (7)$$

superscript  $T$  denotes transpose.  $\mathbf{n}_m[i]$  is the  $N \times 1$  vector of zero-mean complex (circularly symmetric) Gaussian noise samples, having variance  $\sigma^2$  at  $m^{\text{th}}$  antenna in the  $i^{\text{th}}$  interval and i.i.d. across antenna elements. In all, we collect  $KN$  samples per antenna and for  $M$  antennas we aggregate  $KNM$  samples. Let  $\mathbf{r}_m[i]$  be a  $KN$  length vector at the  $m^{\text{th}}$  antenna

$$(\mathbf{r}_m[i] = [\mathbf{r}_{m,1}^T[i], \mathbf{r}_{m,2}^T[i], \dots, \mathbf{r}_{m,K}^T[i]]^T) \text{ and}$$

$\mathbf{r}[i]$  be the overall data vector of length  $KNM$  obtained by stacking the  $M$  antenna outputs during  $i^{\text{th}}$  interval

$$\mathbf{r}[i] = [\mathbf{r}_1^T[i], \mathbf{r}_2^T[i], \dots, \mathbf{r}_M^T[i]]^T \quad (8)$$

### III. ADAPTIVE SPACE-TIME MMSE MULTIUSER DETECTION

#### A. Architecture :

Space-time multiuser detection presented here is suitable for centralized (at the base station) processing. Receiver architecture for 2 users and single antenna case is shown in fig1. Received signals from other antennas are processed using similar structure and then added to the last summer for each user as shown. Sampled output at interval  $T$  denotes soft estimates  $\hat{b}$ . Let  $w_{j,q,m}[n]$  be the  $n^{\text{th}}$  tap (filter coefficient) of  $j^{\text{th}}$  user's chip matched filter output going to  $q^{\text{th}}$  users summer in the  $m^{\text{th}}$

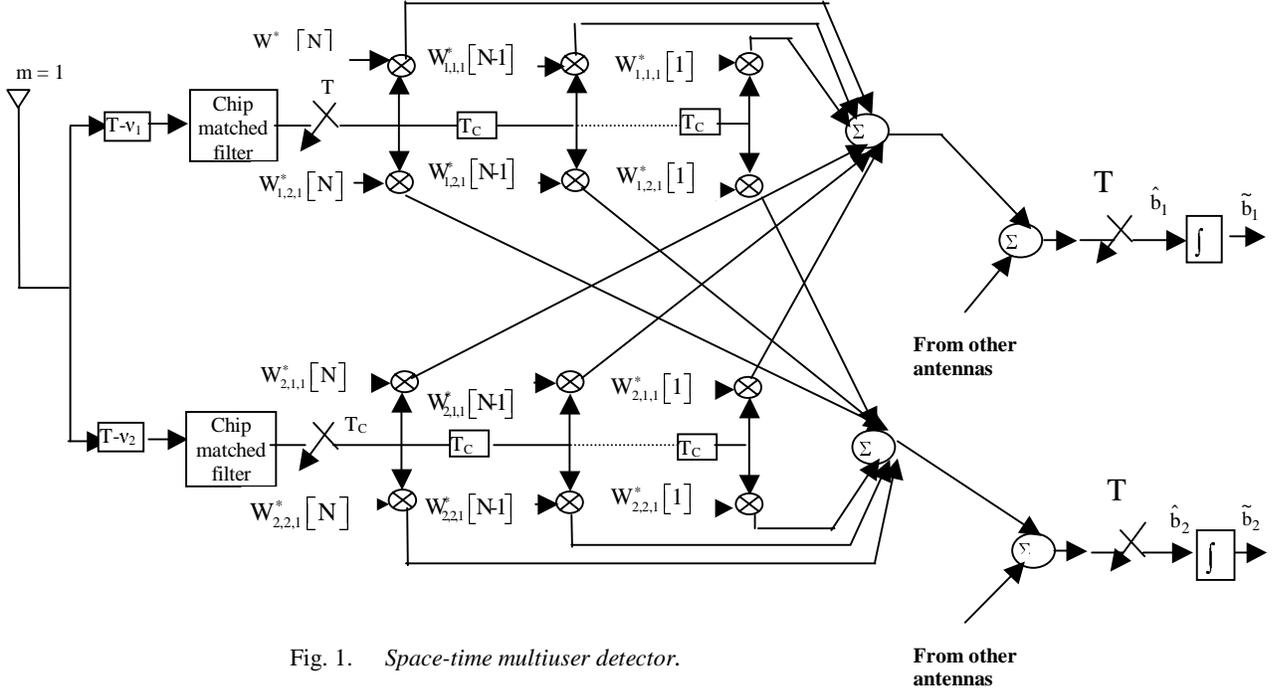


Fig. 1. Space-time multiuser detector.

antenna, where  $1 \leq j, q \leq K$ ,  $1 \leq m \leq M$ , and  $1 \leq n \leq N$ . All  $w$ 's are adapted according to MMSE criteria. The output SINR of an MMSE detector depends on its ability to capture the received energy of the desired user while suppressing MAI and ISI. Choosing  $KNM$  samples, which are shared by all the users, based on MMSE criteria, the detector achieves the optimum tradeoff between RAKE matched filtering and interference mitigation and maximizes the output SINR [1]. For obtaining synchronism between all detected data symbols of all users, we can delay the signal at the input to the  $k^{\text{th}}$  user's chip matched filter by  $cT - v_k$ , where  $c$  is the smallest integer that results in positive delay.

Now we define  $KNM$  length filter coefficients for the  $k^{\text{th}}$  user in  $i^{\text{th}}$  symbol duration as

$$\mathbf{w}_k[i] = [\mathbf{w}_{k,1}^T[i], \mathbf{w}_{k,2}^T[i], \dots, \mathbf{w}_{k,M}^T[i]]^T \quad (9)$$

Where  $KN$  length vector,

$$\mathbf{w}_{k,m}[i] = [\mathbf{w}_{1,k,m}^T[i], \mathbf{w}_{2,k,m}^T[i], \dots, \mathbf{w}_{K,k,m}^T[i]]^T$$

and  $N$  length vector,

$$\mathbf{w}_{j,q,m}[i] = [w_{j,q,m}[1], w_{j,q,m}[2], \dots, w_{j,q,m}[N]]^T$$

We choose  $\mathbf{w}_k[i]$  to minimize the mean-square-error (MSE) between the  $i^{\text{th}}$  data symbol and its soft estimate

$$\begin{aligned} \text{MSE}_k &= E \left\{ \left| b_k[i] - \hat{b}_k[i] \right|^2 \right\} \\ &= 1 + \mathbf{w}_k^H \boldsymbol{\Gamma} \mathbf{w}_k - \mathbf{w}_k^H \mathbf{v}_k - \mathbf{v}_k^H \mathbf{w}_k \end{aligned} \quad (10)$$

where we have defined auto-correlation matrix of data vector  $\boldsymbol{\Gamma}$  and cross correlation vector  $\mathbf{v}_k$  as  $\boldsymbol{\Gamma} = E \{ \mathbf{r}[i] \mathbf{r}^H[i] \}$ , and  $\mathbf{v}_k = E \{ \mathbf{r}[i] b_k^*[i] \}$  (11)  $E, H,$  and  $*$  are expectation, Hermitian transpose and complex conjugate operations respectively. Minimum MSE is achieved by solving Wiener-Hopf equation [4]. i.e.

$$\mathbf{w}_k = \boldsymbol{\Gamma}^{-1} \mathbf{v}_k \quad (12)$$

Grouping the coefficient vector for the  $K$  users, we have the MMSE multiuser coefficient matrix

$$\mathbf{W} = [\mathbf{w}_1, \mathbf{w}_2, \dots, \mathbf{w}_K] = \boldsymbol{\Gamma}^{-1} \mathbf{V} \quad (13)$$

where multiuser cross-correlation matrix  $\mathbf{V}$  of dimension  $KNM \times K$  is given as

$$\mathbf{V} = [\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_K]$$

SINR can be expressed in terms of MSE as

$$\text{SINR} = \frac{\sigma_b^2 - \text{MSE}}{\text{MSE}} \quad (14)$$

where  $\sigma_b^2$  is the variance of input symbols.

#### B. Multiuser APA algorithm:

In the adaptive implementation, filter coefficient can be computed iteratively by transmitting training symbols. APA filtering can be viewed as multiple constraints optimization criterion with number of constraints,  $L$ , is called as the order of the APA filter. We may view the APA filter as

TABLE I. COMPUTATIONAL COMPLEXITY FOR VARIOUS ADAPTIVE ALGORITHMS

Algorithm	Complex Addition / Subtraction	Complex Multiplication / Division
LMS	$2N_T$	$2N_T + 1$
NLMS	$3N_T$	$3N_T + 1$
Regularized APA Algorithm	$N_T(L^2 + 2L + 1) + L^2$	$N_T(L^2 + 2L) + L^2 + 2L + 0(L^2)$
RLS	$3N_T^2 + 3N_T + 1$	$3N_T^2 + 4N_T$

an intermediate adaptive filter between the NLMS filter and RLS filter in terms of both computational complexity and performance. If  $L=1$  then APA filter reduces to NLMS filter and if  $L=KNM$ , APA algorithm reduces to RLS algorithm. In APA filter, cost function for update weight vector for  $k^{\text{th}}$  user is given as [4],

$$\mathbb{C}_k[i] = \|\mathbf{w}_k[i+1] - \mathbf{w}_k[i]\|^2 + \sum_{j=0}^{L-1} \text{Re} \left[ \lambda_j^* (b_k[i-j] - \mathbf{w}_k^H[i+1] \mathbf{r}[i-j]) \right] \quad (15)$$

$\lambda_j$  are the Lagrange multipliers pertaining to multiple constraints. Solving the above problem, we get the following APA algorithm [4],

$$\begin{aligned} \mathbf{A}^H[i] &= [\mathbf{r}[i], \mathbf{r}[i-1], \dots, \mathbf{r}[i-L+1]] \\ \mathbf{B}[i] &= [\mathbf{b}[i], \mathbf{b}[i-1], \dots, \mathbf{b}[i-L+1]]^T \\ \mathbf{E}[i] &= \mathbf{B}[i] - \mathbf{A}[i] \mathbf{W}[i] \\ \mathbf{W}[i+1] &= \mathbf{W}[i] + \mu \mathbf{A}^H[i] (\mathbf{A}[i] \mathbf{A}^H[i] + \delta \mathbf{I})^{-1} \mathbf{E}[i] \end{aligned} \quad (16)$$

where  $\hat{\mathbf{b}}[i] = \mathbf{W}^H[i] \mathbf{r}[i]$ ,  $\tilde{\mathbf{b}}[i] = \text{sgn}(\text{real}(\hat{\mathbf{b}}[i]))$ , during training  $\tilde{\mathbf{b}}[i] = \mathbf{b}[i]$ , where  $\mathbf{b}[i] = [b_1[i], b_2[i], \dots, b_k[i]]^T$ .  $\mu$  is step size which controls the adaptation rate.  $\delta$  is small positive constant used for regularization and  $\mathbf{I}$  is identity matrix of order  $L$ . Computational complexity involved for various algorithms is given in table I, where  $N_T = KNM$ .

#### IV SIMULATION RESULTS

In our simulation, we used Gold sequences of length 31 chips (i.e.  $N=31$ ). Data rate considered is 132 Kbps (to keep simulation time small) which corresponds to chip rate of 4.096Mchips/sec. Multipath spread is taken to be 2.5  $\mu\text{sec.}$ , causing a multipath span of  $10T_c$ . We consider 4 user ( $K=4$ ) system with interferers at 10 dB power advantage as compared to the desired user to illustrate the advantage of MMSE

receiver in near-far situation. Input SNR of the desired user is taken to be 18 dB. In simulation of asynchronous CDMA system, we consider that time delay of initial path of the users' are uniformly distributed between  $[0, T)$ . Exponential power delay profile is assumed and channel is considered static in each independent trial. Mean angle of arrival (AOA) is uniformly distributed between  $[0, 2\pi)$  for each user. Around this mean AOA, Laplacian distributed angular spread [7] is taken as 5 degrees, except when additional diversity gain obtained by increasing angular spread is considered. Order four ( $L=4$ ) APA filter is used. Simulation runs are carried over a large number of independent trials and their mean is plotted. In fig.2 we demonstrate the superiority of APA filtering algorithm over NLMS and simple LMS for single antenna case. For a fair comparison in convergence characteristics we kept the residual MSE same for all the three cases. As seen from the figure, APA filtering algorithm clearly outperforms NLMS algorithm converging in about 180 iterations as against 350-400 iterations taken by NLMS algorithm. RLS algorithm converges in about 60-70 iterations but the computational complexity involved is much higher as seen from the table I. Fig.3 highlights the diversity gain obtained by using multiple antennas at the receiver end. We see from the figure that by doubling the antenna elements we get roughly 3 dB advantage in MSE (or correspondingly in SINR as per eq. (14). Results obtained are in agreement with [3].

In fig.4 we show that by having uncorrelated fading at the antenna elements additional diversity gain of roughly 2 dB can be achieved for four antennas case. In correlated fading with antenna spacing of half wavelength and angular spread of 5 degrees, correlation in excess of 0.98 is observed between adjacent antenna elements and around 0.8 between first and fourth antenna. Through simulation, we have also obtained improvement of 0.5 dB if angular spread is increased from 5 degrees to 15 degrees.

## V CONCLUSIONS

In this paper performance of adaptive MMSE multiuser detector, combined with multiple antennas for the detection of asynchronous WCDMA signals in multipath

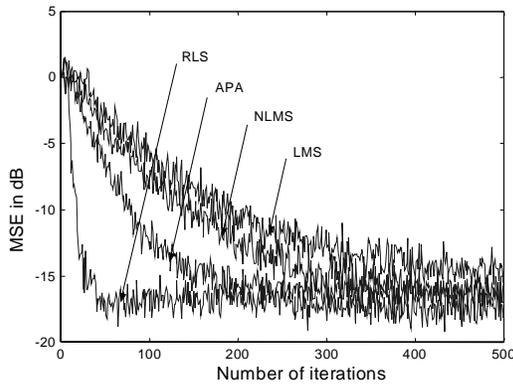


Figure 2. Convergence characteristics of APA, RLS, NLMS, and LMS filtering algorithms

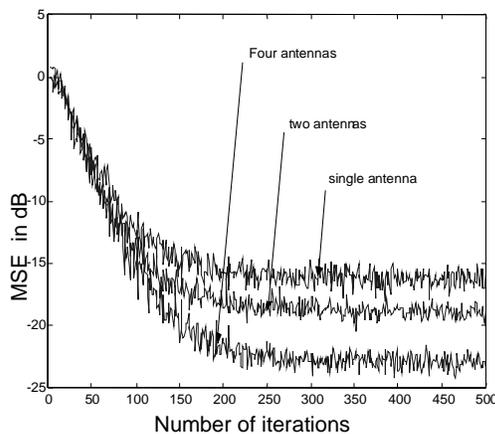


Figure 3. Diversity gain obtained by multiple antennas in correlated fading

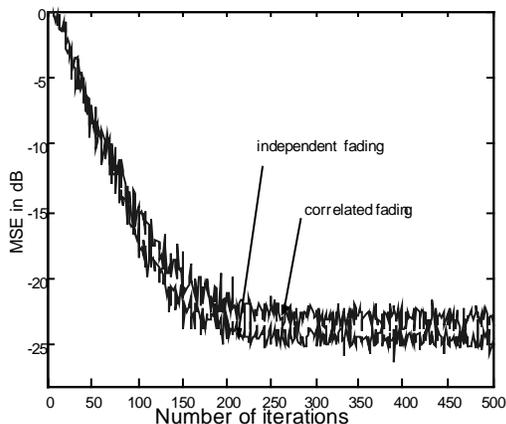


Figure 4. Additional diversity gain obtained with independent fading at the antenna elements (four antenna case)

Rayleigh fading channel is obtained in terms of residual MSE. With the use of APA filtering algorithm, significant improvement can be obtained over NLMS adaptive algorithm in convergence characteristics. We have also demonstrated that using multiple antennas at the receiver, enhances the performance of the temporal processing (multiuser detection) even if the correlation between antenna elements is high. Roughly 3 dB advantage is obtained by doubling the antenna elements. We have also highlighted that additional diversity gain can be achieved if the antennas are placed widely apart, so that independent fading occurs at each antenna element.

## REFERENCES

- [1] John E. Smee, and Stuart C. Schwartz, "Adaptive space-time feed-forward/feed-back detection for high data rate CDMA in frequency-selective fading," *IEEE Trans. Commun.*, vol. 49, no. 2, pp. 317-328, Feb. 2001.
- [2] Constantinos B. Papadias, and Howard Huang, "Linear space-time multiuser detection for multipath CDMA channels," *IEEE J. Select. Areas Commun.*, vol. 19, no. 2, pp. 254-265, Feb. 2001.
- [3] G. J. Foschini and M. J. Gans, "On limits of wireless communications in a fading environment when using multiple antennas," *Wireless Personal Commun.*, vol. 6, no. 3, pp 311-335, March 1998.
- [4] Simon Haykin, *Adaptive Filter Theory* 4<sup>th</sup> ed., Pearson Education Inc., 2002.
- [5] Michael L. Honing, Scott L. Miller, Mark J. Shensa, and Lawrence B. Milstein, "Performance of adaptive linear interference suppression in the presence of dynamic fading," *IEEE Trans. Commun.* vol. 49, no. 4, pp. 635-644, April 2001.
- [6] Kun-Wah Yip, and Tung-Sang Ng, "Efficient simulation of digital transmission over WSSUS channels," *IEEE Trans. Commun.*, vol. 43, no.12, pp. 2907-2912, Dec. 1995.
- [7] Richard B. Ertel, Paulo Cardieri, Kevin W. Sowerby, Theodore S. Rappaport, and Jeffrey H. Reed, "Overview of Spatial Channel Models for Antenna Array Communication Systems," *IEEE Personal Comm. Mag.*, vol. 5, no. 1, pp 10-22, Feb.1998.